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Game option models of convertible bonds: Determinants of call policies

Yue Kuen Kwok

Department of Mathematics
Hong Kong University of Science and Technology
Clear Water Bay, Hong Kong, China

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Abstract

The interaction of bondholder's conversion and issuer's call in a convertible bond leads to a game option model between the two counterparties. Like typical pricing models for corporate debts, the fair value of a convertible bond is highly dependent on issuer's credit risk, tax benefits of coupons and other structural features. The convertible bond pricing models in the literature can be categorized into two approaches: (i) structural firm value models that incorporate dilution effect in the issuer firm's corporate structure upon conversion; and (ii) reduced form models that price convertible bonds based on calibration with market liquid instruments. We review and comment on various pricing formulations of convertible bonds and effectiveness of different numerical schemes for solving the associated optimal stopping problems. Empirical studies on issuers' optimal call policies have revealed discrepancies between the optimal decision rule derived from pricing models and actual market practices. The more refined model formulation of a convertible bond should include corporate finance considerations in the determination of the optimal call policies.

Keywords: Convertible bonds; game option models; optimal stopping problems; optimal call policies.

1. Introduction

Convertible bonds are hybrid securities with both debt and equity like features. Like a debt security, the investor of a convertible bond is entitled to receive periodic coupon payments and the principal repayment at bond's maturity, provided that the issuer does not default, nor the bond is terminated prematurely due

Email address: maykwok@ust.hk

to early conversion or call. On the other hand, the convertible bond exhibits the equity participation feature since the bondholder is allowed to forgo the fixed income component and convert the bond into a preset number of shares of the firm stock at any time during the life of the bond. Upon bondholder's conversion into shares, the screw clause stipulates that the accrued interest between the last coupon date and the conversion date is not paid to the bondholder. The embedded conversion right in a convertible resembles a call option on the underlying issuer's stock with strike price equals the bond component of the convertible. When the stock price is non-performing, the convertible bond almost resembles a straight debt with default risk. For most convertibles, the coupon rate offered by a convertible is lower than that of its straight bond counterpart. The difference in the two coupon rates reflects the conversion premium paid by the bondholder.

1.1. Call feature: Hard and soft call protection

Most convertible bonds contain the call feature, where the issuer can call back the bond at the preset call price (sum of clean call price plus accrued interest). As a common practice, the bondholder's conversion right precedes the issuer's call right. Upon call, the bondholder can choose either to receive the call price or convert into shares (known as forced conversion). The embedded call feature allows firms to use convertibles to get equity in their capital structure by forced conversion. The structuring of the call provisions plays an important role in facilitating optimal convertible bond design and capital investment (Korkeamaki and Moore, 2004).

Usually, the notice period requirement is included in the call provision, where the bondholders make their decision on converting into shares or receiving par at the end of the notice period upon issuance of issuer's call announcement. Besides, in order to protect the conversion premium paid by the bondholder to be called away too soon, the hard call protection feature precludes issuer's call within the call protection period in the early life of the bond. Also, most convertible bonds contain the soft call requirement that places restriction on calling after the early hard call protection period. The bond can be called only if the stock price stays above the hurdle price for certain period of time before the date of calling (say, 20 trading days out of 30 trading days). Also, some convertible bonds may include the "put" option where the bondholder can put the bond to the issuer on specified dates within the bond's life for some preset cash amount (face value plus accrued interest). The book by De Spiegeleer and Schoutens (2011) contains a comprehensive review of product descriptions of various types of convertible bonds.

1.2. Why firms choose convertible bonds for capital financing

There are several good reasons that explain why firms may choose to raise capital via issuance of convertible bonds instead of straight debts or equity. More recent comprehensive studies on the issuance motives and design of convertible bonds, and shareholder wealth effects can be found in Dorion *et al.* (2014) and Dutordoir *et al.* (2014). These incentives and motives include lower coupon rate, less adverse price impact upon issuance, resolution of asset substitution and delayed equity financing. Firms with stronger growth potential can take advantage of a higher conversion premium charged at initiation, which is translated into more significant lowering of the coupon rate in the convertible. Also, issuing equity shares may lead to strong negative announcement effects and more significant adverse price impact on the firm's shares. Due to symmetric information on the firm's quality, investors tend to undervalue a good firm's stock. Convertibles issuance would be greeted with less adverse stock price impact since firms with good growth opportunity would choose convertibles over equity financing (Lewis, 1998; Chakraborty and Yilmaz, 2011). Furthermore, with regard to the issue of "asset substitution" in debt financing, once considerable amount of bonds have been issued, the firm has an incentive to adopt more risky projects at the expense of bondholders' benefit. Therefore, debt financing is not desirable when the asset substitution problem is serious. However, since investors on convertibles share equity growth of the firm, so asset substitution problem is somewhat alleviated (Green, 1984; Lyandres and Zhdanov, 2014). In convertible debt financing, the bondholders can convert their bonds into equity if desired. The issuance of convertibles provides an alternative way for the firm to realize delayed (backdoor) equity financing without significant adverse price impact (Stein, 1992). The option of delayed equity financing allows the firm to have the flexibility to adjust the debt–equity leverage ratio. Also, we observe that an excessive debt level may lead to higher default risk and potential cost of financial distress. With potential costly default losses, a firm that has a high debt–equity leverage ratio will choose convertible financing when the firm is optimistic about its share price performance. Companies may find convertibles an indirect mechanism for implementing equity financing that mitigates the adverse selection costs associated with issuance of equity. Through the call provisions on convertibles, companies may get equity into their capital structures at later times (delayed equity financing). Moreover, a convertible bond's conversion option reduces issue costs while controls the over-investment incentive. Mayers (1998) shows through some detailed empirical studies that convertibles can be a cost-saving financing tool to carry out sequential financing, especially by corporations with large growth opportunities. He argues that a well-structured convertible coupled with a properly designed sequential investment strategy is an efficient approach of raising capital. For example, a firm can finance its first project by issuing convertibles and then force a conversion to finance the second project. Overall speaking, convertible bonds mitigate the contracting costs of moral hazard, adverse selection and financial distress (Krishnaswami and Yaman, 2008). Based on the interviews of financial managers that were conducted to explore the actual market incentives for issuance of convertibles, Dong *et al.* (2013) document various corporate finance considerations, like risk shifting, backdoor equity, perceived equity undervaluation and share dilution, investors' demand as the common rationales that justify why firms choose convertible bonds issuance over equity or straight debts. Interestingly, the interviews indicate that the demand side of the market rather than the supply side (corporate motives of issuers) seems to be more important.

In this paper, we would like to review and provide critical comments on various pricing formulations of convertible bonds. We discuss the advantages and limitations of using the structural firm value approach and reduced form approach for formulating the defaultable game option models of convertible bonds. The discrepancies between the optimal call policies derived from the theoretical pricing models and empirical findings on issuers' calls are examined. We consider various improvements in the model formulation that may enhance the valuation process of convertible bonds. In particular, we examine the determinants of issuer's call policies from corporate finance considerations and consider how some of the realistic market phenomena may be incorporated into the pricing models. The paper is organized as follows. In the next section, we discuss the structural firm value approach that models the game option arising from interaction of the conversion and call rights. Using the firm value process as the stochastic fundamental in the convertible bond pricing formulation, the corporate structure of the issuer firm and dilution effect upon conversion can be modeled directly. We consider the variational inequalities formulation of the game option model and discuss the characterization of the optimal stopping strategies. In Sec. 3, we discuss the reduced form models where the observable stock price process is used as the stochastic fundamental. Credit risk of a convertible is modeled by default intensity under the intensity based framework. We also review different effective numerical pricing algorithms, like the finite difference method, fast Fourier transform method and Monte Carlo simulation method. Besides the straightforward dynamic programming procedure in stochastic optimization, the optimal stopping policies in the game option models can be solved by the least squares regression techniques in Monte Carlo simulation. In Sec. 4, we discuss the determinants of optimal issuers' call policies and the so-called late call phenomena. We consider various factors that may influence convertible bond call policies, like call notice period, cash flow advantage, information signaling, backdoor equity financing, and others. Conclusive remarks are presented in the last section.

2. Structural Firm Value Models

Following the structural firm value approach initiated by Merton (1974) for pricing risky debts, Ingersoll (1977a, 1977b) pioneers the construction of contingent claims models for pricing and analyzing corporate call policies of convertible bonds. Brennan and Schwartz (1977) consider more generalized contingent claims models of convertibles and present finite difference solution of the pricing models. Liao and Huang (2006) present enhanced versions of the firm value contingent claims models with the incorporation of various structural features, like tax benefits, bankruptcy costs, refunding costs, call notice period, etc. In the first generation of contingent claims models, the firm value process is used as the stochastic fundamental and the corporate structure of the issuer firm is assumed to have senior straight debt, convertible bond and equity. Attempt is made to model corporate financing closer to reality via the inclusion of dilution effect upon conversion and seniority of claims of debts with varying seniorities. Though it is appealing from corporate finance considerations that capital structures of the firm can be incorporated into the structural firm value models, there are at least two difficulties in the model implementation. First, a complete structural representation of all corporate securities issued by the firm may pose serious challenge in the modeling process and solution procedure. Second, the firm value process is not directly observable. The calibration of the parameters that characterize the firm value process, like volatility, may be difficult and unreliable.

An important procedure in the valuation process is related to the determination of the optimal strategies of issuer's call and bondholder's conversion. Based on the assumption of symmetric market rationality, each party should pursue an optimal strategy and expects the counterparty to do the same. A pair of equilibrium conversion-call strategies is resulted such that neither party could improve its position by adopting some other strategy. The firm value process is assumed to be determined exogenously, so optimality of call and conversion strategies would not affect the firm value process. The bondholder's (issuer's) optimal conversion strategy is to maximize (minimize) the convertible bond value. This leads to a two-person game option model. Brennan and Schwartz (1977) show that, it is never optimal to convert an uncalled convertible bond except immediately prior to a dividend date or to after an adverse change in the conversion terms or at maturity. The impact of discrete dividends on optimal conversion is similar to that of an American call on a discrete dividend paying underlying stock. Also, the bondholder may be forced to exercise conversion upon adverse change of the conversion terms, at issuer's call or maturity. From the perspective of the bond issuer, the optimal firm's decision rule is to issue call as soon as the bond value if not called equals the call price. This optimal rule is quite intuitive for, if the bond were left uncalled at a market value exceeding the call price, the bond value would clearly not be minimized. On the other hand, if the issuer calls too soon when the bond value is below the call price, the bond value is again not minimized.

Empirical studies reveal that most convertibles were called late. That is, a convertible bond is not called even when its uncalled value has been quite well above the call price. Various forms of market imperfections may give rise to these observed deviations between perfect market decision rules and observed pattern of calls. We will consider the "late" call phenomena in details in Sec. 4.

2.1. Two-person game option models

The convertible bond issuer acts to maximize the equity value of the firm by minimizing the bond value while the bondholder adopts optimal conversion policy (voluntary or forced) in order to maximize the bond value. This leads to a game option model in which one has to solve for a set of interactive optimal stopping decisions made by the two counterparties.

Sirbu and Shreve (2006) initiate a two-person game option model to analyze the optimal calling and conversion policies of convertible bonds under the structural firm value model. Their model is a zero-sum stochastic differential game when the tax effects on interest income are not considered in the model. Let X_t denote the time-t value of the firm asset, which is assumed to consist of equity and single debt (in the form of a convertible bond). The debt value and equity value are seen as financial derivatives of the firm value, X_t. We write $D_t = g(X_t, t)$, where g is the function to be determined in the solution of the model, and observe the accounting rule:

$$X_t = E_t + D_t.$$

The conventional structural firm value approach starts with the assumption that the firm assets are tradeable so that we can price derivatives under the risk neutral measure Q. The discussion of the justification of this assumption based on argument of utility maximization under economic equilibrium can be found in Chen *et al.* (2013). In the formulation of the Sirbu–Shreve game option model, it is assumed that the convertible bondholder receives constant continuous coupons at a rate c and the equity (firm) owner receives constant continuous dividend at a yield δ, with $0 \le \delta < r$, and r is the risk free interest rate. The dynamics of X_t under Q is assumed to be governed by

$$dX_t = rX_t dt - c dt - \delta E_t dt + \sigma X_t dW_t, \tag{1}$$

where W_t is the standard Brownian motion. Let K be the fixed call price and $\gamma \in (0,1)$ be the conversion factor. Upon conversion of the convertible bond into stock, the bondholders receive stock valued at γX_t. The value of the game gives the no-arbitrage price of the convertible bond. The lower value is given by taking the supremum among bondholder's conversion strategies conditional on the infimum of the present value of all cash flows as enforced by equity owner's optimal choice of call policy. On the other hand, the upper value is given by taking the infimum among equity owner's call strategies conditional on the supremum of the present value of all cash flows as enforced by bondholder's optimal choice of conversion policy. One then solves for the price function g of the convertible bond such that equality of the lower and upper values of the game option is resulted. Sirbu and Shreve (2006) show that if $c \le rK$, then the bondholder's conversion should precede the equity owner's call; otherwise, call should precede conversion if $\delta K \le c$. Specifically, they derive the variational inequality formulation for the bond price function g under the following two cases:

(i) If $c \le rK$, the stopping time of optimal *call* is the first time that the conversion value γX_t increases to the level of the call price K. The bond price function g is the unique continuous viscosity solution of the following variational inequality:

$$\min\left\{-\frac{\partial g}{\partial t} - \frac{\sigma^2}{2}x^2\frac{\partial^2 g}{\partial x^2} - (rx - c)\frac{\partial g}{\partial x} + \delta(x-g)\frac{\partial g}{\partial x} + rg - c, g - \gamma x\right\} = 0 \tag{2a}$$

in the domain $[0, K/\gamma] \times [0, T]$.

(ii) If $\delta K \le c$, the stopping time of optimal *conversion* is the first time that the conversion value γX_t increases to the level of the call price K, or at maturity if the conversion value exceeds the par value. The bond price function g is the unique continuous viscosity solution of the following variational inequality:

$$\max\left\{-\frac{\partial g}{\partial t} - \frac{\sigma^2}{2}x^2\frac{\partial^2 g}{\partial x^2} - (rx - c)\frac{\partial g}{\partial x} + \delta(x-g)\frac{\partial g}{\partial x} + rg - c, g - K\right\} = 0 \tag{2b}$$

in the domain $[0, K/\gamma] \times [0, T]$.

As a remark, when $\delta K \le c \le rK$, then optimal call and conversion both occur at the first moment where γX_t reaches K from below. In this case, the price function g is the unique continuous viscosity solution of the following equation:

$$-\frac{\partial g}{\partial t}-\frac{\sigma^2}{2}x^2\frac{\partial^2 g}{\partial x^2}-(rx-c)\frac{\partial g}{\partial x}+\delta(x-g)\frac{\partial g}{\partial x}+rg-c=0. \tag{2c}$$

2.2. Default risk and tax benefits

Like most debt instruments issued by risky companies, convertible bonds are faced with potential default risk. The later versions of the game option models of convertibles commonly incorporate the modeling consideration of default risk. Bielecki *et al.* (2008) construct the model formulation of defaultable convertibles via an appropriate Markovian intensity model of credit risk, where the arrival of default risk is modeled by a default indicator process. The choice of leverage ratio in a firm's capital structure is a tradeoff of tax benefits against bankruptcy costs together with many other considerations, like agency costs. Chen *et al.* (2013) propose a non-zero sum game involving two coupled optimal stopping problems to analyze the interaction of conversion and callable strategies of convertible debts with credit risks and tax benefits. They adopt the endogenous bankruptcy approach to study the impact of credit risk on the optimal call and conversion strategies in convertibles. The random default time is modeled as an optimal stopping time where the equity owner chooses to declare bankruptcy of the firm in order to terminate the due debt obligation of coupon payments paid to the bondholders. Therefore, endogenous bankruptcy occurs only if the firm value falls to some sufficiently low value under some distressed condition. Their non-zero sum game model reviews that avoidance of costly bankruptcy procedure may prompt equity owner to initiate out-of-the-money call, that is, the convertible is called even when the conversion value is below the call price. As a nice theoretical contribution to the literature of defaultable game options, they establish a unique Nash equilibrium of the two-person game in the defaultable convertible bond pricing model. The corresponding Nash equilibrium under the presence of tax benefit and credit risk gives a similar set of optimal conversion and call strategies to that of Brennan and Schwartz (1977). The call strategy is highly dependent on the call price, e.g., the equity owner should never call when the call price is too high and call should precede conversion when the call price is sufficiently low. Their analyses also shows that a high corporate tax will induce the corporate to increase debt level, so issuance of call by the equity owner would be delayed.

As an extension of the two-person game option model with endogenous bankruptcy, Leung *et al.* (2014) examine the market signaling role of the callable feature in convertibles. Using the framework of *Partial Bayesian Equilibrium* in their two-stage sequential dynamic two-person game option models, they illustrate how the belief system of the bondholders may be revealed over the passage of time when the equity owner follows the optimal strategies of declaring call or bankruptcy. They analyze how the callable feature may help lower the adverse selection costs in convertible bonds financing and how a low quality firm may benefit from information asymmetry in raising capital.

For tax effects on convertibles, Realdon (2010) studies the after-tax valuation of convertible bonds in response to the Europe's participation exemption rules and International Financial Reporting Standards. The participation exemption rules exempt capital gains on stocks from corporate taxation so firms may strategically convert the convertible bonds into stocks to enjoy the exemption. Various accounting standards on the taxation of gains may enhance or destroy tax timing options. He shows that the choices of tax rules and accounting standard would have profound impact on the propensity of call in convertible debts. From the perspective of the equity owners, tax deduction benefits induce a higher coupon rate offered by the convertible bond. However, low-coupon convertible bonds may have tax advantages to bond investors in some jurisdictions. The coupons enjoy tax deduction according to certain tax rules. It would be interesting to examine the optimal design of contractual terms in convertibles that enhances tax efficiency of convertible bonds and explore how to strike the balance so that interests of the two counterparties are better aligned.

Also, it would be instructive to apply the structural firm value approach for analyzing the role of call and conversion features in achieving the optimal leverage in firm's capital structure. Hennessy and Tserlukevich (2008) analyze the choice between callable and convertible debts under taxation and agency conflicts. Their model leads to a sequence of non-zero sum stochastic differential games between the bond issuer and bondholders, and equilibrium prices are shown to depend upon the Markov Perfect equilibriums of a sequence of such games. It would be interesting to examine how convertible bond financing may be used to lower the issuance costs of sequential financing, and what roles call feature and conversion feature should play in achieving the optimal capital structure.

3. Reduced Form Models

To circumvent the difficulties in the parameter estimation of the firm value process and simultaneous valuation of more senior liabilities in the firm's corporate structure, the more recent pricing models of convertibles use stock price S_t, as the fundamental state variable. One major challenge is the modeling of default risk since the equity component and bond component of a risky convertible are subject to different default risks. Tsiveriotis and Fernandes (1998) argue that the equity component of the convertible should be discounted at the risk free interest rate r while the bond component should be discounted at a risky rate (equals the credit spread s plus risk free rate r). Let B_c denote the (time dependent) call price paid to the bondholder upon bond issuer's call and B_p denote the (time dependent) put price received by the bondholder upon bondholder's put. Let k be the conversion number of shares upon bondholder's conversion, r_g be the growth rate of stock and q be the dividend yield of stock. Let $V(S,t)$ denote the value of the risky convertible and $B(S,t)$ be the bond component of the convertible. By splitting the bond value into the equity and bond components, the corresponding linear complementarity formulation of the Tsiveriotis–Fernandes model (1998) can be obtained as follows:

$$\frac{\partial V}{\partial t} + \frac{\sigma^2}{2}S^2\frac{\partial^2 V}{\partial S^2} + (r_g - q)S\frac{\partial V}{\partial S} - r(V-B) - (r+s)B = 0 \tag{3a}$$

subject to the constraints:

$$V \geq \max(B_p, kS) \quad \text{and} \quad V \leq \max(B_c, kS). \tag{3b}$$

The bond component satisfies the following partial differential equation:

$$\frac{\partial B}{\partial t} + \frac{\sigma^2}{2}S^2\frac{\partial^2 B}{\partial S^2} + rS\frac{\partial B}{\partial S} - (r+s)B = 0. \tag{4}$$

The last two terms in (3a) indicate the different discount rates applied to the equity component $(V-B)$ and bond component B. The first inequality in (3b) is the lower obstacle condition prescribed by the floor value of V as dictated by the put provision. The second inequality in (3b) is the upper obstacle condition prescribed by the cap on V due to the call provision. The coupled partial differential equations for V and B have to be solved together when the convertible bond remains alive (the state variable S_t stays in the continuation region of the solution domain).

One limitation in the Tsiveriotis–Fernandes model is that the stock price does not jump upon default. Ayache *et al.* (2003) enhance the Tsiveriotis–Fernandes model by allowing jump of the stock price upon default and partial recovery of bond value upon default. This is commonly called the one-and-a-half factor model, where the half factor corresponds to the default intensity h that is taken to be a deterministic function of the stock price. Suppose we assume that the stock price after and before default observe $S^+ = S^-(1-\beta)$, where $\beta \in [0,1]$. We let R denote the recovery rate, where $R \in [0,1]$, and X be the face value. Assuming zero dividend yield, Ayache *et al.* (2003) obtain the following partial differential equation for V:

$$\frac{\partial V}{\partial t} + \frac{\sigma^2}{2}S^2\frac{\partial^2 V}{\partial S^2} + (r+h\beta)S\frac{\partial V}{\partial S} - (r+h)V + h\max(kS(1-\beta), RX) = 0. \tag{5}$$

In Eq. (5), the sum $r + h\beta$ appear in the drift term, where $h\beta$ can be interpreted as the negative dividend yield. Also, the sum $r + h$ appear as the discount rate, where h can be interpreted as the credit spread. The last term in (5) arises from the resulting payoff upon default.

The most comprehensive reduced form convertible bond model is the four-factor affine specification proposed by, Kovalov and Linetsky (2008), where all essential risk factors in hybrid equity–debt pricing models have been incorporated, like stock price under stochastic volatility dynamics, stochastic interest rate, and jump-to-default with dependence on variance and interest rate. They assume the default intensity h_t (arrival rate of the default event), short rate r_t and instantaneous stock return variance V_t to follow the affine specification of stochastic dynamics and the stock price drops to zero (cemetery state) upon default. The risk neutral dynamics of the stock price S_t prior to default is governed by

$$dS_t = (r_t - q + h_t)S_t dt + \sqrt{V_t}S_t dW_t^S, \tag{6}$$

where W_t^S is a standard Brownian motion. The appearance of h_t in the drift term compensates for the possibility of a drop to zero so that the discounted stock price process remains to be a martingale. For the affine specification of the risk factors, they assume the following joint dynamics for r_t, V_t and h_t:

$$\begin{aligned} dr_t &= k_r(\theta_t - r_t)dt + \sigma_r\sqrt{r_t}dW_t^r, \\ dV_t &= k_V(\theta_V - V_t)dt + \sigma_V\sqrt{V_t}dW_t^V, \\ dz_t &= k_z(\theta_z + \gamma\sqrt{V_t} - z_t)dt + \sigma_z\sqrt{z_t}dW_t^z, \\ h_t &= z_t + \alpha V_t + \beta r_t. \end{aligned} \tag{7}$$

The standard Brownian motions observe the following assumptions on their correlation structures:

$$dW_t^S dW_t^V = \rho_{SV}dt,$$
$$dW_t^S dW_t^r = dW_t^S dW_t^z = dW_t^r dW_t^V = dW_t^r dW_t^z = dW_t^V dW_t^z = 0,$$

where ρ_{SV} is assumed to be negative so as to capture the leverage effect. Both the short rate r_t and instantaneous variance V_t are assumed to follow the Cox–Ingersoll–Ross process with positive rate of mean reversion and long-run mean level. The default intensity h_t is modeled as a linear combination of the three factors: a stochastic factor z_t, a stock price variance dependent contribution αV_t and an interest rate dependent contribution βr_t. Their formulation can be considered as the jump-to-default extension of the Heston type stochastic volatility model coupled with stochastic interest rates.

The linear complementarity pricing formulation of a convertible is completed by incorporating the obstacle function constraints and the auxiliary conditions due to the call, conversion and put features. To solve the variational inequalities formulation numerically, Kovalov and Linetsky (2008) also propose the penalty approximation technique, where additional penalty terms are added in the pricing formulations. These penalty terms come into effect when the state variables lie outside the continuation region in the solution domain. With the choice of a large penalty parameter, the penalty terms take the effect of modeling approximately the action of the call and conversion strategies. Similar penalty approximation approach has been applied to pricing optimal stopping problems that are related to the early exercise right in American options, callable right in callable securities and prepayment right in mortgage loans (Dai and Kwok, 2007).

For more realistic specification of a convertible valuation model, one should include discrete coupons, soft call provision and call notice period requirement. The detailed discussion of these contractual specifications under the one-and-a-half reduced form model can be found in Lau and Kwok (2004).

3.1. Numerical algorithms

For numerical solution of the convertible bond pricing models with one or two risk factors, the lattice tree algorithm or finite difference scheme may be the most appropriate choice of numerical scheme (Lau and Kwok, 2004). In a lattice tree algorithm or explicit finite difference scheme, the dynamic programming procedure of comparing continuation value, conversion value and call price at each lattice node provides an efficient mean of solving the associated optimal stopping problem with interaction of optimal calling and conversion rights. Let *cont*, *conv* and *call* denote the continuation value, conversion value and call price in a lattice node. From the perspective of the bondholder, her optimal strategy is exemplified by choosing $\max(\min(cont, call), conv)$.

The maximum reflects the conversion right, which persists with or without call by the issuer. The bond value before potential conversion is seen to be min(*cont*, *call*) since the issuer would always initiate call when the continuation value shoots above the call price. On the other hand, from the perspective of the issuer, an alternative dynamic programming procedure can be constructed as follows:

$$\min(\max(cont, conv), \max(call, conv)).$$

Here, max(*cont*, *conv*) represents the optimal strategy of the bondholder without call. Upon call, the bondholder chooses to receive max(*call*, *conv*). The issuer chooses to call or abstain from call so as to minimize the convertible bond value. The above two dynamic programming procedures can be seen to be mathematically equivalent if we apply the distributive rule of sequencing the order on the "max" and "min" operations.

To deal with the soft call requirement where the bond issuer can call only when the stock price has stayed above some hurdle price for a certain period of time, Lau and Kwok (2004) incorporate the forward shooting grid technique into the lattice tree pricing algorithm. At each lattice tree node, the augmented excursion time variable that governs the soft call requirement is updated through modeling the joint evolution of the excursion time with the stock price path. Initiation of call by the bondholder is allowed only, when the excursion time is above the required threshold specified in the soft call requirement.

When the number of risk factors in the convertible pricing model is more than two, it may be more effective to use the Monte Carlo simulation method in order to avoid the curse of dimensionality in lattice tree algorithms or finite difference schemes. This is because the order of complexity of numerical calculations in lattice tree algorithms is exponential in the dimension of the pricing model while that of the Monte Carlo simulation calculations grows at polynomial order with respect to the number of risk factors. Similar to pricing an American option with early exercise right, the Monte Carlo simulation algorithm for pricing a convertible bond consists of two stages, an optimization stage and a valuation stage (Ammann *et al.*, 2008). In the first stage, one uses the regression based technique to determine the optimal conversion and call strategies based on a first set of simulated paths. In the second stage, the optimal conversion and call strategies obtained from the first stage are applied to a second set of simulated paths to estimate the price of the convertible. By extending the primal-dual simulation method, Beveridge and Joshi (2011) proposed two simulation algorithms that compute unbiased estimates of upper and lower bounds on the convertible bond price. An unresolved challenge is the use of Monte Carlo simulation method for pricing convertible bonds with reset clauses. The reset provision is a sweetener for convertible investors, whereby the conversion ratio is adjusted upwards (equivalently, conversion price is adjusted downwards) if the stock price does not exceed pre-specified trigger prices. Kimura and Shinohara (2006) proposed a preliminary Monte Carlo simulation algorithm to price non-callable convertible bonds with reset clauses. The underlying technical hurdle lies in the combined solution of the optimal conversion and call strategies with the determination of the conversion ratio via realization of the stock price path.

As a final remark on numerical pricing algorithms of convertibles, suppose we model the underlying stock price (or firm value) by a jump diffusion dynamics correlated with stochastic interest rates, the resulting governing equation becomes a two-dimensional partial integral-differential equation, where the integral term arises from the jump component of the stock price process. The numerical solution of the convertible bond pricing model using the finite difference approach becomes computationally infeasible. One may adopt the Fourier Transform Algorithm

Ballotta and Kyriakou (2014) for efficient numerical valuation of the pricing model. The success of the algorithm relies on the known analytic form of characteristic function of bivariate stock price (or firm value) and interest rate process under the jump diffusion framework.

4. Determinants of Issuers' Call Policies

Most pricing models of convertibles adopt the perfect market assumption that it is optimal to call a convertible once the convertible bond value exceeds the effective call price. Consequently, the value of the bondholder's conversion option is minimized, or equivalently, the value of the equity owner is maximized. Empirical studies on calls of convertibles in the financial markets reveal that most calls are "late". In other words, the issuers delay their call decision by waiting until the conversion value of the bond exceeds the effective call price by a wide margin. At the opposite end, for some rare cases, there are "early" calls that are out-of-the-money; that is, the conversion value is lower than the call price. The literature has been rich in both the theoretical and empirical studies of various corporate finance issues that influence the optimal call policies (Sarkar, 2003; King and Mauer, 2014). The challenge is how to incorporate these corporate finance considerations into the pricing models without adding too much complication in the modeling of the corporate structure of the issuer firm and market microstructure.

In the literature, there have been several hypotheses on how the bond issuers make their call decisions. The factors that may influence call policies include: (i) call notice period, (ii) cash flow advantage, (iii) information signaling role and (iv) backdoor equity financing and corporate restructuring. The rationales and justifications behind each of these hypothesized factors are presented in Sec. 4.1.

4.1. Call notice period

When a call is announced, the bondholders are typically given a minimum of 30 days (notice period) to make the decision whether to convert the bond into shares or redeem the bond for the call price. Setting aside the notice period requirement, most valuation models assume that the issuer would call when the conversion value equals the call price (in the form of a fixed cash amount). With the presence of the notice period requirement, the "effective" call price has dependence on the stock price since the holder is essentially given a vested European put option with maturity date being set at the end of the notice period. Due to potential increase in volatility of stock price and negative price reaction upon the announcement of call, the vested put option associated with the notice period would have a higher value and so a higher "effective" call price. A richer set of patterns of interaction of optimal calling and conversion policies are exhibited due to the dependence of the "effective" call price on the stock price level. Dai and Kwok (2005) examine how the notice period requirement may help partly explain the apparent delay in optimal call by the issuer via a more accurate computation of the critical stock price at which the convertible bond should be called. The empirical studies by Altintig and Butler (2005) show that the median call premium associated with "late" call is less than 4% after properly accounting for the call notice period and other factors. This is substantially less than the 26–44% call premium reported in some earlier paper (Ingersoll, 1977b).

Another explanation of "late" calls is the liquidity problem or even financial distress in the event of a failed call (Jaffee and Shleifer, 1990) due to an acute drop in the stock price during the call notice period. Firms with high stock price volatility and low liquidity would tend to delay call until the probability of a failed call is small.

4.2. Cash flow advantage

The cash flow advantage is defined to be the difference between the dividend on the converted shares and the after-tax coupon payment on the convertible bond. From his empirical studies, Asquith (1995) reports that firms choose to call with longer delay when the cash flow advantage is more positive. Indeed, bondholders should have an incentive to convert voluntary, when the dividend is higher than the coupon. When the cash flow advantage is negative, the issuer should call once the conversion value exceeds the call price.

The infamous screw clause stipulates that the convertible bondholder may not receive interest accrued from the last coupon date upon conversion. Therefore, the bond investor may refrain from conversion until after a coupon payment date. The incentive of conversion may be reduced due to the screw clause. In turn, this would influence the issuer's call policy to take advantage of the screw clause.

4.3. Information signaling role of calls

Using signaling equilibrium model, Harris and Raviv (1985) show that forced conversion is associated with bad signal about the firm's future growth opportunities. Therefore, issuers may choose to delay call to avoid spreading the unfavorable information. This also explains the negative stock price reaction to the call announcement. In addition, with reference to subsequent firm performance, firms that have not forced conversion by a given date will subsequently outperform (on average) firms that force conversion on that date. The empirical evidence shows that the market perceives more negatively on an in-the-money-call when the conversion option is more close to being in-the-money.

On the other hand, Cowan *et al.* (1993) argue that managers with positive private information may choose to call early even when the conversion value is lower than the call price (out-of-the-money calls) in order to separate their announcements from those that imply bad news. The market reaction of stock price is positive for these out-of-the-money calls for several reasons. First, the firm has the ability to raise cash to pay the call price. Second, firms may issue new bonds at lower cost by taking advantage of the positive information on firm performance. Third, firms can get rid of unfavorable covenants in existing convertibles sooner. Therefore, corporate managers may choose to issue out-of-the-money calls if the perceived benefits are higher than the premium loss.

4.4. Backdoor equity financing and corporate restructuring

Stein (1992) argues that a convertible can be used as a delayed equity financing that mitigates the adverse selection costs when compared with direct equity or straight debt financing. Convertibles provide attractive means to raise capital without the negative market reaction associated with equity financing and potential costly financial distress associated with debt financing. The call feature provides the flexibility of the firm to force conversion to reduce the debt-to-equity ratio if desired, and so get equity into the firm's capital structure through the *backdoor*. Sequential financing is a commonly used firm's corporate strategy to raise capital. A firm can raise capital by first issuing convertibles and then forces conversion to get debts into equity (Mayers, 1998).

The motivation of delayed equity financing and occurrence of conversion-forcing call are closely correlated. Suppose the firm's financial situation remains in a similar status since initiation of the convertible, Stein (1992) predicts that the firm chooses to call soon after the convertible becomes in-the-money. Also, the decision to call a convertible quite often coincides with an increase in investment activities. Indeed, empirical studies show that a significant fraction of calls occur around corporate restructuring, mergers and other corporate finance events.

5. Conclusion

We have reviewed the structural features of call and conversion in convertibles and discuss why firms choose convertible bonds for capital financing. Medium quality firms may choose to issue convertibles and eventually get equity into the corporate structure through forced conversion. This strategy leads to less negative market reaction at issuance of convertibles when compared with equity financing. Firms with high and strong growth potential may take advantage of higher value of conversion premium in convertibles (reflected by lower coupon rates paid to bondholders) and the chance of achieving delayed equity financing is higher than that of low quality firms.

The pros and cons of the structural firm value approach and the reduced form approach in the construction of convertibles pricing models have been discussed. The reduced form approach is more popularly adopted in valuation models since parameters in the stock price process and credit spreads can be calibrated at relative ease from observable prices of traded financial securities, like stock options and credit default swaps. The inclusion of additional risk factors and more detailed modeling of structural features in convertible models may pose computational challenges in numerical solution. It is worthwhile to explore more effective Monte Carlo simulation methods that deal with interaction of the optimal call and conversion strategies under the setting of multi-factor underlying state variables and non-zero sum game option framework (with the inclusion of tax benefit).

It is desirable to examine whether discrepancies between the call decision rule based on perfect market assumption and actual issuers' call from empirical studies can be narrowed by more refined modeling of the market trading behaviors and practices of convertibles. For example, one may incorporate the phenomena of decline in stock price and increase in its volatility upon call announcement within the call notice period into consideration. It would be instructive to examine the impact of corporate finance considerations and other market factors on the propensity of call decision made by a corporate manager.

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