Real Option Signaling Games of Debt Financing Using Equity Guarantee Swaps

Qiuqi Wang

Department of Statistics and Actuarial Science, University of Waterloo

Yue Kuen Kwok*

Department of Mathematics, Hong Kong University of Science and Technology

June 25, 2020

Abstract

We analyze the real option signaling game models of debt financing of a risky project under information asymmetry, where the firm quality is only known to the firm management but not outsiders. The firm decides on the optimal investment timing of the risky project that requires upfront fixed funding cost and subsequent operating costs. The fixed funding cost is financed via either direct bank loan or entering into a three-party equity guarantee swap (EGS) that involves a bank granting the loan and third party guarantor. Under the EGS agreement, the guarantor is obligated to pay all the future coupon stream to the bank upon default of the firm. In return for the provision of the guarantee, the guarantor obtains certain proportional share of equity of the firm at the time when the swap agreement is signed. The share of equity demanded by the guarantor depends on the outside investors' belief on the firm quality. The low-type firm has the incentive to mimic the investment strategy of being high-type in terms of investment timing and share of equity. The high-type firm may adopt the appropriate separating strategy by speeding up investment or choosing an alternative financing choice. The resulting loss of the real option value of the investment opportunity represents the information cost under separating strategies. We examine the incentive compatibility constraints faced by the firm under different quality types and discuss characterization of the separating and pooling equilibriums. Unlike the usual assumption of perpetuity of investment opportunity, our real option model assumes the time window of the investment opportunity to be finite. We explore how the information cost and nature of separating and pooling equilibriums evolves over the finite time span of the investment opportunity. The information costs and investment thresholds exhibit interesting time dependent behaviors. We examine the firm's investment and financing choice between EGS and the direct bank loan against time and other parameters via comparison of the corresponding information costs and investment thresholds.

Keywords: debt financing, signaling games, separating and pooling equilibriums, real options, information costs

JEL classification: G31, G32

*Correspondence author; E-mail: maykwok@ust.hk.

1   Introduction

Analysis of investment and financing choices under information asymmetry between the firm management and outsiders has been one of the focuses of research in corporate finance. The earliest static investment and financing choices models (Myers, 1984; Myers and Majluf, 1984) establish the pecking order theory under adverse selection, where debt is preferred to equity. The later works extend the model to the dynamic version to consider optimality of debt versus equity-like securities in financing choices. Under their dynamic signaling game model, Strebulaev *et al.* (2014) show that a project is financed by equity if the probability of success is low, otherwise debt financing is preferred. Yang and Zeng (2019) propose a theory of security design in financing entrepreneurial production. They show that debt is optimal when information is not valuable for production and an efficient combination of debt and equity is optimal when information is valuable.

The real option pricing theory for investment problems (Brennan and Schwartz, 1985; McDonald and Siegel, 1986) can be extended to solve dynamic signaling game problems of investment and debt-equity financing choices. Morellec and Schürhoff (2011) develop dynamic real option signaling game models of corporate investment and financing. In their models, a firm of either one of the two quality types is assumed to hold a real investment opportunity with a perpetual life and raises fund by issuing equity or debt. Assuming that the exact firm type is the private information held by the firm, they calculate the real option values and investment thresholds under separating and pooling equilibriums. They also show how the firm may signal its quality type via investment timing and/or financing choices between equity and debt. Clausen and Flor (2015) extend the real option signaling games of Morellec and Schürhoff (2011) by including the abandonment right and assets-in-place. They find that firms are more likely to choose debt against equity when their assets-in-place is higher. In addition, Grenadier and Wang (2005), Grenadier and Malenko (2011), Xu and Li (2010), Bouvard (2014) consider the effect of asymmetric information on the investment and financing policies of different firm types under various real option signaling game models. Besides capital financing, real options signaling game models have been adopted to analyze various corporate finance issues, like dynamic bankruptcy procedure (Nishihara and Shibata, 2018), strategic investment games of incumbent and entrant firms (Watanabe, 2018), decisions on selling out IPO (Nishihara, 2017), mergers and acquisitions strategies of bidder and target firms (Leung and Kwok, 2018).

There exist several potential extensions of the above real option signaling game models in finance. First, most real option models of investment and financing assume investment opportunity to be perpetual, which may be queried since technologies have finite life spans. Gryglewicz *et al.* (2008) discard the perpetuity assumption and assume a finite project life in real option investment models. They observe the acceleration of investment when uncertainty increases as time comes closer to expiry. Wang and Kwok (2019) analyze the real option signaling games of equity financing with information asymmetry under a finite time horizon. Second, most of the previous works on corporate financing consider either debt or equity financing. Such capabilities are limited to listed companies with sufficient resources. However, private small- or medium-sized firms cannot issue equity or debt and can only raise fund via bank loans. In China, the innovative equity guarantee swaps (EGS) are introduced to overcome the difficulties that private small- and medium-sized firms may

not be able to secure bank loans. To hedge the firm default risk faced by the bank, the EGS agreement introduces a third party profit seeking guarantor. The guarantor has the obligation to pay all the remaining coupons and par value to the bank upon default of the firm. In return for the provision of the guarantee, the guarantor obtains a proportional share of equity of the firm at the time when the EGS is signed. Since the EGS involves both the bank loan and share of equity, the nature of EGS is somehow similar to a convertible bond, which is a hybrid of debt financing and equity participation. Wang *et al.* (2015) argue that the EGS in China provides substantial diversification benefits and tax advantages. Luo *et al.* (2016) construct real option models to analyze the optimal investment timing of EGS when the revenue flow dynamics follows the double exponential jump-diffusion models. Shan and Tang (2019) perform empirical studies on the third party loan guarantees for small- and medium-sized enterprises in China. Their studies reveal that loans screened by a third party guarantor have low default rate, indicating the important informational role played by these equity guarantee swaps.

This paper analyze the real option signaling game models of debt financing using either direct bank loans or equity guarantee swaps under asymmetric information. We consider a small- or medium-sized firm facing an investment opportunity for a risky project when a new technology comes into existence. The investment opportunity expires within a finite time horizon. The firm has no ability to issue equity or market bond and can only raise capital to fund the project with a bank loan. The firm can choose to raise fund via direct bank loan or enter into an equity guarantee swap (EGS) with a guarantor. In our real signaling game model, the firm quality is characterized by the level of the revenue flow generated from the project, which is assumed to be the private information held by the firm manager but not accessible to outsiders. The outsiders update their belief on the firm type by observing the investment timing chosen by the firm. We examine how the investment thresholds of different firm types for EGS and direct bank loan under separating and pooling equilibriums evolve over time. We also focus on the financing choice between EGS and direct bank loan under asymmetric information via comparison of the corresponding information costs and investment thresholds.

This paper is organized as follows. Section 2 constructs the real option signaling game model of debt financing under finite time horizon and characterizes the belief system of the loan counterparty. The values of the firm's equity, default right and liability under the direct bank loan and EGS are computed under complete information as the benchmark case. Section 3 discusses the investment behaviors of the firm in separating equilibrium under the direct bank loan and EGS. Incentive compatibility constraints, binding thresholds and investment thresholds for separating strategies are determined for either firm type. Section 4 examines the investment thresholds and belief systems under the pooling equilibrium. Besides the incentive compatibility constraints, optimal investment thresholds and belief systems, we also focus the discussion on the fair share of equity under EGS and coupon rate under the direct bank loan. Section 5 presents the numerical studies on the time evolution of the investment thresholds and value functions. Also, we consider the financing choice between EGS and direct bank loans by observing the corresponding information costs and investment thresholds. Section 6 highlights the main results and concludes the paper.

2   Model formulation of debt financing

We assume that all agents in the financial markets are risk neutral and cash flows are discounted at the constant riskless interest rate r. The investment opportunity is assumed to have a finite life with a fixed time horizon T, while the revenue flow generated by the project have a perpetual life. Once the project is launched, it produces a continuous revenue shock variable that depends on the firm type k. In this paper, we consider two types of the firm quality: high-type ($k = h$) and low-type ($k = l$) firm. At time t, the net revenue shock variable is denoted by $\lambda X_t - f$, where $\lambda > 0$ is a multiplier which takes value of λ_h for high-type or λ_l for low-type ($\lambda_h > \lambda_l > 0$), $f > 0$ is the constant operating expenses of the investment project and X_t is the stochastic shock variable representing the revenue shock variable generated from the project. We assume that X_t is observable and evolves according to the following Geometric Brownian motion:

$$\mathrm{d}X_t = \mu X_t \, \mathrm{d}t + \sigma X_t \, \mathrm{d}Z_t, \quad X_0 > 0, \tag{2.1}$$

where Z_t is the standard Brownian motion, $\mu < r$ is the constant drift rate and $\sigma > 0$ is the constant volatility.

The investment is irreversible and the firm has the option to wait. The firm manager may choose to raise the capital amount I for funding the project by direct bank loan or entering into a three-party equity guarantee swap (EGS). The EGS involves a bank that grants the loan and a third party guarantor.

Conditioned on no default, the present value of the perpetual revenue flow generated by the investment project at time t is given by

$$\mathbb{E}_t \left[\int_t^\infty \mathrm{e}^{-r(u-t)} \lambda X_u \, \mathrm{d}u \, \middle| \, X_t = X \right] = \frac{\lambda X}{r - \mu}, \tag{2.2}$$

where $\mathbb{E}_t$ denotes the expectation based on the information at time t and λ may assume the value λ_h or λ_l. We write

$$\Pi(X) = \frac{X}{r - \mu} \tag{2.3}$$

for notational convenience. Let F denote the present value of the future perpetual stream of operating expenses of the investment project, where

$$F = \int_t^\infty \mathrm{e}^{-r(u-t)} f \, \mathrm{d}u = \frac{f}{r}. \tag{2.4}$$

Upon default, the equity value of the firm becomes zero and the ownership of the firm is transferred to (i) the bank under direct bank loan or (ii) the guarantor under EGS. In either case, a fraction $\alpha \in (0, 1)$ of the firm equity value will be lost due to bankruptcy cost. As a result, the revenue flow received by the bank or guarantor after taking over the firm becomes $(1 - \alpha)\lambda X_t - f$.

2.1   Asymmetric information on firm type and belief system

The asymmetric information in our real signaling game model is the firm quality, which is characterized by the revenue shock variable multiplier λ. The firm quality, either as high-type

or low-type, is the private information held only by the firm manager. Prior to investment undertaken by the firm, the loan counterparty is either the bank (under direct bank loan) or guarantor (under EGS). Let Λ be the discrete Bernoulli random variable that characterizes the multiplier λ. The loan counterparty only forms a belief system of the firm type, denoted by the probabilistic representation: $\mathbb{P}[\Lambda = \lambda_h] = p$ and $\mathbb{P}[\Lambda = \lambda_l] = 1-p$, where $\lambda_h > \lambda_l > 0$; p is deterministic and $p \in (0,1)$. To signify the firm type to outsiders, the signal sent by the firm is the investment threshold of launching the investment project. After receiving the signal, the loan counterparty's belief on Λ can be categorized into the following three types:

(i) $\Lambda = \lambda_l$, the true "low" type of the firm is revealed to the counterparty;

(ii) $\Lambda = \lambda_h$, the true "high" type of the firm is revealed to the counterparty;

(iii) $\Lambda = \lambda_p = p\lambda_h + (1-p)\lambda_l$, a probabilistic belief on Λ since the signal fails to reveal the true type of the firm to the counterparty.

2.2   Notations

To fix the notation in our later discussion, we adopt the following conventions to define various types of variables in our model:

(i) The subscript θ denotes the financing choice of the firm management: $\theta = g$ for the EGS and $\theta = b$ for the direct bank loan.

(ii) The subscript k denotes the firm type: $k = h$ for the high-type firm and $k = l$ for the low-type firm. Under pooling, we only have probabilistic belief on the firm type.

(iii) The superscript γ denotes the type of equilibrium: $\gamma = c$ for complete information, $\gamma = m$ for mimicking, $\gamma = s$ for separating and $\gamma = p$ for pooling.

(iv) The superscript "$*$" denotes the optimal stopping threshold, the "overline" denotes the binding threshold, the "underline" denotes the default threshold and the "hat" denotes the threshold corresponding to zero net value.

For example, $I_{g,l}^m(X)$ represents the intrinsic value I at $X_t = X$ of the low-type firm (l) under mimicking strategy (m) when the financing choice is EGS (g). The firm has to pay the perpetual coupon stream c_θ to the bank under the financing choice θ. Similar to (2.4), the present value of the perpetual continuous coupon stream is found to be $\frac{c_\theta}{r}$, where $\theta = g$ or b.

2.3   Complete information: direct bank loan versus equity guarantee swap

Under the assumption of complete information, the revenue shock variable multiplier λ is known to all parties. We follow the debt financing model of Morellec and Schürhoff (2011), which assumes the equity holder to have the right to default. For simplicity, the tax benefit of

debt is not incorporated in our debt model[1]. First, we derive the value functions of the default right, liability and equity held by various parties when the firm finances the investment via either the direct bank loan or equity guarantee swap under complete information. We provide financial interpretation of various terms in these value functions. We then derive the real option value function before investment under the assumption of investment opportunity of a finite time horizon.

2.3.1 Direct bank loan

Let $D_b(X; \lambda, c_b)$ denote the firm's real option value of the default right of the bank loan at $X_t = X$ and $\Lambda = \lambda$, and c_b is the continuous rate of coupons (in dollar amount) paid by the firm to the bank under the issuance of the perpetual bank loan. Let $\underline{X}_b(\lambda, c_b)$ be the optimal default threshold. The optimal default threshold under the perpetual direct bank loan is given by (Morellec and Schürhoff, 2011)

$$\underline{X}_b(\lambda, c_b) = \frac{\xi}{\xi - 1} \frac{r - \mu}{\lambda} \left(F + \frac{c_b}{r} \right), \tag{2.5}$$

where

$$\xi = \frac{1}{2} - \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{1}{2} - \frac{\mu}{\sigma^2} \right)^2 + \frac{2r}{\sigma^2}} < 0.$$

The default right can be quantified as the option held by the firm to terminate the liabilities of paying the perpetual coupon stream and operating cost, while forfeit the benefit of receiving the revenue flow. The value function of the default right is found to be (Dixit and Pindyck, 1994)

$$D_b(X; \lambda, c_b) = \left[F + \frac{c_b}{r} - \lambda \Pi(\underline{X}_b(\lambda, c_b)) \right] \left[\frac{X}{\underline{X}_b(\lambda, c_b)} \right]^{\xi}, \tag{2.6}$$

where $\left[\frac{X}{\underline{X}_b(\lambda, c_b)} \right]^{\xi}$ is interpreted as the probability of hitting the default threshold $\underline{X}_b(\lambda, c_b)$ from above when the revenue shock variable assumes the level X. Upon default, the bank takes over the ownership of the firm while the continuous coupon stream is discontinued. Let α be the fractional loss on the revenue flow due to the bankruptcy cost, where $\alpha \in (0, 1)$. The liabilities value borne by the bank arising from the firm's default right is given by

$$L_b(X; \lambda, c_b) = \left[F + \frac{c_b}{r} - (1 - \alpha)\lambda \Pi(\underline{X}_b(\lambda, c_b)) \right] \left[\frac{X}{\underline{X}_b(\lambda, c_b)} \right]^{\xi}. \tag{2.7}$$

Note that $L_b(X; \lambda, c_b) > D_b(X; \lambda, c_b)$ due to the fractional loss α arising from the bankruptcy cost. As expected, $L_b(X; \lambda, c_b)$ equals $D_b(X; \lambda, c_b)$ when $\alpha = 0$.

The equity value $E_b(X; \lambda, c_b)$ of the project after investment under the direct bank loan is the sum of the expected value of the revenue and value of the default right minus the sum

---

[1]To incorporate the tax benefit of the debt, we may simply modify the coupon rate c_θ by $(1 - \tau)c_\theta$, where τ is the tax rate. The analysis of the signaling game model remains intact. Leland (1994) deduces that the optimal capital structure under no tax benefit is all equity. However, equity financing is ruled out for small- or medium-sized firms in the current context.

of values of the coupon stream and operating cost. This gives

$$E_b(X; \lambda, c_b) = \lambda\Pi(X) - F - \frac{c_b}{r} + D_b(X; \lambda, c_b). \tag{2.8a}$$

Since we assume that the firm has only one activity: operating on the project and holding the full ownership of the project, the firm's intrinsic value under direct bank loan is the same as the equity value. However, these two values are different under the EGS arrangement since the firm holds partial ownership of the project [see (2.17) later].

The firm raises the fund amount I to finance the investment project via direct bank loan. For simplicity, we assume the bank loan is not negotiable upon bankruptcy[2]. The bank charges the coupon rate c_b under direct bank loan for the loan amount I according to the budget constraint:

$$\frac{c_b}{r} - L_b(X; \lambda, c_b) = I.$$

This gives an implicit equation for finding c_b, which has an implicit dependence on X and λ. Putting all these relations together, we can simplify the firm's equity value as follows:

$$E_b(X; \lambda, c_b) = \lambda\Pi(X) - F - I - L_b(X; \lambda, c_b) + D_b(X; \lambda, c_b)$$
$$= \lambda\Pi(X) - F - I - \alpha\lambda\Pi(\underline{X}_b(\lambda, c_b)) \left[\frac{X}{\underline{X}_b(\lambda, c_b)}\right]^\xi. \tag{2.8b}$$

Under our finite time horizon model, the investment opportunity faces with the mandated maturity date T. For direct bank loan financing under complete information, the real option value $V_b^c(X, t; \lambda)$ of the firm before investment is given by

$$V_b^c(X, t; \lambda) = \sup_{t \leq u \leq T} \mathbb{E}_t\left[e^{-r(u-t)} E_b(X_u; \lambda, c_b) \big| X_t = X\right], \quad 0 < X < X_b^*(t; \lambda), \tag{2.9}$$

where u is a stopping time and $X_b^*(t; \lambda)$ is the first-best threshold for optimal entry into the bank loan at time t. The solution of $V_b^c(X, t; \lambda)$ involves a nonstandard optimal stopping problem since the exercise payoff is $E_b(X; \lambda, c_b)$, which is nonlinear in X.

2.3.2 Equity guarantee swap

The two active parties under the equity guarantee swap (EGS) are the firm and guarantor, while the bank plays a passive role since the bank bears no liability associated with the default right of the firm. Assuming no default risk of the guarantor, the loan is considered to be risk free for the bank. Therefore, the continuous rate of coupons (in dollar amount) charged by the bank on the bank loan under EGS is given by

$$c_g = rI, \tag{2.10}$$

---

[2]Shibata and Nishihara (2015) examine the impact on the choices of debt financing, taking into account that negotiations upon bankruptcy are allowed for bank loans while market bonds are assumed to be non-negotiable. Recall that as a small- or medium-sized enterprise, the firm cannot issue equity or market bond.

which has the nice feature that it is independent of X and λ. The corresponding value functions of the firm's default right and guarantor's liability now take simpler analytic forms, namely,

$$D_g(X;\lambda) = \left[F + I - \lambda\Pi(\underline{X}_g(\lambda))\right]\left[\frac{X}{\underline{X}_g(\lambda)}\right]^{\xi}, \tag{2.11}$$

and

$$L_g(X;\lambda) = \left[F + I - (1-\alpha)\lambda\Pi(\underline{X}_g(\lambda))\right]\left[\frac{X}{\underline{X}_g(\lambda)}\right]^{\xi}, \tag{2.12}$$

where the optimal default threshold under perpetuality of the project is given by

$$\underline{X}_g(\lambda) = \frac{\xi}{\xi - 1}\frac{r - \mu}{\lambda}(F + I). \tag{2.13}$$

The firm's equity value under EGS does not include liability $L_g(X;\lambda)$ since the liability arising from the default of the bank loan is borne by the guarantor. As a result, the corresponding firm's equity value under EGS is given by

$$E_g(X;\lambda) = \lambda\Pi(X) - F - I + D_g(X;\lambda). \tag{2.14}$$

In return, the guarantor is entitled to receive the proportional share $\phi(X;\lambda)$ of the firm's equity value $E_g(X;\lambda)$. To achieve a fair deal, the liability borne by the guarantor is compensated by holding certain proportional share of equity. The fair amount of the proportional share $\phi(X;\lambda)$ is then given by

$$\begin{aligned}
\phi(X;\lambda) &= \frac{L_g(X;\lambda)}{E_g(X;\lambda)} \\
&= \frac{\left[F + I - (1-\alpha)\lambda\Pi(\underline{X}_g(\lambda))\right]\left[\frac{X}{\underline{X}_g(\lambda)}\right]^{\xi}}{\lambda\Pi(X) - F - I + \left[F + I - \lambda\Pi(\underline{X}_g(\lambda))\right]\left[\frac{X}{\underline{X}_g(\lambda)}\right]^{\xi}} \\
&= \frac{\eta + \eta_\alpha}{[\lambda\Pi(X)]^{-\xi}(F + I)^{\xi-1}[\lambda\Pi(X) - F - I] + \eta}, \tag{2.15}
\end{aligned}$$

where

$$\eta = \frac{1}{1 - \xi}\left(\frac{\xi}{\xi - 1}\right)^{-\xi} > 0, \quad \text{and} \quad \eta_\alpha = \alpha\left(\frac{\xi}{\xi - 1}\right)^{1-\xi} > 0. \tag{2.16}$$

Suppose that the firm enters into the EGS to fund the project, the firm's intrinsic value $I_g^c(X;\lambda)$ under EGS is given by

$$\begin{aligned}
I_g^c(X;\lambda) &= [1 - \phi(X;\lambda)]E_g(X;\lambda) = \lambda\Pi(X) - F - I + D_g(X;\lambda) - L_g(X;\lambda) \\
&= \lambda\Pi(X) - F - I - \alpha\lambda\Pi(\underline{X}_g(\lambda))\left[\frac{X}{\underline{X}_g(\lambda)}\right]^{\xi}. \tag{2.17}
\end{aligned}$$

It is seen that $I_g^c(X;\lambda)$ is an increasing function of X. Let $\hat{X}_g(\lambda)$ denote the revenue shock variable level such that $I_g^c(\hat{X}_g(\lambda);\lambda) = 0$. To observe non-negativity of $I_g^c(X;\lambda)$,

we set $I_g^c(X; \lambda) = 0$ when $X \leq \hat{X}_g(\lambda)$. Indeed, non-negativity of $I_g^c(X; \lambda)$ is equivalent to $E_g(X; \lambda) \geq L_g(X; \lambda)$. This is consistent with the observation of the property: $\phi(X; \lambda) \in [0, 1]$.

Similarly, we assume that the investment opportunity lasts until maturity date T. For the EGS under complete information, the real option value of the firm before investment $V_g^c(X, t; \lambda)$ is given by

$$V_g^c(X, t; \lambda) = \sup_{t \leq u \leq T} \mathbb{E}_t \left[e^{-r(u-t)} I_g^c(X_u; \lambda) \big| X_t = X \right], \quad 0 < X < X_g^*(t; \lambda), \tag{2.18}$$

where $X_g^*(t; \lambda)$ is the first-best threshold for optimal investment entry under the EGS at time t. The optimal stopping problem associated with $V_g^c(X, t; \lambda)$ can be solved at relative ease since $I_g^c(X; \lambda)$ only involves power functions of X.

2.3.3 Comparison of value functions and optimal thresholds under direct bank loan and EGS

We would like to compare the relative magnitudes of the value functions and optimal thresholds associated with the financing choices of either the direct bank loan or EGS under complete information. First, it is obvious that $c_b > c_g$ since

$$\frac{c_b}{r} - \frac{c_g}{r} = L_b(X; \lambda, c_b) > 0. \tag{2.19}$$

Without the guarantee offered by the EGS agreement, the bank charges a higher coupon rate to compensate for the liability associated with the default right of the firm. Consequently, the optimal default threshold $\underline{X}_b(\lambda, c_b)$ under the direct bank loan should be higher than $\underline{X}_g(\lambda)$ under EGS since

$$\underline{X}_b(\lambda, c_b) = \frac{\xi}{\xi - 1} \frac{r - \mu}{\lambda} \left(F + \frac{c_b}{r} \right) > \frac{\xi}{\xi - 1} \frac{r - \mu}{\lambda} \left(F + \frac{c_g}{r} \right) = \underline{X}_g(\lambda). \tag{2.20}$$

Furthermore, since the coupon rate is higher under the direct bank loan, the values of the default right and liability would be higher when compared with those under EGS. This is verified mathematically as follows:

$$D_b(X; \lambda, c_b) = \left[F + \frac{c_b}{r} - \lambda \Pi(\underline{X}_b(\lambda, c_b)) \right] \left[\frac{X}{\underline{X}_b(\lambda, c_b)} \right]^{\xi}$$

$$= \eta \lambda^{\xi} \Pi(X)^{\xi} \left(F + \frac{c_b}{r} \right)^{1-\xi} > D_g(X; \lambda), \tag{2.21}$$

and

$$L_b(X; \lambda, c_b) = (\eta + \eta_\alpha) \lambda^{\xi} \Pi(X)^{\xi} \left(F + \frac{c_b}{r} \right)^{1-\xi} > L_g(X; \lambda). \tag{2.22}$$

In a similar manner, since $\underline{X}_b(\lambda, c_b) > \underline{X}_g(\lambda)$, we can establish that the intrinsic (equity) value $E_b(X; \lambda, c_b)$ under the direct bank loan is lower than the intrinsic value $I_g^c(X; \lambda)$ under

9

EGS, where

$$E_b(X; \lambda, c_b) = \lambda\Pi(X) - F - I - \alpha\lambda\Pi(\underline{X}_b(\lambda, c_b)) \left[\frac{X}{\underline{X}_b(\lambda, c_b)}\right]^\xi$$

$$< \lambda\Pi(X) - F - I - \alpha\lambda\Pi(\underline{X}_g(\lambda)) \left[\frac{X}{\underline{X}_g(\lambda)}\right]^\xi = I_g^c(X; \lambda). \qquad (2.23)$$

Since $I_g^c(X; \lambda) > E_b(X; \lambda, c_b)$, it would be more desirable for the firm to enter into the EGS agreement to achieve higher intrinsic value. Lastly, since $E_b(X; \lambda, c_b) < I_g^c(X; \lambda)$, the corresponding thresholds of optimal entry into investment under the above two loan arrangements observe[3]

$$X_b^*(t; \lambda) > X_g^*(t; \lambda). \qquad (2.24)$$

The calculations above show that the manager of the small- or medium-sized firm strictly prefers to find a guarantor to enter into the EGS agreement over the choice of direct bank loan under complete information. The intrinsic value of the firm under either EGS or direct bank loan [shown in (2.8b) and (2.17), respectively] can be expressed as:

$$\lambda\Pi(X) - F - I + D_\theta(X; \lambda) - L_\theta(X; \lambda), \quad \theta = g, b. \qquad (2.25)$$

In both loan arrangements, the firm's direct payoff value from the project is the same. The direct payoff value is given by $\lambda\Pi(X) - F - I$, which is the revenue flow of the project minus the sum of present value of future operating expenses and direct cost. Also the firm holds default right with value $D_\theta(X; \lambda) > 0$ after launching of the project. Upon default, the firm would also be penalized by the liability. We observe

$$D_\theta(X; \lambda) - L_\theta(X; \lambda) = -\alpha\lambda\Pi(\underline{X}_\theta) \left(\frac{X}{\underline{X}_\theta}\right)^\xi < 0, \qquad (2.26)$$

which arises due to the bankruptcy cost upon default with the bankruptcy cost parameter α. For $\alpha > 0$, the firm's intrinsic value is less than the direct payoff value from the project.

In summary, the strategy of entering into the EGS dominates that of the direct bank loan. Since the firm pays lower coupon rate to the bank, this results in lower default threshold and thus lower expected bankruptcy cost when compared with the direct bank loan.

3 Real signaling games under separating equilibrium

This paper focuses on the analysis of the real signaling games on the investment timing and financing choice of the firm with an investment opportunity of a risky project under asymmetric information. When the firm type is regarded as the private information and not accessible to the outsiders, the low-type firm may have an incentive to mimic the investment timing and financing choice of the high-type firm in order to reduce the share of equity under the EGS and/or the coupon rate under the direct bank loan. On the other hand, the

---

[3]This is similar to the optimal stopping rule in an American call option where the early exercise threshold is increasing with higher intrinsic payoff in the call option [see Chapter 5 in Kwok (2008)].

high-type firm has an incentive to speed up its investment or to adopt a different financing choice in order to separate from being visualized as low-type.

In this paper, beyond the usual perpetuality assumption (Morellec and Schürhoff, 2011), we assume finite time horizon of the investment opportunity. First, we examine the incentive compatibility constraints (ICCs) of both firm types and the properties of the corresponding investment thresholds under the EGS agreement. We then characterize the separating equilibrium under the EGS agreement. The high-type firm may choose to signal its true type through investment timing and separate from the low-type by investing earlier. This would impose information cost on the high-type firm under separating equilibrium. Besides EGS, we also consider the direct bank loan as an alternative financing choice under information asymmetry and examine the separating equilibrium when the firm signals its type through financing choice. Under both financing choices, we discuss the nature of separating equilibrium when the investment opportunity comes close to expiry.

3.1   Single crossing condition

In signaling game models, existence of separating equilibrium requires satisfaction of the single crossing condition as necessary condition so that the high-type firm chooses to send higher signal. In the context of real option signaling game investment models, Morellec and Schürhoff (2011) consider the elasticity of substitution between preceived firm quality λ and investment threshold X^*, and obtain the single crossing condition as given by [see Appendix B in Morellec and Schürhoff (2011)]

$$\frac{\partial}{\partial \lambda_k} \left[\frac{\frac{\partial}{\partial \lambda} V_{g,k}^c(X, t; X^*, \lambda)}{\frac{\partial}{\partial X^*} V_{g,k}^c(X, t; X^*, \lambda)} \right] > 0, \quad \text{for all } (X, \lambda, t), \tag{3.1}$$

where

$$V_{g,k}^c(X, t; X^*, \lambda) = \sup_{t \leq u \leq T} \mathbb{E}_t \left[e^{-r(u-t)} [1 - \phi(X_u; \lambda)] E_g(X_u; \lambda_k) \big| X_t = X \right], \quad 0 < X < X^*.$$

The numerator term $\frac{\partial}{\partial \lambda} V_{g,k}^c(X, t; X^*, \lambda)$ represents the increase in real option value due to reduction on proportional share $\phi(\lambda)$ when the firm is perceived with an increased value of λ. The denominator $\frac{\partial}{\partial X^*} V_{g,k}^c(X, t; X^*, \lambda)$ represents the negative effect of investment distortion where lower threshold X^* yields lower real option value $V_{g,k}^c$. The single crossing condition (3.1) gives the necessary condition under which the high-type firm finds it less costly to distort its investment threshold compared to that of the low-type firm. In other words, provided that the single crossing condition (3.1) is satisfied, it is admissible that the high-type firm invests earlier at lower threshold X^* so as to separate itself from being perceived as low-type.

Note that the value function $V_{g,k}^c(X, t; X^*, \lambda)$ is the solution to relevant optimal stopping problem under finite time horizon, which does not admit closed form formula. Mathematically, it is non-tractable to check the necessary condition (3.1) for existence of separating equilibrium in our signaling game model. In our subsequent analysis of separating equilibrium, we derive the sufficient conditions for existence of separating equilibrium through EGS under the assumption that the corresponding single crossing condition is satisfied (see Proposition 2).

3.2 Separating equilibrium through EGS

We first consider the case where the firm finances the project through the EGS agreement. Under the financing choice of EGS, the outsiders can only perceive the firm type through its investment threshold. Under complete information, the low-type firm would enter into the EGS agreement and invest optimally at the threshold $X_g^*(t; \lambda_l)$ at time t. On the other hand, under asymmetric information, the low-type firm has the incentive to mimic the investment behavior of the high-type firm by speeding up its investment in order to reduce the share of equity, which takes the value $\phi(X; \lambda_h)$ under the belief system $\mathbb{P}[\Lambda = \lambda_h] = 1$. The intrinsic value of the low-type firm under the mimicking strategy is given by

$$I_{g,l}^m(X) = [1 - \phi(X; \lambda_h)]E_g(X; \lambda_l) = \frac{E_g(X; \lambda_l)}{E_g(X; \lambda_h)}[E_g(X; \lambda_h) - L_g(X; \lambda_h)]. \qquad (3.2)$$

The domain of definition of $I_{g,l}^m(X)$ is specified based on the following financial phenomena. Since $\phi(X; \lambda_h) \in [0, 1]$, non-negativity of $I_{g,l}^m(X)$ would be ensured. Therefore, the revenue shock variable level X should be above $\hat{X}_g(\lambda_h)$, which is the root obtained by solving (2.17) under $\lambda = \lambda_h$. On the other hand, the equity value of the low-type firm $E_g(X; \lambda_l)$ should be positive, which implies that X should be above the default threshold $\underline{X}_g(\lambda_l)$. Combining these results, we obtain the lower bound of the domain of definition:

$$\hat{\hat{X}}_{g,l} = \max(\hat{X}_g(\lambda_h), \underline{X}_g(\lambda_l)).$$

Besides, since the low-type firm speeds up its investment under the mimicking strategy, the revenue shock variable level X would stay below the first-best investment threshold $X_g^*(t; \lambda_l)$ at time t. This dictates the upper bound of the domain of definition to be given by $X_g^*(t; \lambda_l)$. Therefore, $I_{g,l}^m(X)$ is defined within the interval $[\hat{\hat{X}}_{g,l}, X_g^*(t; \lambda_l)]$.

3.2.1 Incentive compatibility constraint for the low-type firm

At time t, the low-type firm considers the tradeoff between (i) mimicking the high-type firm to attain lower equity share $\phi(X; \lambda_h)$, and (ii) investing optimally at its first-best threshold $X_g^*(t; \lambda_l)$. The low-type firm prefers to mimic the high-type firm only when its corresponding intrinsic value $I_{g,l}^m(X)$ at the revenue shock variable X is larger than $V_g^c(X, t; \lambda_l)$, which is the real option value under complete information [see (2.18)]. In order that the high-type firm can separate from low-type, the incentive compatibility constraint (ICC) for the low-type firm at the revenue shock variable level X and time t is characterized by nonpositivity of the difference of these two value functions, where

$$G_{g,l}(X, t) = I_{g,l}^m(X) - V_g^c(X, t; \lambda_l), \quad \hat{\hat{X}}_{g,l} \leq X \leq X_g^*(t; \lambda_l). \qquad (3.3)$$

Let $\overline{X}_{g,l}(t)$ denote the binding threshold of (3.3), where $G_{g,l}(\overline{X}_{g,l}(t), t) = 0$. Only when $X \leq \overline{X}_{g,l}(t)$ at time t, nonpositivity of $G_{g,l}(X, t)$ is observed. Correspondingly, satisfaction of ICC for the low-type firm [see (3.3)] observes incentive compatibility since the low-type firm prefers not to mimic. In other words, when the revenue shock variable level X is at or below $\overline{X}_{g,l}(t)$, the high-type firm can separate from being perceived as low-type.

Lemma 1 establishes the existence of $\overline{X}_{g,l}(t)$ within $[\hat{\hat{X}}_{g,l}, X_g^*(t; \lambda_l)]$ so that the incentive compatibility constraint for the low-type firm is satisfied when $X \leq \overline{X}_{g,l}(t)$.

Lemma 1. *There exists a binding threshold $\overline{X}_{g,l}(t)$ of (3.3) at a given time t within the interval $[\hat{\hat{X}}_{g,l}, X_g^*(t; \lambda_l)]$ such that*

$$I_{g,l}^m(X) \leq V_g^c(X, t; \lambda_l)$$

when $X \leq \overline{X}_{g,l}(t)$. The incentive compatibility constraint is satisfied for the low-type firm in order for the high-type firm to separate when the revenue shock variable X is at or below $\overline{X}_{g,l}(t)$.

The proof of Lemma 1 is shown in Appendix A. The relations between the binding and first-best thresholds, and satisfaction of the ICC for the low-type firm are depicted in Figure 1. Due to time dependence of the real option value functions and investment thresholds, there is no closed form solution of $V_g^c(X, t; \lambda_l)$ and the binding threshold $\overline{X}_{g,l}(t)$. We resort to numerical methods to compute these two time dependent quantities.
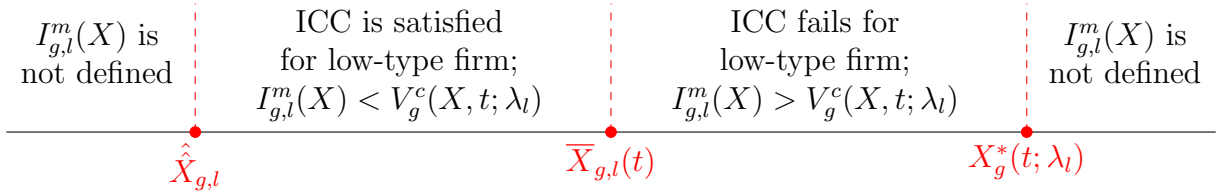

Figure 1: Relations between the binding and first-best thresholds, and satisfaction of the ICC of the low-type firm. Lemma 1 states that $\overline{X}_{g,l}(t)$ exists within $[\hat{\hat{X}}_{g,l}, X_g^*(t; \lambda_l)]$ at any time t.

3.2.2 Incentive compatibility constraint for the high-type firm

The manager of the high-type firm may choose to separate from being perceived as low-type by speeding up investment. On the other hand, if the high-type firm fails to separate from the low-type, the guarantor sets the belief system to be $\Lambda = \lambda_l$. Accordingly, the high-type firm has to accept the share of equity to be $\phi(X; \lambda_l)$. The corresponding intrinsic value upon investment under such scenario is given by

$$I_{g,h}^m(X) = [1 - \phi(X; \lambda_l)] E_g(X; \lambda_h) = \frac{E_g(X; \lambda_h)}{E_g(X; \lambda_l)} [E_g(X; \lambda_l) - L_g(X; \lambda_l)]. \qquad (3.4)$$

Given the intrinsic value $I_{g,h}^m(X)$, the corresponding mimicking real option value function before investment for the high-type firm is given by

$$V_{g,h}^m(X, t) = \sup_{u \in [t,T]} \mathbb{E}_t \left[e^{-r(u-t)} I_{g,h}^m(X_u) \big| X_t = X \right], \quad 0 \leq X \leq X_{g,h}^{m*}(t), \qquad (3.5)$$

where $X_{g,h}^{m*}(t)$ is the optimal investment threshold of the high-type firm under mimicking strategy at time t. The optimal threshold $X_{g,h}^{m*}(t)$ is determined as part of the solution of the

above optimal stopping problem. The procedure is similar to finding the optimal exercise threshold in an American option model.

To analyze the incentive compatibility constraint (ICC) for the high-type firm, the necessary condition (ICC) under which the high-type firm prefers separating rather than mimicking is given by

$$G_{g,h}(X, t) = I_g^c(X; \lambda_h) - V_{g,h}^m(X, t) \geq 0, \quad \hat{X}_g(\lambda_h) \leq X \leq X_{g,h}^{m*}(t). \tag{3.6}$$

Let $\overline{X}_{g,h}(t)$ denote the time-t binding threshold of (3.6) at which $G_{g,h}(\overline{X}_{g,h}(t), t) = 0$. Violation of the ICC implies that the high-type firm would not choose to separate from being perceived as low-type by speeding up investment when the revenue shock variable level X falls below $\overline{X}_{g,h}(t)$.

A separating equilibrium under EGS exists only when $\overline{X}_{g,h}(t) \leq X \leq \overline{X}_{g,l}(t)$; that is, the ICCs of both firm types [see (3.3) and (3.6)] are satisfied. Note that in this paper, we assume that the firm's investment and financing choice is static at given time t^4. The results of the least-cost separating equilibrium and the belief system of the separating equilibrium are summarized as follows:

Proposition 2. *There exists a unique least-cost separating equilibrium under EGS at time t when $\overline{X}_{g,h}(t) < \overline{X}_{g,l}(t)$, where the high-type firm enters into the EGS agreement and invests at the threshold $\min(\overline{X}_{g,l}(t), X_g^*(t; \lambda_h))$ and the low-type firm invests at its first-best threshold $X_g^*(t; \lambda_l)$. The separating equilibrium is sustained under the belief system:*

$$\Lambda(X_{inv}) = \begin{cases} \lambda_l, & \text{if } X_{inv} > \min(\overline{X}_{g,l}(t), X_g^*(t; \lambda_h)) \\ \lambda_h, & \text{otherwise} \end{cases}, \tag{3.7}$$

where X_{inv} is the investment threshold.

The proof of Proposition 2 can be established by following a similar approach as discussed in Wang and Kwok (2019). The separating equilibrium under the EGS agreement holds under the pessimistic belief system of the guarantor. At time t, the high-type firm invests after the revenue shock variable X reaches $\overline{X}_{g,h}(t)$ but before it exceeds $\overline{X}_{g,l}(t)$. The guarantor recognizes the firm as the high-type and takes the lower share of equity $\phi(X; \lambda_h)$, whose positive effect dominates the information cost due to early investment. By speeding up its investment, the high-type firm imposes a mimicking cost to the low-type firm, which is high enough for the low-type firm to choose its first-best threshold $X_g^*(t; \lambda_l)$ and offer higher share of equity to the guarantor. We illustrate the relative positions of the investment thresholds and strategies of both firm types under the separating equilibrium in Figure 2.

---

[4]In other words, the firm's choice at current time is not affected by its preference in the future. As a result, the firm cannot choose to postpone its decision at later time. Though the dynamic optimization of the firm's decision problem can be done through backward induction under the discrete time framework, it becomes non-tractable under the continuous time framework in this paper.
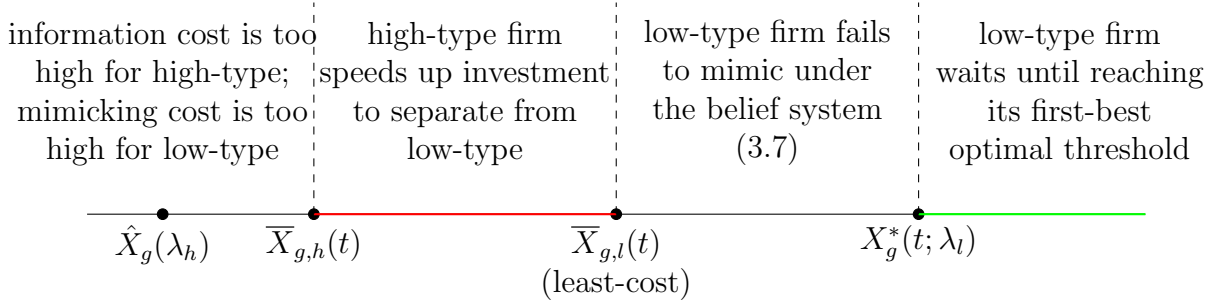
Figure 2: Relative positions of the investment thresholds and behaviors of both firm types under the separating equilibrium.

We define the real option value function of the high-type firm under the least-cost separating equilibrium before investment by $V_{g,h}^s(X,t)$. In the least-cost separating equilibrium, the high-type firm invests at the minimum value between the two following thresholds: the binding threshold $\overline{X}_{g,l}(t)$ of the ICC of the low-type firm and the first-best threshold $X_g^*(t;\lambda_h)$ of the high-type firm under complete information. When $X_g^*(t;\lambda_h) \leq \overline{X}_{g,l}(t)$ at time t, the real-option value $V_{g,h}^s(X,t)$ of the high-type firm takes the same value as the case of complete information. When $X_g^*(t;\lambda_h) > \overline{X}_{g,l}(t)$, the high-type firm may speed up its investment in order to separate from the low-type. Therefore, $V_{g,h}^s(X,t)$ is not always dictated by the optimal stopping rule. In fact, $V_{g,h}^s(X,t)$ can be defined as the real option value with intrinsic value $I_g^c(X;\lambda_h)$ and investment threshold $\min(\overline{X}_{g,l}(t), X_g^*(t;\lambda_h))$. This resembles an up-barrier call option defined on the domain $[0, \min(\overline{X}_{g,l}(t), X_g^*(t;\lambda_h))]$. The percentage drop of the real option value function of the high-type firm in the least-cost separating equilibrium is defined as the (relative) information cost due to investment distortion, namely,

$$C_g^s(X,t) = \frac{V_g^c(X,t;\lambda_h) - V_{g,h}^s(X,t)}{V_g^c(X,t;\lambda_h)}. \tag{3.8}$$

3.3 Separating equilibrium through direct bank loan

According to Section 2.3.3, the EGS dominates the direct bank loan under complete information due to lower bankruptcy cost. However, it may not be the case under information asymmetry since EGS may cause higher information cost for the high-type firm and mimicking cost for the low-type firm. Therefore, it becomes plausible that the high-type firm may be able to separate from the low-type by adopting the direct bank loan. It is seen that under the EGS, the low-type firm may benefit from mimicking to achieve the lower proportional share of equity. Under the direct bank loan, by mimicking the high-type firm, the low-type firm manages to lower the coupon rate. If investing at sufficiently low threshold, the bank would misinterpret the low-type firm as being high-type with $\mathbb{P}[\Lambda = \lambda_h] = 1$. According to Morellec and Schürhoff (2011), the equity value of the low-type firm upon investment is

given by

$$E_b(X; \lambda_l, c_{b,h}) = \lambda_l \Pi(X) - F - \frac{c_{b,h}}{r}$$

$$+ \left[F + \frac{c_{b,h}}{r} - \lambda_l \Pi(\underline{X}_b(\lambda_l, c_{b,h})) \right] \left[\frac{X}{\underline{X}_b(\lambda_l, c_{b,h})} \right]^\xi, \qquad (3.9)$$

where the default threshold is

$$\underline{X}_b(\lambda_l, c_{b,h}) = \frac{\xi}{\xi - 1} \frac{r - \mu}{\lambda_l} \left(F + \frac{c_{b,h}}{r} \right).$$

The coupon rate $c_{b,h}$ is determined by the budget constraint specified in (2.8) by setting $\lambda = \lambda_h$. At the revenue shock variable level X, the low-type firm prefers waiting until its first-best threshold under EGS over mimicking the high-type under the direct bank loan when the following incentive compatibility constraint holds:

$$E_b(X; \lambda_l, c_{b,h}) \leq V_g^c(X, t; \lambda_l). \qquad (3.10a)$$

On the other hand, separating equilibrium exists only when the following incentive compatibility constraint of the high-type firm is satisfied:

$$E_b(X; \lambda_h, c_{b,h}) \geq V_{g,h}^m(X, t). \qquad (3.10b)$$

Let $\overline{X}_{b,l}(t)$ and $\overline{X}_{b,h}(t)$ denote the binding thresholds under the incentive compatibility constraints (3.10a) and (3.10b), respectively. Given the current revenue shock variable level X, the low-type firm would choose not to mimic the high-type through the direct bank loan when its corresponding equity value does not exceed its real option value in the benchmark case of EGS under complete information [shown by (3.10a)]. On the other hand, the high-type firm prefers to separate from the low-type by choosing the direct bank loan when its corresponding equity value is larger than its real option value under the mimicking strategy through EGS [shown by (3.10b)].

As a summary, the low-type firm would choose to wait until its first-best investment threshold $X_b^*(t; \lambda_l)$ only when the current revenue shock variable X is below $\overline{X}_{b,l}(t)$, where the negative effect of investment distortion dominates the positive effect of coupon reduction for the low-type firm. On the other hand, the high-type firm is willing to separate from low-type by imposing a mimicking cost for the low-type firm only when X stays above $\overline{X}_{b,h}(t)$. There is an information cost [see (3.8)] borne by the high-type firm in speeding up its investment in order to separate from the low-type firm.

We summarize the separating equilibrium strategies of both firm types under the direct bank loan in Proposition 3.

Proposition 3. *There exists a unique least-cost separating equilibrium under the direct bank loan at time t whenever $\overline{X}_{b,h}(t) \leq \overline{X}_{b,l}(t)$, where the high-type firm under the bank loan arrangement invests at the threshold $\min(\overline{X}_{b,l}(t), X_b^*(t; \lambda_h))$ and the low-type firm invests at its first-best threshold $X_g^*(t; \lambda_l)$ under the EGS arrangement. The separating equilibrium is sustained under the belief system:*

$$\Lambda(X_{inv}) = \begin{cases} \lambda_l, & \text{if } X_{inv} > \min(\overline{X}_{b,l}(t), X_b^*(t; \lambda_h)) \\ \lambda_h, & \text{otherwise} \end{cases}, \qquad (3.11)$$

where X_{inv} is the investment threshold. We define $V_{b,h}^s(X,t)$ to be the real option value of the high-type firm with intrinsic value $E_b(X; \lambda_h, c_{b,h})$ and investment threshold $\min(\overline{X}_{b,l}(t), X_b^*(t; \lambda_h))$. The (relative) information cost of the high-type firm due to financing distortion is then defined as

$$C_b^s(X,t) = \frac{V_g^c(X,t;\lambda_h) - V_{b,h}^s(X,t)}{V_g^c(X,t;\lambda_h)}. \tag{3.12}$$

Proposition 3 holds under the pessimistic belief system of the bank. In the separating equilibrium, the bank perceives the firm as high-type when it invests at or earlier than the threshold $\min(\overline{X}_{b,l}(t), X_b^*(t; \lambda_h))$. Under the direct bank loan, the high-type firm has incentive to invest earlier since the resulting information cost of the high-type firm by investment distortion can be compensated by reduction of the coupon rate via signaling its quality type to the bank. On the other hand, by investing at the same threshold as the high-type, the low-type firm will be considered as high-type and benefits from paying lower coupon rate. However, it may find this mimicking strategy too costly due to significant investment distortion which offsets the positive effect of lower coupon rate through mimicking.

3.4 Separating equilibrium near maturity of investment opportunity

We investigate the separating equilibrium of the firm through investment timing when the investment opportunity is close to expiry. The first-best thresholds $X_g^*(T^-; \lambda)$ and $X_b^*(T^-; \lambda)$ can be found by solving the following algebraic equations:

$$\lambda X + \left[\frac{\sigma^2}{2}\xi(\xi-1) + \mu\xi - r\right]\eta_\alpha \lambda^\xi \Pi(X)^\xi (F+I)^{1-\xi} - r(F+I) = 0, \tag{3.13}$$

$$\frac{\eta + \eta_\alpha}{\eta_\alpha} \frac{\lambda X - r(F+I)}{c_b/r - I} = \frac{\frac{\sigma^2}{2}(1-\xi)\xi \left[\frac{r(F+I)}{c_b+rF}\right]^2}{\left[1 - (1-\xi)\frac{c_b-rI}{c_b+rF}\right]^3} - \frac{\mu\xi}{1 - (1-\xi)\frac{c_b-rI}{c_b+rF}} + r, \tag{3.14}$$

respectively. The derivation of the above algebraic equations is presented in Appendix B.

Similar to American options, when $t \to T^-$, the real option value of the EGS converges to the intrinsic value due to continuity. Under complete information, we have

$$\lim_{t \to T^-} V_g^c(X,t;\lambda) = I_g^c(X;\lambda). \tag{3.15}$$

At $t \to T^-$, the separating equilibrium exists only when the incentive compatibility constraint [(3.3)] of the low-type firm is violated, namely,

$$G_{g,l}(X, T^-) = I_{g,l}^m(X) - V_g^c(X, T^-; \lambda_l) = I_{g,l}^m(X) - I_g^c(X; \lambda_l) \leq 0. \tag{3.16}$$

It is seen from Appendix A that $I_{g,l}^m(X) \geq I_g^c(X; \lambda_l)$, so (3.16) holds only when $I_{g,l}^m(X) = I_g^c(X; \lambda_l) = 0$. The binding threshold $\overline{X}_{g,l}(T^-)$ of (3.16) is the revenue shock level $\hat{X}_{g,l} = \max(\hat{X}_g(\lambda_h), \underline{X}_g(\lambda_l))$, where $I_{g,l}^m(X)$ first decreases to zero. Proposition 2 states that $\overline{X}_{g,l}(T^-)$

equals the investment threshold of the high-type firm in the least-cost separating equilibrium. When the investment opportunity comes close to expiry, we deduce the following investment behavior of both firm types under the least-cost separating equilibrium:

1. When $\hat{\hat{X}}_{g,l} = \hat{X}_g(\lambda_h)$, (3.16) holds only for $X \leq \hat{X}_g(\lambda_h)$; that is, X falls below the zero intrinsic value threshold of the high-type firm. It implies that the high-type firm can only separate from the low-type by investing earlier than $\hat{X}_g(\lambda_h)$ and receives a nonpositive payoff. Consequently, the high-type firm would rather choose not to invest. Therefore, separating equilibrium does not exist in this case.

2. When $\hat{\hat{X}}_{g,l} = \underline{X}_g(\lambda_l)$, (3.16) holds whenever $X \leq \underline{X}_g(\lambda_l)$. The high-type firm invests at $\underline{X}_g(\lambda_l)$ in order to separate from the low-type. Since this investment threshold is higher than $\hat{X}_g(\lambda_h)$, the high-type firm earns a positive payoff. Therefore, there exists a least-cost separating equilibrium where the high-type firm invests at $\min(X_g^*(T^-; \lambda_h), \underline{X}_g(\lambda_l))$ and the low-type firm invests at its first-best threshold $X_g^*(T^-; \lambda_l)$.

4   Real signaling games under pooling equilibrium

When it is too costly for the high-type firm to separate from low-type, it may rather choose to pool with the low-type firm. In the pooling equilibrium, the firm of either quality type adopts the same investment timing and financing choice. The outsiders are unable to distinguish the true firm quality. As a result, the corresponding belief on Λ is given by

$$\Lambda = \lambda_p = p\lambda_h + (1-p)\lambda_l.$$

We start with the discussion of the ICCs and investment thresholds under the respective pooling equilibrium for the EGS agreement and direct bank loan. In particular, we analyze the pooling equilibrium when the investment opportunity comes closer to expiry.

4.1   Pooling equilibrium through EGS

Under pooling equilibrium, the guarantor in the EGS is unable to determine the exact firm type. For the guarantor, the expected liability $L_g^p(X)$ upon default is given by

$$L_g^p(X) = \mathbb{P}[\Lambda = \lambda_h]L_g(X; \lambda_h) + \mathbb{P}[\Lambda = \lambda_l]L_g(X; \lambda_l)$$

$$= p[F + I - (1-\alpha)\lambda_h\Pi(\underline{X}_g(\lambda_h))]\left[\frac{X}{\underline{X}_g(\lambda_h)}\right]^\xi$$

$$+ (1-p)[F + I - (1-\alpha)\lambda_l\Pi(\underline{X}_g(\lambda_l))]\left[\frac{X}{\underline{X}_g(\lambda_l)}\right]^\xi$$

$$= (\eta + \eta_\alpha)\lambda_{p,\xi}\Pi(X)^\xi (F + I)^{1-\xi},$$

where

$$\lambda_{p,\xi} = p\lambda_h^\xi + (1-p)\lambda_l^\xi.$$

As a fair deal, the guarantor demands the proportion share $\phi_p(X)$ of the equity value $E_g^p(X)$ of the firm as dictated by

$$\phi_p(X)E_g^p(X) = L_g^p(X). \tag{4.1}$$

The expected equity value of the firm right after investment is given by

$$E_g^p(X) = \mathbb{P}[\Lambda = \lambda_h]E_g(X; \lambda_h) + \mathbb{P}[\Lambda = \lambda_l]E_g(X; \lambda_l)$$

$$= p\left\{\lambda_h\Pi(X) - F - I + [F + I - \lambda_h\Pi(\underline{X}_g(\lambda_h))]\left[\frac{X}{\underline{X}_g(\lambda_h)}\right]^\xi\right\}$$

$$+ (1-p)\left\{\lambda_l\Pi(X) - F - I + [F + I - \lambda_l\Pi(\underline{X}_g(\lambda_l))]\left[\frac{X}{\underline{X}_g(\lambda_l)}\right]^\xi\right\}$$

$$= \lambda_p\Pi(X) - F - I + \eta\lambda_{p,\xi}\Pi(X)^\xi (F + I)^{1-\xi}.$$

Under pooling equilibrium, the EGS agreement specifies the same proportional share of equity for both firm types. To achieve a fair deal as dictated by (4.1), we obtain

$$\phi_p(X) = \frac{L_g^p(X)}{E_g^p(X)} = \frac{\eta + \eta_\alpha}{\lambda_{p,\xi}^{-1}\Pi(X)^{-\xi}(F + I)^{\xi-1}[\lambda_p\Pi(X) - F - I] + \eta}, \tag{4.2}$$

where the two coefficients η and η_α are defined in (2.16).

4.1.1 Incentive compatibility constraints

The pooling equilibrium exists only when the pooling strategies dominate the separating strategies for both firm types. The low-type firm prefers to pool with the high-type at time t when the following ICC holds:

$$I_{g,l}^p(X) \geq V_g^c(X, t; \lambda_l), \quad 0 \leq X \leq X_g^*(t; \lambda_l), \tag{4.3}$$

where the intrinsic value of the low-type firm under the pooling strategy is given by

$$I_{g,l}^p(X) = [1 - \phi_p(X)]E_g(X; \lambda_l) = \frac{E_g(X; \lambda_l)}{E_g^p(X)}[E_g^p(X) - L_g^p(X)].$$

Let $\overline{X}_{g,l}^p(t)$ denote the corresponding binding threshold such that equality holds for ineq. (4.3). When the ICC (4.3) is satisfied, the low-type firm prefers to lower its investment threshold rather than wait until its first-best threshold corresponding to the separating equilibrium. Non-negativity of $I_{g,l}^p(X)$ requires the share of equity $\phi_p(X) \in [0, 1]$ and equity value $E_g(X; \lambda_l)$ to be nonnegative. The binding threshold $\hat{X}_{g,p}$ is determined by solving

$$E_g^p(\hat{X}_{g,p}) - L_g^p(\hat{X}_{g,p}) = 0.$$

At $\hat{X}_{g,p}$, the expected equity value and liability are the same and the share of equity demanded by the guarantor is zero. Similar to the discussion on the domain of definition of $I_{g,l}^m(X)$ [see (3.2)], the intrinsic value $I_{g,l}^p(X)$ stays positive when X is above the lower bound

$\hat{\underline{X}}_{g,p} = \max(\hat{X}_{g,p}, \underline{X}_g(\lambda_l))$, and it is set to be zero when X is below $\hat{\underline{X}}_{g,p}$. The relations among the thresholds and satisfaction of the ICC of the low-type firm under pooling equilibrium are depicted in Figure 3.
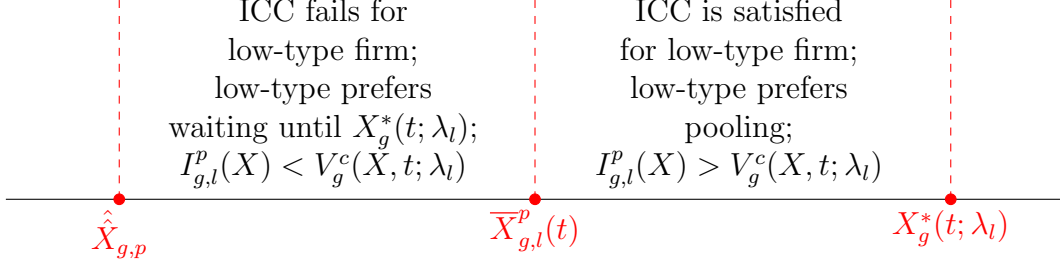
ICC fails for
low-type firm;
low-type prefers
waiting until $X_g^*(t; \lambda_l)$;
$I_{g,l}^p(X) < V_g^c(X, t; \lambda_l)$

ICC is satisfied
for low-type firm;
low-type prefers
pooling;
$I_{g,l}^p(X) > V_g^c(X, t; \lambda_l)$

$\hat{\underline{X}}_{g,p}$ $\overline{X}_{g,l}^p(t)$ $X_g^*(t; \lambda_l)$

Figure 3: Relations among the thresholds and satisfaction of the ICC of the low-type firm under pooling equilibrium.

4.1.2 Pooling equilibrium strategies

When the pooling strategy dominates the least-cost separating strategy, the high-type firm prefers pooling equilibrium to separating equilibrium. As stated in Proposition 2, the high-type firm chooses to invest at the threshold $\min(\overline{X}_{g,l}(t), X_g^*(t; \lambda_h))$ under the least-cost separating equilibrium. When $X_g^*(t; \lambda_h) \leq \overline{X}_{g,l}(t)$ at time t, the high-type firm always prefers investing at $X_g^*(t; \lambda_h)$ under the pooling strategy due to optimality of the first-best threshold $X_g^*(t; \lambda_h)$. Therefore, the pooling equilibrium exists only when the binding threshold $\overline{X}_{g,l}(t)$ is below $X_g^*(t; \lambda_h)$; that is, $I_g^c(X_g^*(t; \lambda_h), t) > V_{g,h}^m(X_g^*(t; \lambda_h), t)$. The high-type firm prefers to adopt the pooling strategy when the following ICC holds at the threshold $\overline{X}_{g,l}(t)$:

$$I_g^c(\overline{X}_{g,l}(t); \lambda_h) \leq V_{g,h}^p(\overline{X}_{g,l}(t), t), \tag{4.4}$$

where the left-hand side is the intrinsic value of the high-type firm under the separating strategy [see (2.17)]. Here, the right-hand side is the real option value of the high-type firm under the pooling strategy with optimal threshold $X_g^{p*}(t)$ and intrinsic value as given by

$$I_{g,h}^p(X) = [1 - \phi_p(X)]E_g(X; \lambda_h) = \frac{E_g(X; \lambda_h)}{E_g^p(X)}[E_g^p(X) - L_g^p(X)]. \tag{4.5}$$

Let $\overline{X}_{g,h}^p(t)$ denote the binding threshold at which equality holds for ineq. (4.4). The ICC depicted in (4.4) shows the choice of the high-type firm between the pooling and separating strategies. When the current revenue shock variable X reaches the separating threshold $\overline{X}_{g,l}(t)$, the high-type firm finds it too costly to separate from the low-type and prefers to wait until the optimal investment threshold $X_g^{p*}(t)$ of the pooling strategy. However, if the high-type firm waits too long such that the revenue shock variable X exceeds the optimal threshold $X_g^{p*}(t)$, the guarantor may misinterpret it as a low-type and request for higher share of equity. This implies that the high-type firm maximizes its value when it invests exactly at the threshold $X_g^{p*}(t)$, which constitutes a Pareto-dominant equilibrium.

We summarize the nature and belief system of the pooling equilibrium in Proposition 4.

Proposition 4. *There exists a Pareto-dominant pooling equilibrium at time t when $\overline{X}_{g,l}(t) < X_g^*(t; \lambda_h)$ and the ICCs in (4.3) and (4.4) hold. In the pooling equilibrium, the firm of either quality type chooses to enter into the EGS agreement and invest when the revenue shock variable X reaches the optimal threshold $X_g^{p*}(t)$ of the high-type firm. The pooling equilibrium can be sustained under the belief system*

$$
\Lambda(X_{inv}) = \begin{cases} \lambda_h, & \text{if } X_{inv} \leq \overline{X}_{g,l}(t) \\ \lambda_p, & \text{if } \overline{X}_{g,l}(t) < X_{inv} \leq X_g^{p*}(t) \ , \\ \lambda_l, & \text{otherwise} \end{cases} \tag{4.6}
$$

where X_{inv} is the investment threshold.

The pooling equilibrium stated in Proposition 4 should observe Pareto-dominance, where the high-type firm chooses the investment threshold in order to maximize its payoff. The optimal threshold $X_g^{p*}(t)$ is the highest value in the interval $(\overline{X}_{g,l}(t), X_g^{p*}(t)]$, where the guarantor cannot distinguish the firm type and the belief system is $\mathbb{P}[\Lambda = \lambda_h] = p$. Therefore, the high-type firm has no incentive to deviate from the equilibrium. On the other hand, the low-type firm also maximizes its payoff through investing at the same threshold $X_g^{p*}(t)$. It cannot deviate to its optimal pooling threshold $X_{g,l}^{p*}(t) > X_g^{p*}(t)$ since it falls into the region where the guarantor's belief system is $\mathbb{P}[\Lambda = \lambda_h] = 0$. Moreover, provided that ineqs. (4.3) and (4.4) are satisfied, the pooling equilibrium also dominates the separating equilibrium. Under Pareto-dominance, the guarantor only demands the lower share of equity from the low-type firm. This yields higher value of the low-type firm when compared with that under complete information, which dominates the mimicking cost of the low-type firm arising from investment distortion. Compared to the separating equilibrium, the high-type firm also achieves a higher payoff by pooling with the low-type by lowering the negative effect of investment distortion, though it may be charged with higher share of equity by the guarantor. The relative positions of the investment thresholds and investment strategies of both firm types under the pooling equilibrium are illustrated in Figure 4.
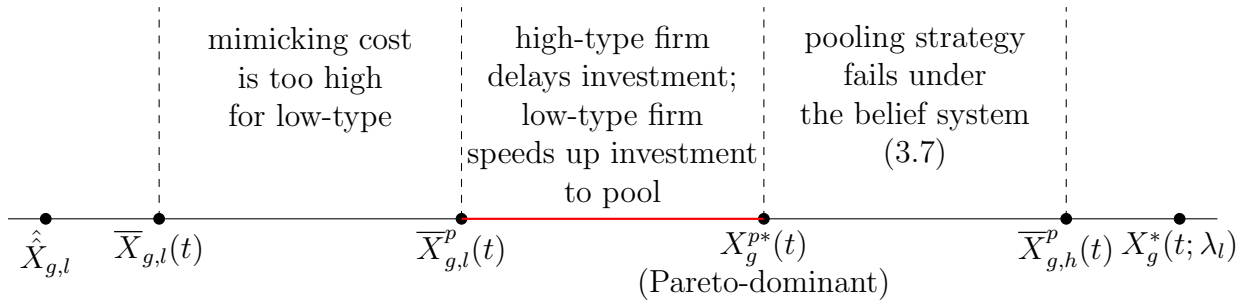

Figure 4: Relative positions of the investment thresholds and behaviors of both firm types under separating equilibrium.

The optimal investment threshold $X_g^{p*}(t)$ under the pooling equilibrium strategy lies between the first-best thresholds $X_g^*(t; \lambda_h)$ and $X_g^*(t; \lambda_l)$. This implies that under the pooling equilibrium, the low-type firm invests more aggressively while the high-type firm delays its investment when compared with that under complete information. The high-type firm

21

therefore incurs an information cost due to this form of investment distortion. The level of investment distortion can be quantified by the (relative) information cost of the high-type firm under pooling equilibrium through the EGS agreement, which is defined by

$$C_g^p(X, t) = \frac{V_g^c(X, t; \lambda_h) - V_{g,h}^p(X, t)}{V_g^c(X, t; \lambda_h)}. \tag{4.7}$$

4.2 Pooling equilibrium under direct bank loan

When the high-type firm fails to separate from the low-type under the direct bank loan due to high information cost, it may resort to the pooling strategy. Since the bank cannot differentiate the type of the firm, the bank charges the same coupon rate $c_{b,p} \in (c_{b,h}, c_{b,l})$ as determined by the following budget constraint:

$$\frac{c_{b,p}}{r} - L_b^p(X; c_{b,p}) = I. \tag{4.8}$$

Here, the bank evaluates the default risk of the firm as quantified by the following expected liability value

$$
\begin{aligned}
L_b^p(X; c_{b,p}) &= \mathbb{P}[\Lambda = \lambda_h] L_b(X; \lambda_h, c_{b,p}) + \mathbb{P}[\Lambda = \lambda_l] L_b(X; \lambda_l, c_{b,p}) \\
&= p \left[F + \frac{c_{b,p}}{r} - (1 - \alpha)\lambda_h \Pi(\underline{X}_b(\lambda_h, c_{b,p})) \right] \left[\frac{X}{\underline{X}_b(\lambda_h, c_{b,p})} \right]^\xi \\
&\quad + (1 - p) \left[F + \frac{c_{b,p}}{r} - (1 - \alpha)\lambda_l \Pi(\underline{X}_b(\lambda_l, c_{b,p})) \right] \left[\frac{X}{\underline{X}_b(\lambda_l, c_{b,p})} \right]^\xi \\
&= (\eta + \eta_\alpha)\lambda_{p,\xi} \Pi(X)^\xi \left(F + \frac{c_{b,p}}{r} \right)^{1-\xi}.
\end{aligned}
$$

The characterization of the pooling equilibrium for the direct bank loan is different from that of the EGS since the common coupon rate $c_{b,p}$ is dependent on the expected liability value $L_b^p(X; c_{b,p})$. However, it is still feasible to calculate the expected liability value in eq. (4.8) since $c_{b,p}$ is independent of X.

Under the same coupon rate $c_{b,p}$, the equity values of the high-type and low-type firm right after investment are given by

$$
\begin{aligned}
E_b(X; \lambda_h, c_{b,p}) &= \lambda_h \Pi(X) - F - \frac{c_{b,p}}{r} \\
&\quad + \left[F + \frac{c_{b,p}}{r} - \lambda_h \Pi(\underline{X}_b(\lambda_h, c_{b,p})) \right] \left[\frac{X}{\underline{X}_b(\lambda_h, c_{b,p})} \right]^\xi
\end{aligned}
$$

and

$$
\begin{aligned}
E_b(X; \lambda_l, c_{b,p}) &= \lambda_l \Pi(X) - F - \frac{c_{b,p}}{r} \\
&\quad + \left[F + \frac{c_{b,p}}{r} - \lambda_l \Pi(\underline{X}_b(\lambda_l, c_{b,p})) \right] \left[\frac{X}{\underline{X}_b(\lambda_l, c_{b,p})} \right]^\xi,
\end{aligned}
$$

respectively. The low-type firm prefers pooling with the high-type at time t provided that the following ICC is satisfied:

$$E_b(X; \lambda_l, c_{b,p}) \geq V_g^c(X, t; \lambda_l), \quad \underline{X}_b(\lambda_l, c_{b,p}) \leq X \leq X_g^*(t; \lambda_l). \qquad (4.9)$$

By adopting the financing choice of the direct bank loan and the same investment timing as the high-type, the low-type firm is charged by the bank at the common coupon rate $c_{b,p}$, which is lower than the coupon rate $c_{b,l}$ charged under complete information. The low-type firm prefers the pooling strategy since its corresponding equity value exceeds the real option value of waiting until its first best threshold $X_g^*(t; \lambda_l)$ under the benchmark case of EGS.

Similar to Section 4.1, the high-type firm always prefers separating when $X_b^*(t; \lambda_h) \leq \overline{X}_{b,l}(t)$ and it may choose to invest at its first-best optimal threshold. Therefore, pooling equilibrium under direct bank loan exists only when $\overline{X}_{b,l}(t) < X_b^*(t; \lambda_h)$. At the separating threshold $\overline{X}_{b,l}(t)$ of the high-type firm, the ICC of the high-type firm is given by

$$E_b(\overline{X}_{b,l}(t); \lambda_h, c_{b,h}) \leq V_{b,h}^p(\overline{X}_{b,l}(t), t), \qquad (4.10)$$

where the real option value $V_{b,h}^p(X, t)$ of the high-type firm under the pooling strategy is governed by

$$V_{b,h}^p(X, t) = \sup_{t \leq u \leq T} \mathbb{E}_t[e^{-r(u-t)} E_b(X_u; \lambda_h, c_{b,p}) \mid X_t = X], \quad 0 \leq X \leq X_b^{p*}(t). \qquad (4.11)$$

Here, $X_b^{p*}(t)$ is defined as the optimal threshold of the high-type firm under the pooling strategy. The high-type firm chooses not to adopt the separating strategy at the threshold $\overline{X}_{b,l}(t)$ when (4.10) holds since the real option value of waiting until the optimal pooling threshold $X_b^{p*}(t)$ is higher than the equity value of investing immediately, though it can separate from the low-type by doing so.

The pooling equilibrium under the direct bank loan is summarized as follows:

Proposition 5. *There exists a Pareto-dominant pooling equilibrium under the direct bank loan at time t when $\overline{X}_{b,l}(t) < X_b^*(t; \lambda_h)$ and (4.9) and (4.10) hold. In the pooling equilibrium, both firm types choose to invest when the revenue shock variable X reaches the optimal pooling threshold $X_b^{p*}(t)$. The bank charges the common coupon rate $c_{b,p}$ [see (4.8)] for both firm types. The (relative) information cost of the high-type firm under the pooling equilibrium through the direct bank loan is given by*

$$C_b^p(X, t) = \frac{V_g^c(X, t; \lambda_h) - V_{b,h}^p(X, t)}{V_g^c(X, t; \lambda_h)}. \qquad (4.12)$$

As a summary, when pooling equilibrium prevails, the low-type firm pays a lower coupon rate from the bank, which results in higher equity value since this dominates the mimicking cost due to investment distortion. On the other hand, the high-type firm also adopts the pooling equilibrium since it chooses to delay its investment further in order to avoid facing the high cost of investment distortion when separating strategies are adopted, at the cost of being charged at a higher coupon rate compared to that under complete information.

4.3 Pooling equilibrium near maturity of investment opportunity under EGS agreement

We consider the pooling strategy of the firm under the EGS agreement when the investment opportunity is going to expire very soon. Similar to Section 3.4, by virtue of continuity of the real option value function $V_{g,h}^p(X,t)$ with respect to t, the asymptotic value of $V_{g,h}^p(X,t)$ when $t \to T^-$ is given by

$$V_{g,h}^p(X,T^-) = I_{g,h}^p(X). \tag{4.13}$$

To examine the pooling strategy of the firm near maturity of the investment opportunity, we notice that the ICC of the low-type firm [see (4.3)] always holds for $t \to T^-$ since $I_{g,l}^p(X) \geq I_g^c(X;\lambda_l) = V_g^c(X,T^-;\lambda_l)$ is always true. We deduce that the low-type firm always prefers to pool with the high-type to achieve higher equity value when the investment opportunity expires soon. Under $\overline{X}_{g,l}(T^-) < X_g^*(T^-;\lambda_h)$, it remains to check the satisfaction of the ICC of the high-type firm at $t \to T^-$, namely,

$$I_g^c(\overline{X}_{g,l}(T^-);\lambda_h) \leq I_{g,h}^p(\overline{X}_{g,l}(T^-)). \tag{4.14}$$

The high-type firm chooses whether to invest or not when the current revenue shock variable X reaches the separating threshold near maturity. Recall from Section 3.4 that $\overline{X}_{g,l}(T^-) = \hat{\hat{X}}_{g,l} = \max(\hat{X}_g(\lambda_h), \underline{X}_g(\lambda_l))$. We then determine the pooling equilibrium near maturity by considering the following two cases:

1. When $\hat{X}_g(\lambda_h) \geq \underline{X}_g(\lambda_l)$, the separating equilibrium does not exist since the binding threshold $\overline{X}_{g,l}(T^-)$ for the separating equilibrium equals the zero-NPV threshold $\hat{X}_g(\lambda_h)$ and the high-type firm chooses not to invest. After the revenue flow shock variable X exceeds $\hat{X}_g(\lambda_h)$, the guarantor's belief system becomes $\mathbb{P}[\lambda = \lambda_p]$. Then the high-type firm will rather wait until the revenue flow shock variable reaches the optimal pooling threshold $X_g^{p*}(T^-)$. Therefore, there exists a Pareto-dominant pooling equilibrium where the firm chooses to invest when X reaches the threshold $X_g^{p*}(T^-)$ regardless of its type.

2. When $\hat{X}_g(\lambda_h) < \underline{X}_g(\lambda_l)$, the separating threshold of the high-type firm is $\overline{X}_{g,l}(T^-) = \underline{X}_g(\lambda_l)$, where $I_g^c(\overline{X}_{g,l}(T^-);\lambda_h)$ is positive and larger than $I_{g,h}^p(\overline{X}_{g,l}(T^-))$. Then the high-type firm chooses to invest whenever the revenue flow shock variable X reaches $\underline{X}_g(\lambda_l)$, where it is still perceived as high-type. In this case, a pooling equilibrium fails to exist.

In summary, Pareto-dominant pooling equilibrium prevails if and only if $\hat{X}_g(\lambda_h) \geq \underline{X}_g(\lambda_l)$. Under such condition, separating equilibrium does not exist and the high-type firm prefers to adopt the pooling strategy and waits until the optimal pooling threshold $X_g^{p*}(T^-)$ to invest.

5 Numerical studies on investment thresholds and information costs

In this section, we present the numerical studies on the investment thresholds and information costs of either firm type under both separating and pooling equilibriums. We focus on the time dependence of the firm's separating and pooling strategies, emphasizing the firm's financing choice between the EGS agreement and direct bank loan by comparing the investment thresholds and information costs. To compute the value functions and optimal thresholds in the associated stopping models, the numerical calculations are based on the fully implicit finite difference scheme coupled with the Projected Successive-Over-Relaxation method (Kwok, 2008; Wang and Kwok, 2019). The base parameters for the numerical plots are set to be $r = 5\%$, $\mu = 1\%$, $\sigma = 25\%$, $\lambda_h = 1.25$, $\lambda_l = 0.8$, $F = 200$, $I = 100$, $T = 5$ and $p = 0.5$.

5.1 Separating equilibrium

Figures 5 and 6 plot the investment thresholds of the high-type firm in the least-cost separating equilibrium with respect to time $t \in [0, T]$ at three levels of λ_l. The high-type firm chooses to invest at the minimum among its first-best optimal threshold and binding threshold in the separating equilibrium. The plots show that the first-best and binding thresholds of the firm are decreasing as time gets closer to maturity of investment opportunity. This implies that the low-type firm's incentive of mimicking increases with time, which provides stronger incentive for the high-type firm to invest earlier. The reason is that the firm's value of the option to invest decreases and the firm tends to invest earlier when the remaining life span of the investment opportunity becomes smaller. We observe a concave function of the first-best threshold and a convex function of the binding threshold with respect to time. By virtue of these properties, Figure 5(a) shows two intersection points of the first-best and the binding thresholds. This implies that the high-type firm chooses its first-best threshold $X_g^*(t; \lambda_h)$ at the earlier time, changes to the binding threshold $\overline{X}_{g,l}(t)$ at the intermediate time and reverts to its first-best threshold when time is close to maturity. Intuitively, the first-best threshold is affected by the value of the firm's option to invest, which decreases at a slower rate in time when t is sufficiently far from maturity T but the rate of decrease rises sharply when t is close to the maturity date. On the other hand, the binding threshold of the high-type firm in the separating equilibrium is affected by the low-type firm's incentive of mimicking, which reaches a level that is sufficiently high and does not change much when time is close to expiry of the investment opportunity. By examining Figures $5(a) - (c)$ and Figures $6(a) - (c)$ for $\lambda_l = 0.4$, 0.5 and 0.8, we find that the binding threshold increases with decreasing λ_l. The numerical plots are consistent with the financial intuition that the low-type firm finds it more costly to mimic the high-type for a smaller multiplier λ_l. Correspondingly, the high-type firm is allowed to invest optimally at its first-best threshold in order to separate from the low-type.
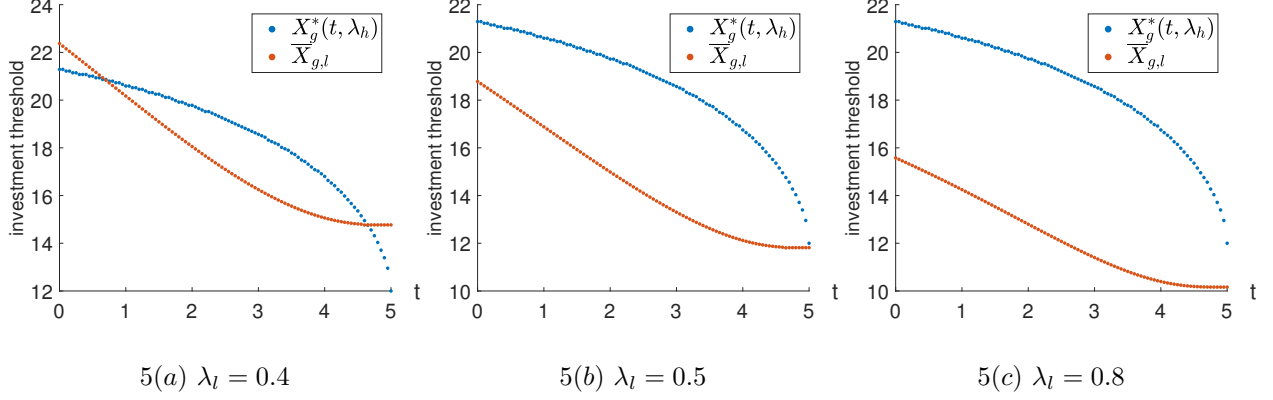
5(a) $\lambda_l = 0.4$ 5(b) $\lambda_l = 0.5$ 5(c) $\lambda_l = 0.8$

Figure 5: The binding threshold $\overline{X}_{g,l}(t)$ and first-best investment threshold $X_g^*(t; \lambda_h)$ are plotted against time t at three values of λ_l under separating equilibrium.
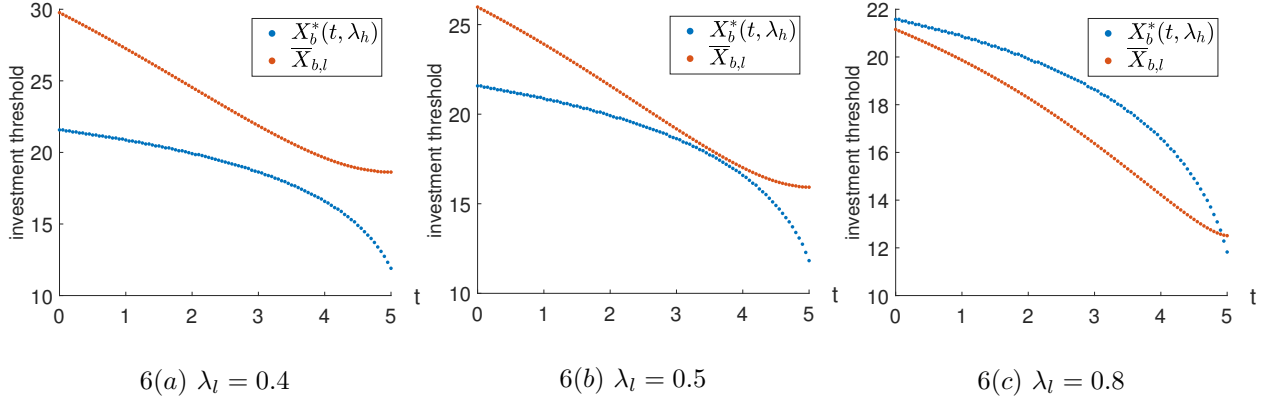

6(a) $\lambda_l = 0.4$ 6(b) $\lambda_l = 0.5$ 6(c) $\lambda_l = 0.8$

Figure 6: The binding threshold $\overline{X}_{b,l}(t)$ and first-best investment threshold $X_b^*(t; \lambda_h)$ are plotted against time t at three values of λ_l under separating equilibrium.

Figure 7 plots the information costs (in percents) of the high-type firm under the least-cost separating equilibrium against time $t \in [0, T]$ at three different levels of λ_l. The plots reveal strong time dependence of the firm's information costs under separating equilibrium. Correspondingly, the financing choice of the firm between the EGS and direct bank loan may change as time evolves. The information costs under both financing choices are calculated at the zero-NPV threshold of the high-type firm $\hat{X}_g(\lambda_h) = 10.16$ under the EGS arrangement. Figures 7(a, b) show that at low values of the multiplier λ_l, the information cost of the high-type firm under EGS first increases until reaching its maximum at intermediate time, and then decreases for time close to maturity. This property is dictated by the gap between the binding and first-best thresholds of the high-type firm under EGS, which becomes larger at longer time to expiry and narrower when time is closer to maturity. It implies that the high-type firm is more aggressive to separate from the low-type at intermediate time but becomes less aggressive at earlier time and close to maturity. In other cases, the information cost increases as time proceeds and drops to zero at maturity T. We can deduce from the plots that the high-type firm prefers to separate from low-type through the EGS when the multiplier λ_l is low, while separate through the direct bank loan when λ_l is high. At intermediate level of λ_l, the high-type firm prefers to separate through the direct bank loan

26

when time is far from maturity and changes to EGS when time is near maturity. This is because when λ_l is low enough, it becomes more difficult for the low-type firm to distort its investment strategy. Also, the effect of investment distortion on the high-type firm lessens at some time before maturity so that the high-type firm becomes easier to separate through investment timing.
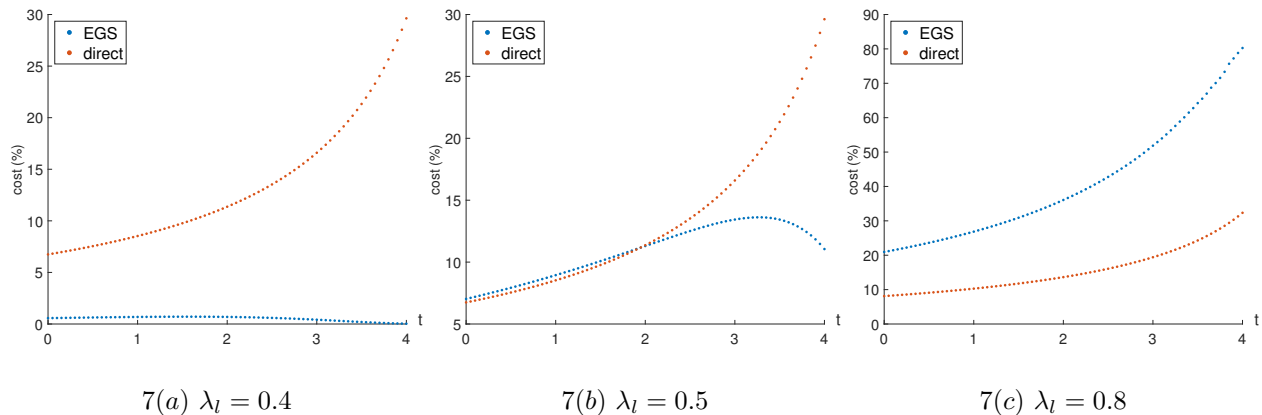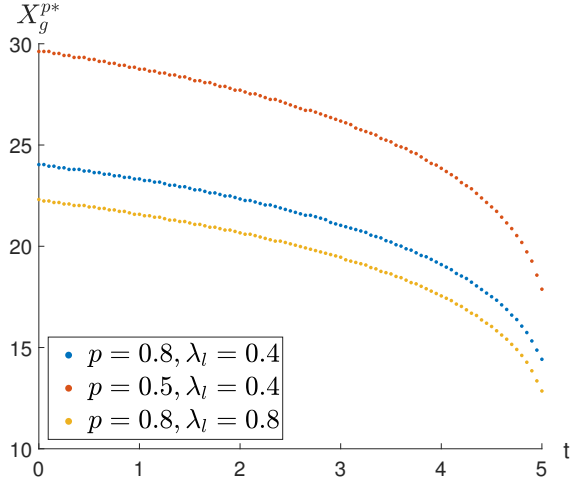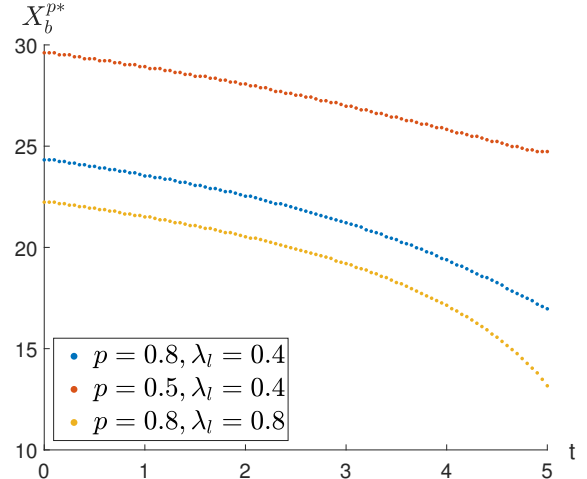

7(a) $\lambda_l = 0.4$ 7(b) $\lambda_l = 0.5$ 7(c) $\lambda_l = 0.8$

Figure 7: The information costs (in percents) in the least-cost separating equilibrium are plotted against time t at three values of λ_l for financing choices under the EGS and direct bank loan.

5.2   Pooling equilibrium

Figures 8 and 9 plot the investment thresholds and information costs (in percents) of the high-type firm under the pooling equilibrium with respect to time under different sets of parameter values: (i) $p = 0.8$, $\lambda_l = 0.4$, (ii) $p = 0.5$, $\lambda_l = 0.4$ and (iii) $p = 0.8$, $\lambda_l = 0.8$. The two financing choices of EGS and direct bank loan are plotted separately. Figures 8 and 9 show that for higher probability p, the optimal threshold of the high-type firm under the pooling strategy decreases in value and the information cost becomes lower; while for lower multiplier λ_l, both the optimal threshold and the information cost become higher. Similar to Figure 5, the information cost of the high-type firm under pooling strategy increases with time and finally drops to zero at maturity T.
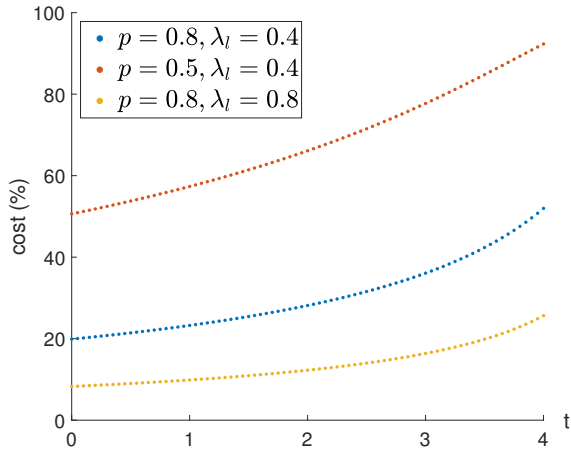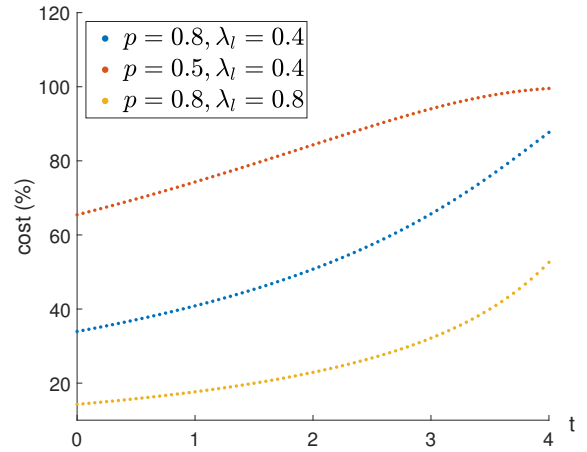
8(a) EGS

8(b) direct bank loan

Figure 8: The optimal thresholds $X_g^{p*}(\lambda_h)$ and $X_b^{p*}(\lambda_h)$ are plotted against time t for (a) EGS, (b) direct bank loan under pooling equilibrium with three sets of parameter values of λ_l and p.

9(a) EGS

9(b) direct bank loan

Figure 9: The information costs (in percents) of the high-type firm under the Pareto-dominant pooling equilibrium are plotted for (a) EGS, (b) direct bank loan against time t at three sets of parameter values of λ_l and p.

Table 1 lists the information costs (in percents) of the high-type firm under EGS separating, EGS pooling, direct bank loan separating and direct bank loan pooling with respect to the multiplier λ_l and two maturity dates. Recall that we assume the firm's investment and financing choice to be static at given time t. Thus the firm chooses the investment and financing strategy that maximizes its real option value (or equivalently, minimizes its information cost) at current time $t = 0$. The plots show that the high-type firm chooses to separate through EGS when λ_l is low since the gap between the revenue flows of the two firm types is large and the high-type firm finds it less costly to separate by speeding up

28

its investment. In particular, the high-type firm invests at its first-best threshold so that the corresponding information cost is zero. When the multiplier λ_l is high and close to λ_h, the high-type firm resorts to the pooling strategy through EGS since the information cost of separating is large. At intermediate level of λ_l, the high-type firm chooses to separate through the direct bank loan. Table 1 also shows the effect of the maturity date of the investment opportunity. At intermediate value of the multiplier λ_l ($\lambda_l = 0.5$ for instance), the high-type firm would choose to separate through the financing choice of the direct bank loan for shorter maturity ($T = 5$) but separate through the EGS for longer maturity ($T = 10$). The result is consistent with that showed in Figure 7(b).

Table 1: The information costs (in percents) of the high-type firm under EGS separating, EGS pooling, direct bank loan separating and direct bank loan pooling are listed against the multiplier λ_l with two choices of maturity.

	$T = 5$				$T = 10$			
λ_l	EGS_s	EGS_p	loan_s	loan_p	EGS_s	EGS_p	loan_s	loan_p
0.125	0	78.67	6.68	99.08	0	58.31	3.07	88.20
0.250	0	64.55	6.68	87.83	0	43.77	3.07	61.16
0.375	0.12	52.75	6.66	69.09	0.03	33.77	3.06	41.31
0.500	7.14	42.42	6.67	51.93	2.38	26.04	3.07	28.69
0.625	18.80	33.22	6.94	38.62	6.68	19.75	3.15	20.43
0.750	21.36	25.00	7.76	28.62	8.21	14.49	3.42	14.73
0.875	19.28	17.61	8.34	20.98	8.05	10.01	3.63	10.57
1.000	14.47	11.06	7.85	15.15	6.55	6.18	3.51	7.46
1.125	7.99	5.19	6.68	10.46	3.90	2.87	3.08	5.01

In Table 2, we show the effect of probability p in the belief system and fractional bankruptcy loss α on information costs of the high-type firm under different strategies. It suggests that the high-type firm would choose to separate from the low-type for low probability p since it is more likely to be perceived as low-type under the pooling strategy. The firm would instead choose to adopt the pooling strategy for high probability p. The EGS agreement is shown to be more sensitive to the level of probability compared with the direct bank loan. Considering the pooling equilibrium, the firm prefers to pool through the direct bank loan for low probability p while through EGS for high probability p. Table 2 also suggests that the high-type firm prefers separating for low bankruptcy cost while pooling for high bankruptcy cost, since pooling with the low-type will mitigate the loss upon default when the bankruptcy cost is too high. The information costs under direct bank loan are shown to be more sensitive to change to the fractional bankruptcy loss α. Under pooling equilibrium, the firm prefers the direct bank loan for low bankruptcy cost and adopt the financing choice of EGS for high bankruptcy cost.

Table 2: The information costs (in percents) of the high-type firm with varying values of probability p and fractional loss α due to bankruptcy using the loan arrangement of either EGS or direct bank loan under separating and pooling equilibriums.

p	EGS_s	EGS_p	$loan_s$	$loan_p$	α	EGS_s	EGS_p	$loan_s$	$loan_p$
0.1	20.95	42.34	8.11	39.18	0.05	18.69	21.24	3.32	17.38
0.2	20.95	36.98	8.11	35.76	0.10	19.29	21.42	4.47	19.35
0.3	20.95	31.89	8.11	32.43	0.15	19.86	21.59	5.66	21.33
0.4	20.95	26.85	8.11	28.93	0.20	20.41	21.76	6.87	23.31
0.5	20.95	21.92	8.11	25.29	0.25	20.95	21.92	8.11	25.29
0.6	20.95	17.19	8.11	21.65	0.30	21.46	22.07	9.38	27.26
0.7	20.95	12.66	8.11	18.00	0.35	21.98	22.30	10.68	29.35
0.8	20.95	8.28	8.11	14.27	0.40	22.46	22.44	12.02	31.30
0.9	20.95	4.05	8.11	10.50	0.45	22.94	22.57	13.36	33.23

6 Conclusion

In this paper, we have analyzed the real option signaling game models of debt financing of a risky project under information asymmetry, where the firm quality is private information to the firm manager but not outside investors. Our signaling game real option models extend other similar models in two aspects (i) finite time horizon of the life span of the investment opportunity instead of the usual perpetuity assumption, and (ii) new financing choice via the equity guarantee swap (EGS). The EGS arrangement involves the third party guarantor in the bank loan arrangement, which gains popularity in China in recent years. This is because the EGS arrangement facilitates the small- and medium-sized firms to secure bank loans in China. The guarantor takes partial share of equity value of the firm, in return to the guaranteed coupon and par payments to the bank upon default of the firm. The EGS thus exhibits the hybrid nature of equity and debt. The signals sent by the firm to outside investors involve investment timing and financial choice between the direct bank loan and EGS. Our analysis shows that the low-type firm may have an incentive to mimic the investment timing and financing choice of high-type in order to take advantage of the lower proportional share of equity in EGS and coupon rate in direct bank loan. Conversely, the high-type firm may have an incentive to separate from being perceived as low-type by speeding up investment and / or choosing different financing choice. Under the separating equilibrium, the high-type firm faces information costs due to investment distortion. On the other hand, suppose the high-type firm fails to separate due to high information costs, pooling equilibrium is resulted where both firm types invest at the same investment threshold and adopt the same financing choice. In this case, the outsiders cannot differentiate the firm type quality from the signal of investment threshold sent by the firm. We have performed characterization of the separating and pooling equilibriums under the EGS and direct bank loan. Also, we have examined the time dependent behaviors of the binding and optimal investment thresholds, in particular, the investment decisions at time close to expiry of investment opportunity under both separating and pooling equilibriums.

References

[1] Bouvard, M. (2014) Real option financing under asymmetric information, *Review of Financial Studies*, 27(1): 180-210.

[2] Brennan, M.J., Schwartz, E.S. (1985) Evaluating natural resource investments, *Journal of Business*, 58(2): 135-157.

[3] Clausen, S., Flor, C.R. (2015) The impact of assets-in-place on corporate financing and investment decision, *Journal of Banking and Finance*, 61: 64-80.

[4] Dixit, A., Pindyck, R. (1994) *Investment under Uncertainty*, Princeton University Press, Princeton, New Jersey.

[5] Grenadier, S.R., Malenko, A. (2011) Real option signaling games with applications to corporate finance, *Review of Financial Studies*, 24(12):3993-4036.

[6] Grenadier, S.R., Wang, N. (2005) Investment timing, agency and information, *Journal of Financial Economics*, 75: 493-533.

[7] Gryglewicz, S., Huisman, K.J.M., Kort, P.M. (2008) Finite project life and uncertainty effects on investment, *Journal of Economic Dynamics and Control*, 32: 2191-2213.

[8] Kwok, Y.K. (2008) *Mathematical Models of Financial Derivatives*, second edition, Springer.

[9] Leland, H.E. (1994) Corporate debt value, bond covenants and optimal capital structure, *Journal of Finance*, 49(4): 1213-1252.

[10] Leung, C.M., Kwok, Y.K. (2018) Real options signaling game models for dynamic acquisition under information asymmetry, *Decisions in Economics and Finance*, 41: 35-63.

[11] Luo, P.F., Wang, H.M., Yang, Z.J. (2016) Investment and financing for SMEs with a partial guarantee and jump risk, *European Journal of Operational Research*, 249: 1161-1168.

[12] McDonald, R., Siegel, D. (1986) The value of waiting to invest, *Quarterly Journal of Economics*, 101(4): 707-727.

[13] Morellec, E., Schürhoff, N. (2011) Corporate investment and financing under asymmetric information, *Journal of Financial Economics*, 99: 262-288.

[14] Myers, S.C. (1984) The capital structure puzzle, *Journal of Finance*, 39: 575-592.

[15] Myers, S.C., Majluf, N.S. (1984) Corporate financing and investment decisions when firms have information that investors do not have, *Journal of Financial Economics*, 13: 187-221.

[16] Nishihara, M. (2017) Selling out or going public? A real options signaling approach, *Finance Research Letters*, 22: 146-152.

[17] Nishihara, M., Shibata, T. (2018) Dynamic bankruptcy procedure with asymmetric information between insiders and outsiders, *Journal of Economic Dynamics and Control*, 90: 118-137.

[18] Shan, S.C., Tang, D.Y. (2019) Outsourcing bank loan screening: the economics of third-party loan guarantees, *Working paper of Shanghai University of Finance and Economics*.

[19] Shibata, T., Nishihara, M. (2015) Investment timing, debt structure, and financing constraints, *European Journal of Operational Research*, 241:513-526.

[20] Strebulaev, I.A., Zhu, H.X., Zryumov, P. (2014) Dynamic information asymmetry, financing, and investment decisions, *Working paper of Stanford University*.

[21] Wang, H.M., Yang, Z.J., Zhang, H. (2015) Entrepreneurial finance with equity-for-guarantee swap and idiosyncratic risk, *European Journal of Operational Research*, 241: 863-871.

[22] Wang, Q.Q., Kwok, Y.K. (2019) Signaling game models of equity financing under information asymmetry and finite project life, *International Journal of Financial Engineering*, 6(1): 1950002.

[23] Watanabe, T. (2018) Real options and signaling in strategic investment games, *Journal of the Operations Research Society of Japan*, 61(1):18-39.

[24] Xu, R.X., Li, S.H. (2010) Belief updating, debt pricing and financial decisions under asymmetric information, *Research in International Business and Finance*, 24(2): 123-137.

[25] Yang, M., Zeng Y. (2019) Financing entrepreneurial production: security design with flexible information acquisition, *Review of Financial Studies*, 32(3): 819-863.

Appendix A - Proof of Lemma 1

We would like to prove the existence of a root for $G_{g,l}(X,t) = 0$ [see (3.3)], where

$$G_{g,l}(X,t) = I_{g,l}^m(X) - V_g^c(X,t;\lambda_l), \quad \hat{\hat{X}}_{g,l} \le X \le X_g^*(t;\lambda_l).$$

Since $I_l^m(\hat{\hat{X}}_{g,l}) = 0$, and the real option value function $V_g^c(X,t;\lambda_l)$ is non-negative, we have $G_{g,l}(\hat{\hat{X}}_{g,l},t) < 0$. At $X = X_g^*(t;\lambda_l)$, the low-type firm invests at its first-best threshold under complete information. According to the value-matching condition [see (2.17) and (2.18)], we have

$$V_g^c(X_g^*(t;\lambda_l),t;\lambda_l) = I_g^c(X_g^*(t;\lambda_l);\lambda_l) = \lambda_l\Pi(X_g^*(t;\lambda_l)) - F - I - \alpha\lambda_l\Pi(\underline{X}_g(\lambda_l))\left[\frac{X_g^*(t;\lambda_l)}{\underline{X}_g(\lambda_l)}\right]^\xi.$$

By replacing $\frac{c_b}{r}$ in $L_b(X;\lambda)$ by I, the value of liabilities under EGS can be deduced to be

$$L_g(X;\lambda) = (\eta + \eta_\alpha)\lambda^\xi\Pi(X)^\xi(F+I)^{1-\xi}.$$

Obviously, $L_g(X;\lambda_h) < L_g(X;\lambda_l)$ and $E_g(X_g^*(t;\lambda_l);\lambda_l) < E_g(X_g^*(t;\lambda_l);\lambda_h)$ since $\lambda_h > \lambda_l$ and $\xi < 0$. We consider

$$I_{g,l}^m(X_g^*(t;\lambda_l))$$
$$= \lambda_l\Pi(X_g^*(t;\lambda_l)) - F - I + D_g(X_g^*(t;\lambda_l);\lambda_l) - \frac{E_g(X_g^*(t;\lambda_l);\lambda_l)}{E_g(X_g^*(t;\lambda_l);\lambda_h)}L_g(X_g^*(t;\lambda_l);\lambda_h)$$
$$> \lambda_l\Pi(X_g^*(t;\lambda_l)) - F - I + D_g(X_g^*(t;\lambda_l);\lambda_l) - \frac{E_g(X_g^*(t;\lambda_l);\lambda_l)}{E_g(X_g^*(t;\lambda_l);\lambda_h)}L_g(X_g^*(t;\lambda_l);\lambda_l)$$
$$> \lambda_l\Pi(X_g^*(t;\lambda_l)) - F - I + D_g(X_g^*(t;\lambda_l);\lambda_l) - L_g(X_g^*(t;\lambda_l);\lambda_l)$$
$$= \lambda_l\Pi(X_g^*(t;\lambda_l)) - F - I + \left[F + I - \lambda_l\Pi(\underline{X}_g(\lambda_l))\right]\left[\frac{X_g^*(t;\lambda_l)}{\underline{X}_g(\lambda_l)}\right]^\xi$$
$$\quad - \left[F + I - (1-\alpha)\lambda_l\Pi(\underline{X}_g(\lambda_l))\right]\left[\frac{X_g^*(t;\lambda_l)}{\underline{X}_g(\lambda_l)}\right]^\xi$$
$$= \lambda_l\Pi(X_g^*(t;\lambda_l)) - F - I - \alpha\lambda_l\Pi(\underline{X}_g(\lambda_l))\left[\frac{X_g^*(t;\lambda_l)}{\underline{X}_g(\lambda_l)}\right]^\xi = V_g^c(X_g^*(t;\lambda_l),t;\lambda_l).$$

Therefore, we have

$$G_{g,l}(X_g^*(t;\lambda_l),t) = I_{g,l}^m(X_g^*(t;\lambda_l)) - V_g^c(X_g^*(t;\lambda_l),t;\lambda_l) > 0.$$

According to the mean value theorem, there exists a solution of the algebraic equation: $G_{g,l}(X,t) = 0$ within the interval $[\hat{\hat{X}}_{g,l}, X_g^*(t;\lambda_l)]$ since the function $G_{g,l}(X,t)$ is continuous with respect to X. In other words, there exists a threshold $\overline{X}_{g,l}(t)$ binding (3.3) within the interval $[\hat{X}_g(\lambda_h), X_g^*(t;\lambda_l)]$ such that the high-type firm can separate from the low-type at time t when $X \le \overline{X}_{g,l}(t)$.

Appendix B - Optimal thresholds near maturity

At $t \to T^-$, the investment opportunity remains alive and thus the real option value function V satisfies the following equation:

$$\frac{\partial V}{\partial t} + \frac{\sigma^2}{2} X^2 \frac{\partial^2 V}{\partial X^2} + \mu X \frac{\partial V}{\partial X} = rV.$$

The continuity of the real option value function implies that it converges to its corresponding exercise payoffs when the investment opportunity comes closer to expiry. Similar to the analysis of the optimal exercise threshold at time close to expiry for an American option (Kwok, 2008), the optimal stopping threshold X^* when $t \to T^-$ is obtained by setting

$$\left. \frac{\partial V(X,t)}{\partial t} \right|_{t=T^-} = -\frac{\sigma^2}{2} X^2 \frac{\partial^2 V(X,T^-)}{\partial X^2} - \mu X \frac{\partial V(X,T^-)}{\partial X} + rV\left(X,T^-\right) = 0, \qquad \text{(B1)}$$

where $V(X,T^-)$ is set to be the exercise payoff.

First-best threshold under equity guarantee swap

If the firm chooses EGS, the real option value under complete information at $t = T^-$ becomes $I_g^c(X;\lambda)$, where

$$V_g^c(X,T^-;\lambda) = I_g^c(X;\lambda) = \lambda \Pi(X) - F - I - \eta_\alpha \lambda^\xi \Pi(X)^\xi (F+I)^{1-\xi}.$$

The first- and second-order derivatives of $V_g^c(X,T^-;\lambda)$ with respect to X are found to be

$$\frac{\partial I_g^c(X)}{\partial X} = \frac{\lambda}{r-\mu} - \xi \eta_\alpha \lambda^\xi X^{\xi-1}(r-\mu)^{-\xi}(F+I)^{1-\xi},$$

$$\frac{\partial^2 I_g^c(X)}{\partial X^2} = -\xi(\xi-1)\eta_\alpha \lambda^\xi X^{\xi-2}(r-\mu)^{-\xi}(F+I)^{1-\xi}.$$

The exercise payoff $I_g^c(X;\lambda)$ and its derivatives are functions of X. The first-best threshold $X_g^*(T^-;\lambda)$ is determined such that (B1) is satisfied at $X = X_g^*(T^-;\lambda)$. Substituting these relations into (B1), the first-best threshold $X_g^*(T^-;\lambda)$ is given by the solution of the following algebraic equation:

$$\lambda X + \left[\frac{\sigma^2}{2}\xi(\xi-1) + \mu\xi - r \right] \eta_\alpha \lambda^\xi \Pi(X)^\xi (F+I)^{1-\xi} - r(F+I) = 0. \qquad \text{(B2)}$$

First-best threshold under direct bank loan

If the firm chooses the direct bank loan, the real option value under complete information becomes $E_b(X;\lambda,c_b)$ at $t \to T^-$:

$$V_b^c(X,T^-;\lambda) = E_b(X;\lambda,c_b) = \lambda \Pi(X) - F - I - \eta_\alpha \lambda^\xi \Pi(X)^\xi \left(F + \frac{c_b}{r} \right)^{1-\xi}, \qquad \text{(B3)}$$

where the coupon rate c_b is determined by the following budget constraint:

$$\frac{c_b}{r} - (\eta + \eta_\alpha)\lambda^\xi \Pi(X)^\xi \left(F + \frac{c_b}{r}\right)^{1-\xi} = I. \tag{B4}$$

Since c_b has implicit dependence on X, we apply the Implicit Function theorem to (B4) to express $\frac{\partial c_b}{\partial X}$ in terms of c_b and X, where

$$\frac{\partial c_b}{\partial X} = \left[\frac{X}{\xi(c_b - rI)} - \frac{(1/\xi - 1)X}{c_b + rF}\right]^{-1}.$$

Combining (B3) and (B4), the value function can be expressed as

$$V_b^c(X, T^-; \lambda) = \lambda\Pi(X) - F - I - \frac{\eta_\alpha}{\eta + \eta_\alpha}\left(\frac{c_b}{r} - I\right).$$

The first- and second-order derivatives of $V_b^c(X, T^-; \lambda)$ are found to be

$$\frac{\partial V_b^c}{\partial X} = \frac{\lambda}{r - \mu} - \frac{\frac{\eta_\alpha}{\eta + \eta_\alpha}\xi X^{-1}\left(\frac{c_b}{r} - I\right)}{1 - (1 - \xi)\frac{c_b - rI}{c_b + rF}},$$

$$\frac{\partial^2 V_b^c}{\partial X^2} = \frac{\frac{\eta_\alpha}{\eta + \eta_\alpha}(1 - \xi)\xi X^{-2}\left(\frac{c_b}{r} - I\right)\left[\frac{r(F+I)}{c_b + rF}\right]^2}{\left[1 - (1 - \xi)\frac{c_b - rI}{c_b + rF}\right]^3}.$$

Substituting V_b^c, $\frac{\partial V_b^c}{\partial X}$ and $\frac{\partial^2 V_b^c}{\partial X^2}$ into (B1), the first-best threshold $X_b^*(T^-; \lambda)$ is given by the solution of the following algebraic equation:

$$\frac{\eta + \eta_\alpha}{\eta_\alpha}\frac{\lambda X - r(F + I)}{c_b/r - I} = \frac{\frac{\sigma^2}{2}(1 - \xi)\xi\left[\frac{r(F+I)}{c_b + rF}\right]^2}{\left[1 - (1 - \xi)\frac{c_b - rI}{c_b + rF}\right]^3} - \frac{\mu\xi}{1 - (1 - \xi)\frac{c_b - rI}{c_b + rF}} + r. \tag{B5}$$

Since the coupon rate c_b is a function of X, we resort to numerical method to solve the above algebraic equation.