Mathematical Excalibur

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Olympiad Corner

Below are the problems of the Fourth Round of the 53rd Ukrainian National Math Olympiad for 10-th Graders.

Problem 1. Suppose that for real x,y,z,t the following equalities hold: $\{x+y+z\} = \{y+z+t\} = \{z+t+x\} = \{t+x+y\} = 1/4$. Find all possible values of $\{x+y+z+t\}$. (Here $\{x\}=x-[x]$.)

Problem 2. Let M be the midpoint of the side BC of $\triangle ABC$. On the side AB and AC the points F and E are chosen. Let K be the point of the intersection of BF and CE and L be chosen in a way that $CL \| AB$ and $BL \| CE$. Let N be the point of intersection of AM and CL. Show that KN is parallel to FL.

Problem 3. It is known that for natural numbers a,b,c,d and n the following inequalities hold: $a+c < n$ and $a/b+c/d < 1$. Prove that $a/b+c/d < 1-1/n^3$.

Problem 4. There are 100 cards with numbers from 1 to 100 on the table. Andriy and Nick took the same number of cards in a way that the following condition holds: if Andriy has a card with a number n then Nick has a card with a number $2n+2$. What is the maximal number of cards could be taken by the two guys?

(continued on page 4)

Acknowledgment: Thanks to Elina Chiu, Math. Dept., HKUST for general assistance.

On-line:
http://www.math.ust.hk/mathematical_excalibur/

The editors welcome contributions from all teachers and students. With your submission, please include your name, address, school, email, telephone and fax numbers (if available). Electronic submissions, especially in MS Word, are encouraged. The deadline for receiving material for the next issue is ***April 12, 2014***.

For individual subscription for the next five issues for the 13-14 academic year, send us five stamped self-addressed envelopes. Send all correspondence to:

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Using Tangent Lines to Prove Inequalities (Part II)

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We offer a continuation of the paper by Kin-Yin Li (cf. *Math Excalibur*, *vol.* 10, *no.* 5) where he considers using tangent lines to prove inequalities.

Example 1. Suppose that a, b, and c are positive real numbers satisfying $a+b+c=3$. Find the minimum of the expression $a^4+2b^4+3c^4$.

Solution. Let $f_k(x)=kx^4$, where $x\in(0,3)$, $k = 1, 2, 3$. As $f_k''(x) = 12kx^2 > 0$, where $x>0$, so functions f_k are convex, which means that their graphs do not fall below their tangents drawn at any point $x_k\in(0,3)$ (k=1,2,3). Points x_1, x_2 and x_3 are chosen such that $f_1'(x_1) = f_2'(x_2) = f_3'(x_3)$ and $x_1+x_2+x_3=3$. That is,

$$4x_1^3=8x_2^3=12x_3^3 \text{ and } x_1+x_2+x_3=3.$$

Hence,

$$x_1=\frac{3\sqrt[3]{6}}{\sqrt[3]{2}+\sqrt[3]{3}+\sqrt[3]{6}},\quad x_2=\frac{x_1}{\sqrt[3]{2}},\quad x_3=\frac{x_1}{\sqrt[3]{3}}$$

and for any $x\in(0,3)$, we have the inequalities (k = 1,2,3, see Fig. 1)

$$kx^4 \ge f_k(x_k)+f_k'(x_k)(x-x_k). \qquad (1)$$

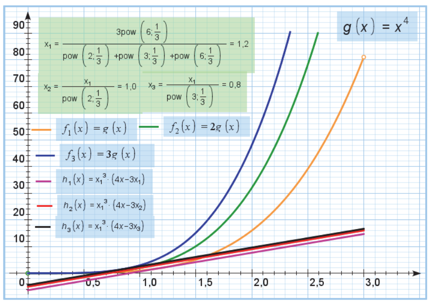

Fig. 1

Adding inequalities (1) for x equals a, b and c, we obtain

$$a^4+2b^4+3c^4$$
$$\ge x_1^4\left(1+\frac{1}{2\sqrt[3]{2}}+\frac{1}{3\sqrt[3]{3}}\right)+f_1'(x_1)(3-\sum_{k=1}^{3}x_k)$$
$$=\frac{81(6\sqrt[3]{3}+3\sqrt[3]{3}+2\sqrt[3]{2})}{(\sqrt[3]{2}+\sqrt[3]{3}+\sqrt[3]{6})^4},$$

which is the minimum (with equality holding at $a=x_1$, $b=x_2$ and $c=x_3$).

Example 2. Let a, b, $c > 0$ be real numbers such that $ab+bc+ca = 1$. Prove the inequality

$$\frac{a^2}{b+c}+\frac{b^2}{c+a}+\frac{c^2}{a+b}\ge\frac{\sqrt{3}}{2}.$$

Solution. Let $S=a+b+c$. Based on the inequality $(a+b+c)^2 \ge 3(ab+bc+ca)$, which is equivalent to $(a-b)^2 + (b-c)^2 + (c-a)^2 \ge 0$, we find that $S \ge \sqrt{3}$.

Let $f(x) = x^2/(S-x)$ for $x\in(0,S)$. Let us construct the tangent equation at the point $x_0=S/3$ (see Fig. 2a,b):

$$y=f\left(\frac{S}{3}\right)+f'\left(\frac{S}{3}\right)\left(x-\frac{S}{3}\right)=\frac{S}{6}+\frac{5}{4}\left(x-\frac{S}{3}\right)=\frac{5x-S}{4}.$$

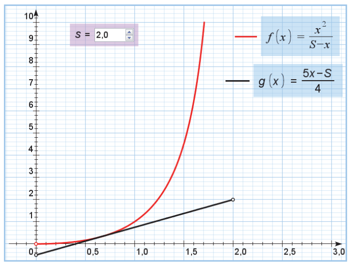

Fig. 2a

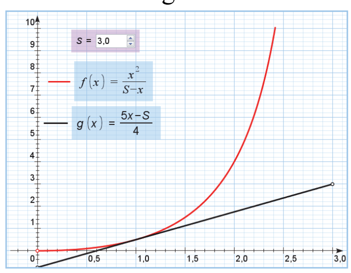

Fig. 2b

Since the inequality $x^2/(S-x) \ge (5x-S)/4$ is equivalent to $(S-3x)^2 \ge 0$ on the interval $(0,S)$, applying it thrice, based on the previously proved inequality $S \ge \sqrt{3}$, we find that

$$\frac{a^2}{b+c}+\frac{b^2}{c+a}+\frac{c^2}{a+b}$$
$$=\frac{a^2}{S-a}+\frac{b^2}{S-b}+\frac{c^2}{S-c}$$
$$\ge\frac{5(a+b+c)-3S}{4}=\frac{S}{2}\ge\frac{\sqrt{3}}{2}.$$

(continued on page 2)

Example 3. Let a, b, $c \geq 0$ be real numbers. Prove the inequality

$$\sqrt{a^2+1}+\sqrt{b^2+1}+\sqrt{c^2+1} \geq \sqrt{6(a+b+c)}.$$

Solution. Assume that $S=a+b+c$ and $f(x)=\sqrt{x^2+1}$ for $x\in(0,S)$. We form the tangent equation at the point $x_0=S/3$:

$$y = f\left(\frac{S}{3}\right)+f'\left(\frac{S}{3}\right)\left(x-\frac{S}{3}\right)$$
$$=\frac{\sqrt{S^2+9}}{3}+\frac{S}{\sqrt{S^2+9}}\left(x-\frac{S}{3}\right)$$
$$=\frac{Sx+3}{\sqrt{S^2+9}}.$$

Since on the interval $(0,S)$, the inequality

$$\sqrt{x^2+1} \geq \frac{Sx+3}{\sqrt{S^2+9}} \qquad (2)$$

is equivalent to the inequality $(x - S/3)^2 \geq 0$, we find that

$$\sqrt{a^2+1}+\sqrt{b^2+1}+\sqrt{c^2+1}$$
$$\geq \sqrt{S^2+9}+\frac{S}{\sqrt{S^2+9}}(a+b+c-S)$$
$$=\sqrt{S^2+9}$$
$$\geq \sqrt{6S}=\sqrt{6(a+b+c)}.$$

Example 4. Let a, b and c be positive real numbers such that $a+2b+3c \geq 20$. Prove the inequality

$$a+b+c+\frac{3}{a}+\frac{9}{2b}+\frac{4}{c} \geq 13.$$

Solution. Note that if a=2, b=3, c=4, the inequality becomes equality. Let $f(x)=1/x$ for $x>0$. Then f is convex in the interval $(0,+\infty)$. Hence the graph of the function f does not go below the tangent line drawn at any point $x_0 > 0$. Thus, the following inequalities are valid (see Fig. 3):

$$\frac{1}{a} \geq \frac{1}{2}-\frac{1}{4}(a-2)=1-\frac{a}{4},$$
$$\frac{1}{b} \geq \frac{1}{3}-\frac{1}{9}(b-3)=\frac{2}{3}-\frac{b}{9},$$
$$\frac{1}{c} \geq \frac{1}{4}-\frac{1}{16}(c-4)=\frac{1}{2}-\frac{c}{16}.$$

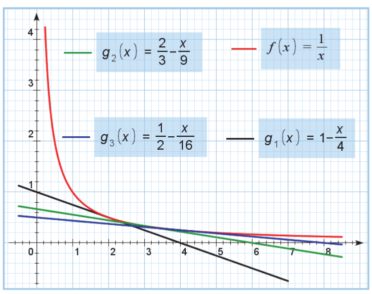

Fig. 3

As given in the statement of the problem, we find that

$$a+b+c+\frac{3}{a}+\frac{9}{2b}+\frac{4}{c}$$
$$\geq a+b+c+3-\frac{3a}{4}+3-\frac{b}{2}+2-\frac{c}{4}$$
$$=8+\frac{a+2b+3c}{4} \geq 8+\frac{20}{4}=13.$$

Example 5. (Pham Kim Hung) Let a, b and c be positive real numbers such that $a^2+b^2+c^2=3$. Prove the inequality

$$\frac{1}{2-a}+\frac{1}{2-b}+\frac{1}{2-c} \geq 3.$$

Solution. Note that when $a=b=c=1$, the inequality becomes an equality. Consider $f(x) = 1/(2-x)$ and $g(x) = kx^2+m$, where $x\in(0,\sqrt{3})$. The numbers k and m are to be chosen so that $f(1) = g(1)$ and $f'(1) = g'(1)$. That is, $1=k+m$ and $1=2k$. Hence, $k=m=1/2$ and $g(x)=(x^2+1)/2$. Since the inequality $1/(2-x) \geq (x^2+1)/2$ is equivalent to $x(x-1)^2 \geq 0$, it is true for any $x\in(0,\sqrt{3})$ (see Fig. 4). Hence,

$$\frac{1}{2-a}+\frac{1}{2-b}+\frac{1}{2-c} \geq \frac{a^2+b^2+c^2+3}{2}=3.$$

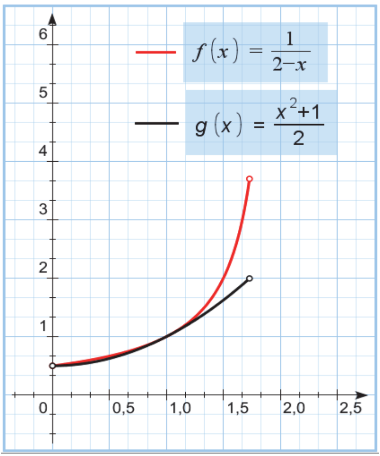

Fig. 4

Example 6. Let a, b and c be positive real numbers. Prove the inequality

$$(a^5-a^2+3)(b^5-b^2+3)(c^5-c^2+3) \geq (a+b+c)^3.$$

Solution. Note that when $a=b=c=1$, the inequality becomes an equality. Consider $f(x)=x^5-x^2+3$ and $g(x) = kx^3+m$, where $x>0$. The numbers k and m are to be chosen so that $f(1) = g(1)$ and $f'(1) = g'(1)$. That is, $3=k+m$ and $3=3k$. Hence, $k=1$, $m=1/2$ and $g(x)=x^3+2$. The inequality (see Fig. 5)

$$x^5-x^2+3 \geq x^3+2 \qquad (3)$$

is true for any $x>0$ as it can be represented in the form $(x-1)^2(x^3+2x^2+2x+1) \geq 0$.

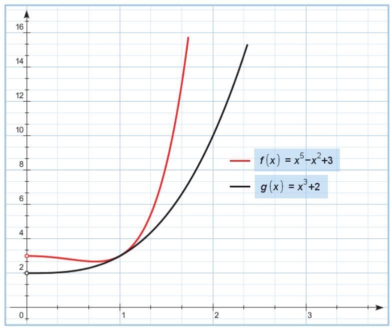

Fig. 5

Example 7. Let a, b, c, d and e be positive real numbers such that

$$\frac{1}{4+a}+\frac{1}{4+b}+\frac{1}{4+c}+\frac{1}{4+d}+\frac{1}{4+e}=1.$$

Prove the inequality

$$\frac{a}{4+a^2}+\frac{b}{4+b^2}+\frac{c}{4+c^2}+\frac{d}{4+d^2}+\frac{e}{4+e^2} \leq 1.$$

Solution. Consider $f(x) = x/(4+x^2)$ and $g(x) = m + k/(4+x)$, where $x \geq 0$. The numbers k and m are to be chosen so that $f(1) = g(1)$ and $f'(1) = g'(1)$. Hence $k = -3$ and $m = 4/5$. Since the inequality

$$\frac{x}{4+x^2} \leq \frac{4}{5}-\frac{3}{4+x}$$

is equivalent to $(x-1)^2(x+1) \geq 0$, it is true for any $x \geq 0$ (see Fig. 6).

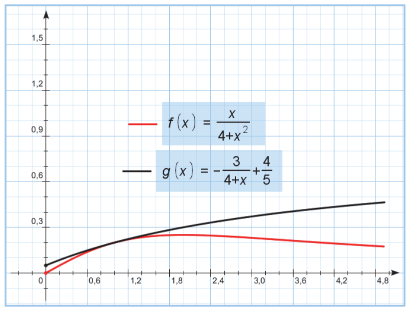

Fig. 6

Applying this inequality, we have

$$\frac{a}{4+a^2}+\frac{b}{4+b^2}+\frac{c}{4+c^2}+\frac{d}{4+d^2}+\frac{e}{4+e^2}$$
$$\leq 4-3\left(\frac{1}{4+a}+\frac{1}{4+b}+\frac{1}{4+c}+\frac{1}{4+d}+\frac{1}{4+e}\right)$$
$$=1.$$

Finally, we have some exercises for the readers.

Exercise 1. (Gabriel Dospinescu) Let $a_1, a_2, \ldots, a_n$ be positive real numbers such that $a_1a_2\cdots a_n = 1$. Prove that

$$\sqrt{1+a_1^2}+\sqrt{1+a_2^2}+\cdots+\sqrt{1+a_n^2}$$
$$\leq \sqrt{2}(a_1+a_2+\cdots+a_n).$$

(continued on page 4)

Problem Corner

We welcome readers to submit their solutions to the problems posed below for publication consideration. The solutions should be preceded by the solver's name, home (or email) address and school affiliation. Please send submissions to *Dr. Kin Y. Li, Department of Mathematics, The Hong Kong University of Science & Technology, Clear Water Bay, Kowloon, Hong Kong.* The deadline for sending solutions is ***April 12, 2014.***

Problem 441. There are six circles on a plane such that the center of each circle lies outside of the five other circles. Prove there is no point on the plane lying inside all six circles.

Problem 442. Prove that if $n > 1$ is an integer, then n^5+n+1 has at least two distinct prime divisors.

Problem 443. Each pair of n ($n\geq 6$) people play a game resulting in either a win or a loss, but no draw. If among every five people, there is one person beating the other four and one losing to the other four, then prove that there exists one of the n people beating all the other $n-1$ people.

Problem 444. Let D be on side BC of equilateral triangle ABC. Let P and Q be the incenters of $\triangle ABD$ and $\triangle ACD$ respectively. Let E be the point so that $\triangle EPQ$ is equilateral and D, E are on opposite sides of line PQ. Prove that lines BC and DE are perpendicular.

Problem 445. For each positive integer n, prove there exists a polynomial $p(x)$ of degree n with integer coefficients such that $p(0)$, $p(1)$, …, $p(n)$ are distinct and each is of the form $2\times 2014^k+3$ for some positive integer k.

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Solutions

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Problem 436. Prove that for every positive integer n, there exists a positive integer $p(n)$ such that the interval $[1, p(n)]$ can be divided into n pairwise disjoint intervals with each contains at least one integer and the sum of the integers in each of these intervals is the square of some integer.

Solution. **Jerry AUMAN, Math Activity Center** (Carmel Alison Lam Foundation Secondary School), **Jon GLIMMS** (Vancouver, Canada) and **ZOLBAYAR Shagdar** (Orchlon International School, Ulaanbaatar, Mongolia).

We look for a pattern. Since $1=1^2$, let $p(1)=1$. Since $2+3+4=3^2$, let $p(2)=1+3=4$ and divide $[1,4]$ into $[1,1]$ and $(1,4]$. Since

$$5+6+7+8+9+10+11+12+13 = 9^2,$$

let $p(3) = 1+3+9=13$ and divide $[1,13]$ into $[1,1]$, $(1,4]$, $(4,13]$.

This suggests we let $p(n) = 1 + 3 + 3^2 + \cdots + 3^{n-1} = (3^n-1)/2$ and divide $[1, p(n)]$ into $[1, p(1)]$, $(p(1), p(2)]$, …, $(p(n-1), p(n)]$. The integers in $(p(k), p(k+1)]$ are from $(3^k+1)/2$ to $(3^{k+1}-1)/2$, which sums to 3^{2k}. So we are done.

Other commended solvers: **Kaustav CHATTERJEE** (MCKV Institute of Engineering College, India) and **SP47** (Hanoi, Vietnam).

Problem 437. Determine all real numbers x satisfying the condition that $\cos x$, $\cos 2x$, $\cos 4x$, …, $\cos 2^nx$, … are all negative.

***Solution* 1. Jerry AUMAN, T. W. LEE** (Alumni of New Method College) and **Math Activity Center** (Carmel Alison Lam Foundation Secondary School).

For such x, we have $2^nx=2\pi(k_n+\theta_n)$, where $k_n\in\mathbb{Z}$ and $1/4 < \theta_n < 3/4$. In base 2 this is $.01_2< \theta_0=.d_1d_2d_3\ldots_2<.10111\ldots_2$. No d_nd_{n+1} can be 00 or 11, otherwise $\theta_{n-1}=.00\ldots_2$ or $.11\ldots_2$ would not be in $(1/4,3/4)$. So $\theta_0 = .010101\ldots_2=1/3$ or $.101010\ldots_2=2/3$. Then $x=2\pi(k_0+1/3)$ or $2\pi(k_0+2/3)$ and for all $n = 0,1,2,\cdots$, $\cos 2^nx = -1/2$.

Solution 2. **Ioan Viorel CODREANU** (Secondary School Satulung, Maramures, Romania) and **GLIMMS** (Vancouver, Canada).

Let $t=\cos 2\theta$. Suppose $\cos\theta$, $\cos 2\theta$ and $\cos 4\theta$ are negative. Then $t<0$ and $2t^2-1<0$ imply $-\sqrt{2}/2<t=2\cos^2\theta-1<0$. We get

$$\cos\theta<-\frac{\sqrt{2-\sqrt{2}}}{2}<-\frac{1}{4}.$$

Suppose $s_n = \cos 2^nx < 0$ for $n = 0,1,2,3,\cdots$. Then $s_n\in[-1, -1/4)$. So $|s_n-1/2| > 3/4$. Using this and $s_{n+1}=2s_n^2-1$, we have

$$\left|s_{n+1}+\frac{1}{2}\right|=2\left|s_n^2-\frac{1}{4}\right|=2\left|\left(s_n-\frac{1}{2}\right)\left(s_n+\frac{1}{2}\right)\right|\geq\frac{3}{2}\left|s_n+\frac{1}{2}\right|.$$

Repeating this, since $-1\leq s_{n+1}<0$, we get

$$\frac{1}{2}\geq\left|s_{n+1}+\frac{1}{2}\right|\geq\left(\frac{3}{2}\right)^n\left|s_0+\frac{1}{2}\right|.$$

Then $|s_0+1/2| < (2/3)^n$. Taking limit, we see $\cos x = s_0= -1/2$, i.e. $x = \pm 2\pi/3+2k\pi$, where k is integer. Conversely, $s_0= -1/2$ implies $s_n= -1/2$ for $n = 1,2,3,\cdots$.

Other commended solvers: **Henry LEUNG Kai Chung** (Graduate of HKUST Maths).

Problem 438. Suppose $P(x)$ is a polynomial with integer coefficients such that for every integer n, $P(n)$ is divisible by at least one of the positive integers $a_1, a_2,\ldots, a_m$. Prove that there exists one of the a_i such that for all integer n, $P(n)$ is divisible by that a_i.

Solution. **Jerry AUMAN, Jon GLIMMS** (Vancouver, Canada) and **Math Activity Center** (Carmel Alison Lam Foundation Secondary School).

Assume the contrary that for each a_i, there exists integer n_i such that $P(n_i)$ is not divisible by a_i. Consider the prime factorizations of a_i and $|P(n_i)|$. Then there exists a prime divisor p_i of a_i such that $d_i = p_i^{e_i}$ is the greatest power of p_i dividing a_i, however d_i does not divide $|P(n_i)|$. If two of the d_i's are powers of the same prime, then eliminate the one with the larger exponent. (In this way, each of $a_1, a_2,\ldots, a_m$ is still divisible by one of the remaining d_i's.)

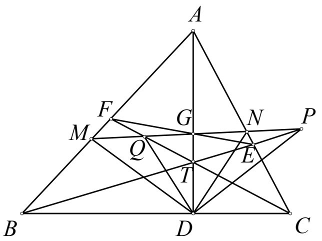
By the Chinese remainder theorem, there exist integers n such that $n \equiv n_i$ (mod d_i) for the remaining d_i's. Now $P(n)-P(n_i)$ is divisible by $n-n_i$, which is divisible by d_i. Since $P(n_i)$ is is not divisible by d_i. So $P(n)$ is not divisible by any d_i's, contradicting $P(n)$ is divisible by at least one of the positive integers $a_1, a_2,\ldots, a_m$, hence also divisible by at least one d_i.

Problem 439. In acute triangle ABC, T is a point on the altitude AD (with D on side BC). Lines BT and AC intersect at E, lines CT and AB intersect at F, lines EF and AD intersect at G. A line ℓ passing through G intersects side AB, side AC, line BT, line CT at M, N, P, Q respectively.

Prove that $\angle MDQ = \angle NDP$.

Solution. **William FUNG** and **Math Activity Center** (Carmel Alison Lam Foundation Secondary School).

Set the origin at D and A, B, C at $(0,a)$, $(b,0)$, $(c,0)$ respectively.



Let T be at $(0,1)$. The equations of the lines BT, CT, AB, AC are

$$y = -(x/b) + 1,\quad y = -(x/c) + 1,$$

$$y = -(ax/b) + a,\quad y = -(ax/c) + a$$

respectively. Since $E = BT \cap AC$ and $F = CT \cap AB$, we can solve the equations of the lines to get

$$E = \left(\frac{(a-1)bc}{ab-c}, \frac{a(b-c)}{ab-c}\right)$$

and $$F = \left(\frac{(a-1)cb}{ac-b}, \frac{a(c-b)}{ac-b}\right).$$

From the y-intercept of line EF, we get $G=(0,\ 2a/(a+1))$. Let the equation of ℓ be $y=mx+2a/(a+1)$. Then $M = \ell \cap AB$ is at

$$\left(\frac{a(a-1)b}{(a+mb)(a+1)}, \frac{a(2a+(a+1)mb)}{(a+mb)(a+1)}\right).$$

Using role symmetry of B and C, we can replace b by c in the coordinates of M to get coordinates of N. Similarly, $P = \ell \cap BT$ is at

$$\left(\frac{-(a-1)b}{(a+mb)(a+1)}, \frac{2a+(a+1)mb}{(a+mb)(a+1)}\right).$$

The coordinates of Q can be found by replacing b by c in the coordinates of P.

Since D is the origin, the slopes of lines DM and DP can be found by taking the y-coordinates of M and P dividing by their respective x-coordinates, which turn out to be the negative of each other! So lines DM and DP are symmetric with respect to the y-axis! Similarly, lines DN and DQ are symmetric with respect to the y-axis. Therefore, $\angle MDQ = \angle NDP$.

Comments: There is a pure geometry solution using a number of equations from applying Menelaus' theorem to different triangles. There is also a solution using harmonic division and cross-ratios from projective geometry.

Other commended solvers: **Georgios BATZOLIS** (Mandoulides High School, Thessaloniki, Greece), **Andrea FANCHINI** (Cantu, Italy), **T. W. LEE** (Alumni of New Method College), **SP47** (Hanoi, Vietnam), **Titu ZVONARU** (Comăneşti, Romania) and **Neculai STANCIU** ("George Emil Palade'' Secondary School, Buzău, Romania).

Problem 440. There are n schools in a city. The i-th school will send C_i students to watch a performance at a field. It is <u>known</u> that $0 \le C_i \le 39$ for $i=1, 2, \ldots, n$ and $C_1+C_2+\cdots+C_n=1990$. The seats will be put in a rectangle arrangement with each row having 199 seats. Determine the least number of rows needed to satisfy the condition that all students from the same school must sit in the same row for all possibilities of the known conditions above.

Solution. **Adnan ALI** (9th Grade, Atomic Energy Central School 4 (AECS4), Mumbai, India), **Jerry AUMAN** and **Jon GLIMMS** (Vancouver, Canada).

Let k be the minimal number of rows needed. For $m = 1, 2, \ldots, k$, let there be a_m students in row m. If there are no more than 160 students in some row, then since each school sends at most 39 students, we can put in students from one more school in that row. So we may assume $a_m \ge 161$. Now

$$1990 = a_1 + a_2 + \cdots + a_k \ge 161k,$$

which implies $k \le 12$.

Next, we show 11 rows may not be enough. Suppose there are $n = 80$ schools with $C_i = 25$ for $i = 1, 2, \ldots, 79$ and $C_{80} = 15$. This totals to 1990 students. Then there can only be one row with $25\times7+15 = 190$ students and the other 10 rows with $25\times7=175$ students. This only totals to 1940 students.

So the least number of rows needed to satisfy the condition that all students from the same school must sit in the same row for all possibilities of the known conditions is 12.

Other commended solvers: **T. W. LEE** (Alumni of New Method College) and **Math Activity Center** (Carmel Alison Lam Foundation Secondary School).

Olympiad Corner

(*Continued from page 1*)

Problem 5. Find the values of x such that the following inequality holds

$$\min\{\sin x, \cos x\} < \min\{1-\sin x, 1-\cos x\}.$$

Problem 6. Find all pairs of prime numbers p and q that satisfy the following equation

$$3p^q - 2q^{p-1} = 19.$$

Problem 7. Is it possible to choose 24 points in the space, such that no three of them lie on the same line and choose 2013 planes in a way that each plane passes through at least 3 of the chosen points and each triple of points belongs to at least one of the chosen planes?

Problem 8. Let M be the midpoint of the internal bisector AD of $\triangle ABC$. Circle ω_1 with diameter AC intersects BM at E and circle ω_2 with diameter AB intersects CM at F. Show that B, E, F, C belong to the same circle.

Using Tangent Lines …

(*Continued from page 2*)

<u>Exercises 2.</u> Let a, b and c be non-negative real numbers. Prove that

$$\frac{a}{b^2+c^2+d^2} + \frac{b}{c^2+d^2+a^2} + \frac{c}{d^2+a^2+b^2} + \frac{d}{a^2+b^2+c^2} \ge \frac{3\sqrt{3}}{2}\cdot\frac{1}{\sqrt{a^2+b^2+c^2+d^2}}.$$

<u>Exercise 3.</u> Let a, b and c be positive real numbers. Determine the minimal value of

$$\frac{3a}{b+c} + \frac{4b}{c+a} + \frac{5c}{a+b}.$$

<u>Exercise 4.</u> Let a, b and c be positive real numbers such that $ab+bc+ca=3$. Prove that

$$(a^7 - a^4 + 3)(b^5 - b^2 + 3)(c^4 - c + 3) \ge 27.$$

<u>References</u>

[1] Chetkovski, Z., <u>*Inequalities. Theorems, Techniques and Problems.*</u> Springer Verlag, Berlin Heidelberg, (2012).

[2] Pham Kim Hung, <u>*Secrets in Inequalities (volume 1).*</u> Editura Gil, Zalău, (2007).