Mathematical Excalibur

Volume 12, Number 3 *September 2007 – October 2007*

Olympiad Corner

Below were the problems of the 2007 International Math Olympiad, which was held in Hanoi, Vietnam.

Day 1 (*July 25, 2007*)

Problem 1. Real numbers $a_1, a_2, \ldots, a_n$ are given. For each i ($1 \le i \le n$) define

$$d_i = \max\{a_j : 1 \le j \le i\} - \min\{a_j : 1 \le j \le n\}$$

and let $d = \max\{d_i : 1 \le i \le n\}$.

(a) Prove that, for any real numbers $x_1 \le x_2 \le \ldots \le x_n$,

$$\max\{|x_i - a_i| : 1 \le i \le n\} \ge \frac{d}{2}. \qquad (*)$$

(b) Show that there are real numbers $x_1 \le x_2 \le \ldots \le x_n$ such that equality holds in (*).

Problem 2. Consider five points A, B, C, D and E such that $ABCD$ is a parallelogram and $BCED$ is a cyclic quadrilateral. Let ℓ be a line passing through A. Suppose that ℓ intersects the interior of the segment DC at F and intersects line BC at G. Suppose also that $EF=EG=EC$. Prove that ℓ is the bisector of angle DAB.

(continued on page 4)

Acknowledgment: Thanks to Elina Chiu, Math. Dept., HKUST for general assistance.

On-line:
http://www.math.ust.hk/mathematical_excalibur/

The editors welcome contributions from all teachers and students. With your submission, please include your name, address, school, email, telephone and fax numbers (if available). Electronic submissions, especially in MS Word, are encouraged. The deadline for receiving material for the next issue is ***November 25, 2007***.

For individual subscription for the next five issues for the 05-06 academic year, send us five stamped self-addressed envelopes. Send all correspondence to:

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Convex Hull

Kin Yin Li

A set S in a plane or in space is convex if and only if whenever points X and Y are in S, the line segment XY must be contained in S. The intersection of any collection of convex sets is convex. For an arbitrary set W, the convex hull of W is the intersection of all convex sets containing W. This is the smallest convex set containing W. For a finite set W, the boundary of the convex hull of W is a polygon, whose vertices are all in W.

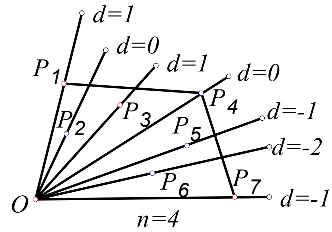
In a previous article (see *pp*. 1-2, *vol*. 5, *no*. 1 of *Math. Excalibur*), we solved problem 1 of the 2000 IMO using convex hull. Below we will discuss more geometric combinatorial problems that can be solved by studying convex hulls of sets.

Example 1. There are $n > 3$ coplanar points, no three of which are collinear and every four of them are the vertices of a convex quadrilateral. Prove that the n points are the vertices of a convex n-sided polygon.

Solution. Assume one of these points, say P, is inside the convex hull of the n points. Let Q be a vertex of the convex hull. The diagonals from Q divide the convex hull into triangles. Since no three points are collinear, P is inside some $\triangle QRS$, where RS is a side of the boundary. Then P,Q,R,S cannot be the vertices of a convex quadrilateral, a contradiction. So all n points can only be the vertices of the boundary polygon.

Example 2. (*1979 Putnam Exam*) Let A be a set of $2n$ points in the plane, no three of which are collinear, n of them are colored red and the other blue. Prove that there are n line segments, no two with a point in common, such that the endpoints of each segment are points of A having different colors.

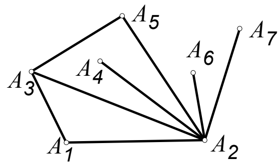
Solution. The case $n = 1$ is true. Suppose all cases less than n are true. For a vertex O on the boundary polygon of the convex hull of these $2n$ points, it is one of the $2n$ points, say its color is red. Let P_1, P_{2n-1} be adjacent vertices to O. If one of them, say P_1, is blue, then draw line segment OP_1 and apply induction to the other $2(n-1)$ points to finish. Otherwise,



let $d = 1$ and rotate the line OP_1 toward line OP_{2n-1} about O hitting the other $2n-3$ points one at a time. When the line hits a red point, increase d by 1 and when it hits a blue point, decrease d by 1. When the line hits P_{2n-1}, $d = (n-1) - n = -1$. So at some time, $d = 0$, say when the line hits P_j. Then $P_1,\ldots,P_j$ are on one side of line OP_j and $O, P_{j+1},\ldots, P_{2n-1}$ are on the other side. The inductive step can be applied to these two sets of points, which leads to the case n being true.

Example 3. (*1985 IMO Longlisted Problem*) Let A and B be finite disjoint sets of points in the plane such that any three distinct points in $A\cup B$ are not collinear. Assume that at least one of the sets A, B contains at least five points. Show that there exists a triangle all of whose vertices are contained in A or in B that does not contained in its interior any point from the other set.

Solution. Suppose A has at least five points. Take a side A_1A_2 of the boundary of the convex hull of A. For any other A_i in A, let $\alpha_i=\angle A_1A_2A_i$, say $\alpha_3 < \alpha_4 < \cdots < 180^\circ$. Then the convex hull H of A_1, A_2, A_3, A_4, A_5 contains no other points of A.



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Perpendicular Lines

Kin Yin Li

In geometry, sometimes we are asked to prove two lines are perpendicular. If the given facts are about right angles and lengths of segments, the following theorem is often useful.

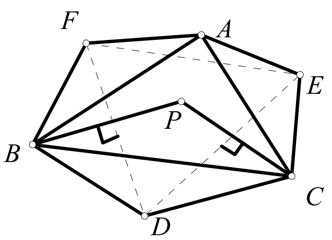
Theorem. On a plane, for distinct points R, S, X, Y, we have $RX^2-SX^2=RY^2-SY^2$ if and only if $RS \perp XY$.

Proof. Let P and Q be the feet of the perpendicular from X and Y to line RS respectively. If $RS \perp XY$, then $P=Q$ and $RX^2-SX^2=RP^2-SP^2=RY^2-SY^2$.

Conversely, if $RX^2-SX^2=RY^2-SY^2=m$, then $m=RP^2-(SR\pm RP)^2$. So $RP=\mp(SR^2+m)/2SR$. Replacing P by Q, we get $RQ=\mp(SR^2+m)/2SR$. Hence, $RP=RQ$. Interchanging R and S, we also get $SP=SQ$. So $P=Q$. Therefore, $RS \perp XY$.

Here are a few illustrative examples.

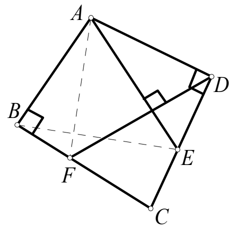
Example 1. (*1997 USA Math Olympiad*) Let ABC be a triangle, and draw isosceles triangles BCD, CAE, ABF externally to ABC, with BC, CA, AB as their respective bases. Prove the lines through A, B, C, perpendicular to the lines EF, FD, DE, respectively, are concurrent.



Solution. Let P be the intersection of the perpendicular line from B to FD with the perpendicular line from C to DE. Then $PB \perp FD$ and $PC \perp DE$. By the theorem above, we have $PF^2-PD^2=BF^2-BD^2$ and $PD^2-PE^2=CD^2-CE^2$.

Adding these and using $AF=BF$, $BD=CD$ and $CE=AE$, we get $PF^2-PE^2=AF^2-AE^2$. So $PA \perp EF$ and P is the desired concurrent point.

Example 2. (*1995 Russian Math Olympiad*) $ABCD$ is a quadrilateral such that $AB=AD$ and $\angle ABC$ and $\angle CDA$ are right angles. Points F and E are chosen on BC and CD respectively so that $DF \perp AE$. Prove that $AF \perp BE$.

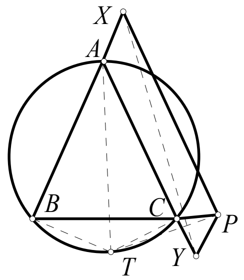


Solution. We have $AE \perp DF$, $AB \perp BF$ and $AD \perp DE$, which are equivalent to

$$AD^2-AF^2=ED^2-EF^2, \quad \text{(a)}$$
$$AB^2-AF^2=-BF^2, \quad \text{(b)}$$
$$AD^2-AE^2=-DE^2. \quad \text{(c)}$$

Doing (a) − (b) + (c) and using $AD=AB$, we get $AB^2-AE^2=BF^2-EF^2$, which implies $AF \perp BE$.

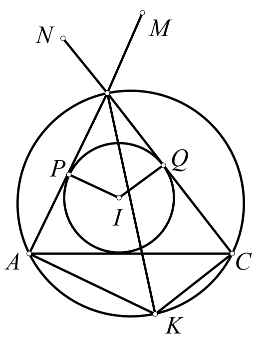
Example 3. In acute $\triangle ABC$, $AB=AC$ and P is a point on ray BC. Points X and Y are on rays BA and AC such that $PX \| AC$ and $PY \| AB$. Point T is the midpoint of minor arc BC on the circumcircle of $\triangle ABC$. Prove that $PT \perp XY$.



Since AT is a diameter, $\angle ABT = 90^\circ = \angle ACT$. Then $TX^2=XB^2+BT^2$ and $TY^2=TC^2+CY^2$. So $TX^2-TY^2=BX^2-CY^2$.

Since $PX \| AC$, we have $\angle ABC = \angle ACB = \angle XPB$, hence $BX=PX$. Similarly, $CY=PY$. Therefore, $TX^2-TY^2=PX^2-PY^2$, which is equivalent to $PT \perp XY$.

Example 4. (1994 Jiangsu Province Math Competition) For $\triangle ABC$, take a point M by extending side AB beyond B and a point N by extending side CB beyond B such that $AM=CN=s$, where s is the semiperimeter of $\triangle ABC$. Let the inscribed circle of $\triangle ABC$ have center I and the circumcircle of $\triangle ABC$ have diameter BK. Prove that $KI \perp MN$.



Solution. Let the incircle of $\triangle ABC$ touch side AB at P and side BC at Q. We will show $KM^2-KN^2=IM^2-IN^2$.

Now since $\angle MAK = \angle BAK = 90^\circ$ and $\angle NCK = \angle BCK = 90^\circ$, we get

$$\begin{aligned} KM^2-KN^2 &= (KA^2+AM^2)-(KC^2+CN^2) \\ &= KA^2-KC^2 \\ &= (KA^2-KB^2)+(KB^2-KC^2) \\ &= BC^2-AB^2. \end{aligned}$$

Also, since $\angle MPI = \angle BPI = 90^\circ$ and $\angle NQI = \angle BQI = 90^\circ$, we get

$$\begin{aligned} IM^2-IN^2 &= (IP^2+PM^2)-(IQ^2+QN^2) \\ &= PM^2-QN^2 \\ &= (AM-AP)^2+(CN-QC)^2. \end{aligned}$$

Now

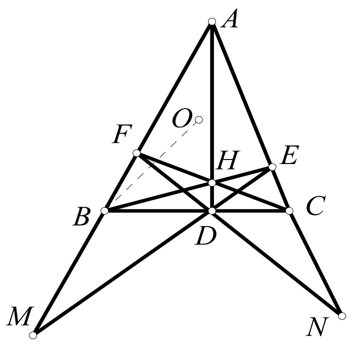
$$AM-AP = s-\frac{AB+CA-BC}{2} = BC$$

and

$$CN-QC = s-\frac{CA+BC-AB}{2} = AB.$$

So $IM^2-IN^2=BC^2-AB^2=KM^2-KN^2$.

Example 5. (*2001 Chinese National Senior High Math Competition*) As in the figure, in $\triangle ABC$, O is the circumcenter. The three altitudes AD, BE and CF intersect at H. Lines ED and AB intersect at M. Lines FD and AC intersect at N. Prove that (1) $OB \perp DF$ and $OC \perp DE$; (2) $OH \perp MN$.



Solution. (1) Since $\angle AFC = 90^\circ = \angle ADC$, so A,C,D,F are concyclic. Then $\angle BDF = \angle BAC$. Also,

$$\begin{aligned} \angle OBC &= \tfrac{1}{2}(180^\circ - \angle BOC) \\ &= 90^\circ - \angle BAC = 90^\circ - \angle BDF. \end{aligned}$$

So $OB \perp DF$. Similarly, $OC \perp DE$.

(2) Now $CH \perp MA$, $BH \perp NA$, $DA \perp BC$, $OB \perp DF = DN$ and $OC \perp DE = DM$. So

$$MC^2-MH^2=AC^2-AH^2 \quad \text{(a)}$$
$$NB^2-NH^2=AB^2-AH^2 \quad \text{(b)}$$
$$DB^2-DC^2=AB^2-AC^2 \quad \text{(c)}$$
$$BN^2-BD^2=ON^2-OD^2 \quad \text{(d)}$$
$$CM^2-CD^2=OM^2-OD^2. \quad \text{(e)}$$

Doing (a) − (b) + (c) + (d) − (e), we get $NH^2-MH^2=ON^2-OM^2$. So $OH \perp MN$.

Problem Corner

We welcome readers to submit their solutions to the problems posed below for publication consideration. The solutions should be preceded by the solver's name, home (or email) address and school affiliation. Please send submissions to *Dr. Kin Y. Li, Department of Mathematics, The Hong Kong University of Science & Technology, Clear Water Bay, Kowloon, Hong Kong.* The deadline for submitting solutions is ***November 25, 2007.***

Problem 281. Let N be the set of all positive integers. Prove that there exists a function $f : N \to N$ such that $f(f(n)) = n^2$ for all n in N. (*Source: 1978 Romanian Math Olympiad*)

Problem 282. Let a, b, c, A, B, C be real numbers, $a \neq 0$ and $A \neq 0$. For every real number x,

$$|ax^2+bx+c| \leq |Ax^2+Bx+C|.$$

Prove that $|b^2-4ac| \leq |B^2-4AC|$.

Problem 283. P is a point inside $\triangle ABC$. Lines AC and BP intersect at Q. Lines AB and CP intersect at R. It is known that $AR=RB=CP$ and $CQ=PQ$. Find $\angle BRC$ with proof. (*Source: 2003 Japanese Math Olympiad*)

Problem 284. Let p be a prime number. Integers x, y, z satisfy $0 < x < y < z < p$. If x^3, y^3, z^3 have the same remainder upon dividing by p, then prove that $x^2+y^2+z^2$ is divisible by $x+y+z$. (*Source: 2003 Polish Math Olympiad*)

Problem 285. Determine the largest positive integer N such that for every way of putting all numbers 1 to 400 into a 20×20 table (1 number per cell), one can always find a row or a column having two numbers with difference not less than N. (*Source: 2003 Russian Math Olympiad*)

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Solutions

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Problem 276. Let n be a positive integer. Given a $(2n-1)\times(2n-1)$ square board with exactly one of the following arrows ↑, ↓, →, ← at each of its cells. A beetle sits in one of the cells. Per year the beetle creeps from one cell to another in accordance with the arrow's direction. When the beetle leaves the cell, the arrow at that cell makes a counterclockwise 90-degree turn. Prove that the beetle leaves the board in at most $2^{3n-1}(n-1)!-3$ years.
(*Source: 2001 Belarussian Math Olympiad*)

Solution. **Jeff CHEN** (Virginia, USA), **GRA20 Problem Solving Group** (Roma, Italy), **PUN Ying Anna** (HKU, Math Year 1) and **Fai YUNG**.

Let $a(n)$ be the maximum number of years that the beetle takes to leave the $(2n-1)\times(2n-1)$ board. Then $a(1) = 1$. For $n > 1$, apart from 1 year necessary for the final step, the beetle can stay

(1) in each of the 4 corners for at most 2 years (two directions that do not point outside)

(2) in each of the other $4(2n-3)$ cells of the outer frame for at most 3 years (three directions that do not point outside)

(3) in the inner $(2n-3)\times(2n-3)$ board for at most $a(n-1)$ years (when the starting point is inside the inner board) plus $4(2n-3)a(n-1)$ years (when the arrow in a cell of the outer frame points inward the beetle enters the inner board).

Therefore, $a(n) \leq 1 + 4\cdot 2 + 3\cdot 4(2n-3) + (4(2n-3)+1)a(n-1)$. Since $a(n-1) \geq 0$,

$$\begin{aligned} a(n)+3 &\leq 8(n-1)(a(n-1)+3) - 3a(n-1) \\ &\leq 8(n-1)(a(n-1)+3). \end{aligned}$$

Since $a(1) + 3 = 4$, we get $a(n) + 3 \leq 2^{3n-1}(n-1)!$ and so $a(n) \leq 2^{3n-1}(n-1)!-3$.

Problem 277. (*Due to Koopa Koo, Univ. of Washington, Seattle, WA, USA*) Prove that the equation

$$x^2 + y^2 + z^2 + 2xyz = 1$$

has infinitely many integer solutions (then try to get all solutions – Editors).

Solution. **Jeff CHEN** (Virginia, USA), **FAN Wai Tong** and **GRA20 Problem Solving Group** (Roma, Italy).

It is readily checked that if n is an integer, then $(x, y, z) = (n, -n, 1)$ is a solution.

Comments: Trying to get all solutions, we can first rewrite the equation as

$$(x^2-1)(y^2-1) = (xy+z)^2.$$

For any solution (x, y, z), we must have $x^2-1 = du^2$, $y^2-1 = dv^2$, $xy+z = \pm duv$ for some integers d, u, v. The cases d is negative, 0 or 1 lead to trivial solutions. For $d > 1$, we may suppose it is square-free (that is, no square divisor greater than 1). Then we can find all solutions of Pell's equation $s^2-dt^2 = 1$ (see *vol.* 6, *no.* 3 of Math Excalibur, *page* 1). Any two solutions (s_0, t_0) and (s_1, t_1) of Pell's equation yield a solution $(x, y, z)=(s_0, s_1, \pm dt_0t_1-s_0s_1)$ of

$$x^2 + y^2 + z^2 + 2xyz = 1.$$

Commended solvers: **PUN Ying Anna** (HKU, Math Year 1) and **WONG Kam Wing** (TWGH Chong Ming Thien College).

Problem 278. Line segment SA is perpendicular to the plane of the square $ABCD$. Let E be the foot of the perpendicular from A to line segment SB. Let P, Q, R be the midpoints of SD, BD, CD respectively. Let M, N be on line segments PQ, PR respectively. Prove that AE is perpendicular to MN.

Solution 1. **Stephen KIM** (Toronto, Canada).

Below when we write $XY \perp IJK\ldots$, we mean line XY is perpendicular to the plane containing $I, J, K,\ldots$. Also, we write $XY \perp WZ$ for vectors XY and WZ to mean their dot product is 0.

Since $SA \perp ABCD$, so $SA \perp BC$. Since $AB \perp BC$, so $BC \perp SAB$. Since A, E are in the plane of SAB, $AE \perp BC$. This along with the given fact $AE \perp SB$ imply $AE \perp SBC$.

Since P, Q are midpoints of SD, BD respectively, we get $PQ \| SB$. Similarly, we have $QR \| BC$. Then the planes SBC and PQR are parallel. Since MN is on the plane PQR, so MN is parallel to the plane SBC. Since $AE \perp SBC$ from the last paragraph, so $AE \perp MN$.

Solution 2. **Kelvin LEE** (Winchester College, England) and **PUN Ying Anna** (HKU, Math Year 1).

Let A be the origin, AD be the x-axis, AB be the y-axis and AS be the z-axis. Let $B = (0, a, 0)$ and $S = (0, 0, s)$. Then $C = (a, a, 0)$ and $E = (0, rs, ra)$ for some r. The homothety with center D and ratio 2 sends P to S, Q to B and R to C. Let it send M to M' and N to N'. Then M' is on SB, N' is on SC and $M'N' \| MN$. So $M' = (0, 0, s) + (0, a, -s)u = (0, au, s(1-u))$ for some u and $N' = (0, 0, s) + (a, a, -s)v = (av, av, s(1-v))$ for some v. Now the dot product of AE and $M'N'$ is

$$(0, rs, ra)\cdot(av, a(v-u), s(u-v)) = 0.$$

So $AE \perp M'N'$. Therefore, $AE \perp MN$.

Commended solvers: **WONG Kam Wing** (TWGH Chong Ming Thien College).

Problem 279. Let R be the set of all real numbers. Determine (with proof) all functions $f: R\to R$ such that for all real x and y,

$$f(f(x)+y)=2x+f(f(f(y))-x).$$

Solution. **Jeff CHEN** (Virginia, USA), **Salem MALIKIĆ** (Sarajevo College, 3rd Grade, Sarajevo, Bosnia and Herzegovina) and **PUN Ying Anna** (HKU, Math Year 1).

Setting $y = 0$, we get

$$f(f(x)) = 2x + f(f(f(0)) - x). \qquad (1)$$

Then putting $x = 0$ into (1), we get

$$f(f(0)) = f(f(f(0))). \qquad (2)$$

In (1), setting, $x = f(f(0))$, we get

$$f(f(f(f(0)))) = 2f(f(0)) + f(0).$$

Using (2), we get $f(f(0)) = 2f(f(0)) + f(0)$. So $f(f(0)) = -f(0)$. Using (2), we see $f^{(k)}(0) = -f(0)$ for $k = 2,3,4,\ldots$.

In the original equation, setting $x = 0$ and $y = -f(0)$, we get

$$\begin{aligned} f(0) &= -2f(0) + f(f(f(-f(0)))) \\ &= -2f(0) + f^{(5)}(0) \\ &= -2f(0) - f(0) = -3f(0). \end{aligned}$$

So $f(0) = 0$. Then (1) becomes

$$f(f(x)) = 2x + f(-x). \qquad (3)$$

In the original equation, setting $x = 0$, we get $f(y) = f(f(f(y)))$. (4)

Setting $x = f(y)$ in (3), we get

$$f(y) = f(f(f(y))) = 2f(y) + f(-f(y)).$$

So $f(-f(y)) = -f(y)$. Setting $y = -f(x)$ in the original equation, we get

$$0 = 2x + f(f(f(-f(x))) - x).$$

For every real number w, setting $x = -w/2$, we see $w = f(f(f(-f(x))) - x)$. Hence, f is surjective. Then by (4), $w = f(f(w))$ for all w. By (3), setting $x = -w$, we get $f(w) = w$ for all w. Substituting this into the original equation clearly works. So the only solution is $f(w) = w$ for all w.

Commended solvers: **Kelvin LEE** (Winchester College, England),

Problem 280. Let n and k be fixed positive integers. A basket of peanuts is distributed into n piles. We gather the piles and rearrange them into $n + k$ new piles. Prove that at least $k + 1$ peanuts are transferred to smaller piles than the respective original piles that contained them. Also, give an example to show the constant $k + 1$ cannot be improved.

Solution. **Jeff CHEN** (Virginia, USA), **Stephen KIM** (Toronto, Canada) and **PUN Ying Anna** (HKU, Math Year 1).

Before the rearrangement, for each pile, if the pile has m peanuts, then attach a label of $1/m$ to each peanut in the pile. So the total sum of all labels is n.

Assume that only at most k peanuts were put into smaller piles after the rearrangement. Since the number of piles become $n + k$, so there are at least n of these $n + k$ piles, all of its peanuts are now in piles that are larger or as large as piles they were in before the rearrangement. Then the sum of the labels in just these n piles is already at least n. Since there are $k > 0$ more piles, this is a contradiction.

For an example to show $k + 1$ cannot be improved, take the case originally one of the n piles contained $k + 1$ peanuts. Let us rearrange this pile into $k + 1$ piles with 1 peanut each and leave the other $n - 1$ piles alone. Then only these $k + 1$ peanuts go into smaller piles.

Commended solvers: **WONG Kam Wing** (TWGH Chong Ming Thien College).

Olympiad Corner

(continued from page 1)

Problem 3. In a mathematical competition some competitors are friends. Friendship is always mutual. Call a group of competitors a *clique* if each two of them are friends. (In particular, any group of fewer than two competitors is a clique.) The number of members of a clique is called its *size*.

Given that, in this competition, the largest size of a clique is even, prove that the competitors can be arranged in two rooms such that the largest size of a clique contained in one room is the same as the largest size of a clique in the other room.

Day 2 (*July 26, 2007*)

Problem 4. In triangle ABC the bisector of angle BCA intersects the circumcircle again at R, the perpendicular bisector of BC at P, and the perpendicular bisector of AC at Q. The midpoint of BC is K and the midpoint of AC is L. Prove that the triangles RPK and RQL have the same area.

Problem 5. Let a and b be positive integers. Show that if $4ab - 1$ divides $(4a^2 - 1)^2$, then $a = b$.

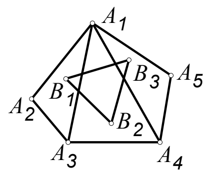
Problem 6. Let n be a positive integer. Consider

$$S = \{(x,y,z) : x,y,z \in \{0,1,\ldots,n\}, x+y+z>0\}$$

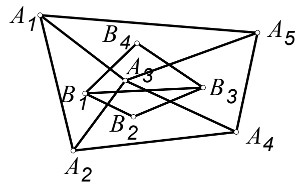
as a set of $(n+1)^3 - 1$ points in the three-dimensional space. Determine the smallest possible number of planes, the union of which contains S but does not include (0, 0, 0).

Convex Hull

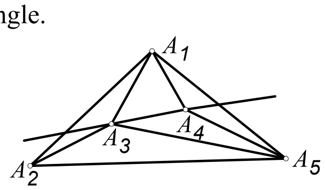
(continued from page 1)



Case 1: (The boundary of H is the pentagon $A_1A_2A_3A_4A_5$.) If $\triangle A_1A_2A_3$ or $\triangle A_1A_3A_4$ or $\triangle A_1A_4A_5$ does not contain any point of B in its interior, then we are done. Otherwise, there exist B_1, B_2, B_3 in their interiors respectively. Then we see $\triangle B_1B_2B_3$ is a desired triangle.



Case 2: (The boundary of H is a quadrilateral, say $A_1A_2A_4A_5$ with A_3 inside.) If $\triangle A_1A_3A_2$ or $\triangle A_2A_3A_4$ or $\triangle A_4A_3A_5$ or $\triangle A_5A_3A_1$ does not contain any point of B in its interior, then we are done. Otherwise, there exist B_1, B_2, B_3, B_4 in their interiors respectively. Then either $\triangle B_1B_2B_3$ or $\triangle B_3B_4B_1$ does not contain A_3 in its interior. That triangle is a desired triangle.



Case 3: (The boundary of H is a triangle, say $A_1A_2A_5$ with A_3, A_4 inside, say A_3 is closer to line A_1A_2 than A_4.) If $\triangle A_1A_2A_3$ or $\triangle A_1A_3A_4$ or $\triangle A_1A_4A_5$ or $\triangle A_2A_3A_5$ or $\triangle A_3A_4A_5$ does not contain any point of B in its interior, then we are done. Otherwise, there exists a point of B in each of their interiors respectively. Then three of these points of B lie on one side of line A_3A_4. The triangle formed by these three points of B is a desired triangle.