**Olympiad Corner**

*Below are the problems of the Team Selection Test 1 for the Dutch IMO team held in June, 2014.*

**Problem 1.** Determine all pairs \((a,b)\) of positive integers satisfying 
\[
a^2 + b^2 + ab + a \quad \text{and} \quad b^2 - a \quad \text{are all non-zero and for which we have}
\]
\[
\frac{1}{a + b} + \frac{1}{b + ac} = \frac{1}{a + b}.
\]
Prove that \((c-3)(c+1)\) is rational.

**Problem 2.** Let \(\triangle ABC\) be a triangle. Let \(M\) be the midpoint of \(BC\) and let \(D\) be a point on the interior of side \(AB\). The intersection of \(AM\) and \(CD\) is called \(E\). Suppose that \(|AD| = |DE|\). Prove that \(|AB| = |CE|\).

**Problem 3.** Let \(a\), \(b\) and \(c\) be rational numbers for which \(a + bc, b + ac\) and \(a + b\) are all non-zero and for which we have
\[
\frac{1}{a + bc} + \frac{1}{b + ac} = \frac{1}{a + b}.
\]
Prove that \(\sqrt{(c-3)(c+1)}\) is rational.

**Problem 4.** Let \(\triangle ABC\) be a triangle with \(|AC| = |2AB|\) and let \(O\) be its circumcenter. Let \(D\) be the intersection of the angle bisector of \(\angle A\) and \(BC\). Let \(E\) be the orthogonal projection of \(O\) on \(AD\) and let \(F\) be a point on \(AD\) satisfying \(|CD| = |CF|\). Prove that \(|EBF| = |ECF|\).

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**Polygons Problems**

*Kin Yin Li*

In geometry textbooks, we often come across problems about triangles and quadrilaterals. In this article we will present some problems about \(n\)-sided polygons with \(n > 4\). This type of problem appears every few years in math olympiads of many countries.

**Example 1.** Prove that if \(ABCDE\) is a convex pentagon with all sides equal and \(\angle A \geq \angle B \geq \angle C \geq \angle D \geq \angle E\), then it is a regular pentagon.

**Solution.**

Since 
\[
\angle B = \frac{180^\circ - \angle D}{2},
\]
we get \(\angle EAC \geq \angle EAC\). Next,
\[
\angle EAC = \frac{180^\circ - \angle B}{2} = \angle A + \angle B - 90^\circ.
\]
Hence, \(\angle EAC = \angle EAC\). Then equality holds everywhere above so that \(\angle A = \angle E\) and we are done.

**Example 2.** (Bulgaria, 1979) In convex pentagon \(ABCDE\), \(\angle ABC\) and \(\angle CDE\) are equilateral. Prove that if \(O\) is the center of \(\triangle ABC\) and \(M, N\) are midpoints of \(BD, AE\) respectively, then \(\triangle OME \sim \triangle OND\).

**Solution.**

Let \(P, Q\) be the midpoints of \(BC, AC\) respectively. Observe that \(\angle COP = 60^\circ\), \(OC = 2OP\), \(PM \parallel CD\), \(\angle DCE = 60^\circ\) and \(EC = DC = 2MP\). Then rotating about \(O\) by \(60^\circ\) clockwise and follow by doubling distance from \(O\), we see \(\triangle OPM\) goes to \(\triangle OCE\). Hence \(\angle EOM = \angle COP = 60^\circ\) and \(OE = 2OM\). Similarly we can rotate about \(O\) by \(60^\circ\) counterclockwise and double distance from \(O\) to bring \(\triangle OQD\) to \(\triangle OCD\). Then \(\angle DON = 60^\circ\), \(OD = 2ON\) and so \(\triangle OME \sim \triangle OND\).

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The editors welcome contributions from all teachers and students. With your submission, please include your name, address, school, email, telephone and fax numbers (if available). Electronic submissions, especially in MS Word, are encouraged. The deadline for receiving material for the next issue is April 10, 2015.

For individual subscription for the next five issues for the 14-15 academic year, send us five stamped self-addressed envelopes. Send all correspondence to:

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Example 4. (Czechoslovakia, 1974) Prove that if a circumscribed hexagon $ABCD$ satisfies

$$AB = BC, \ CD = DE \text{ and } EF = FA,$$

then the area of $\triangle ACE$ is less than or equal to the area of $\triangle BDF$.

**Solution.** Let $O$ be the circumcenter of hexagon $ABCDEF$ and $R$ be the radius of the circumcircle. Let

$$\alpha = \angle CAE, \beta = \angle AEC, \gamma = \angle ACE.$$

From the given conditions on the sides, we get

$$\angle AOB = \angle BOC = \beta, \quad \angle COD = \angle DOE = \alpha, \quad \angle EOF = \angle FOA = \gamma.$$

Let $[XYZ]$ denote the area of $\triangle XYZ$. We have

$$[ACE] = \frac{EC \cdot CA \cdot AE}{4R} = \frac{2R \sin \alpha \cdot 2R \sin \beta \cdot 2R \sin \gamma}{4R} = 2R^2 \sin \alpha \sin \beta \sin \gamma.$$

Similarly,

$$[BDF] = 2R^2 \sin \frac{\alpha + \beta + \gamma}{2}.$$

Now for positive $\alpha, \beta, \gamma$ satisfying $\alpha + \beta + \gamma = 180^\circ$, we have

$$\sin^2 \alpha \sin^2 \beta \sin^2 \gamma = \sin(\alpha \sin \beta)(\sin \gamma \sin \alpha)(\sin \beta \sin \gamma) = \prod_{\text{cyclic}} \frac{\cos(\alpha - \beta) - \cos(\alpha + \beta)}{2} \leq \prod_{\text{cyclic}} \frac{1 - \cos(\alpha + \beta)}{2} = \sin^2 \frac{\alpha + \beta + \gamma}{2} \sin^2 \frac{\beta + \gamma + \alpha}{2} \times \frac{\gamma + \alpha}{2}.$$

Therefore, $[ACE] \leq [BDF]$.

**Example 5. (IMO 1996)** Let $ABCD$ be a convex hexagon such that $AB$ is parallel to $DE$, $BC$ is parallel to $EF$ and $CD$ is parallel to $FA$. Let $R_a, R_c, R_e$ be the circumradii of triangles $FAB, BCD, DEF$ respectively, and let $P$ denote the perimeter of the hexagon. Prove that

$$R_a + R_c + R_e \geq \frac{P}{2}.$$

**Solution.** Let $a, b, c, d, e, f$ denote the lengths of the sides $AB, BC, CD, DE, EF, FA$ respectively. By the parallel conditions, we have $\angle A = \angle D, \angle B = \angle E, \angle C = \angle F$.

Consider rectangle $PQRS$ such that $A$ is on $PQ$, $E$ is on $QR$, $D$ is on $RS$ and $B, C$ are on $SP$.

We have $BF \geq PQ = SR$. So $2BF \geq PQ + SR$, which is the same as

$$2BF \geq (asinB + f \sin C) + (c \sin C + d \sin B).$$

Similarly,

$$2BD \geq (cis A + b \sin B) + (b \sin B + f \sin A),$$

$$2DF \geq (c \sin C + d \sin A) + (a \sin A + b \sin C).$$

Next, by the extended sine law,

$$R_a = \frac{BF}{2 \sin A}, \quad R_c = \frac{BD}{2 \sin C}, \quad R_e = \frac{DE}{2 \sin E}.$$

Then using the inequalities and equations above, we have

$$R_a + R_c + R_e \geq \frac{a \sin B + b \sin A + f}{4 \sin A + b \sin B} + \frac{c \sin C + d \sin A + e}{4 \sin C + d \sin A} \geq \frac{a + b + c + d + e + f}{2} = \frac{P}{2}.$$

**Example 6. (Great Britain, 1988)** Let four consecutive vertices $A, B, C, D$ of a regular polygon satisfy

$$\frac{1}{AB} = \frac{1}{AC} + \frac{1}{AD}.$$

Determine the number of sides of the polygon.

**Solution.** Let the circumcircle of the polygon have center $O$ and radius $R$. Let $\alpha = \angle AOB$, then $0 < 3\alpha = \angle AOD < 360^\circ$. So $0 < \alpha < 120^\circ$. Also, from

$$AB = 2R \sin \alpha, \quad AC = 2R \alpha,$$

$$AD = 2R \sin \frac{3\alpha}{2},$$

we get

$$\frac{1}{2 \sin \alpha} = \frac{1}{2 \sin \alpha} + \frac{1}{\sin \frac{3\alpha}{2}}.$$

Clearing denominators, we have

$$0 = \sin \alpha \cos \frac{3\alpha}{2} - \sin \alpha \sin \frac{3\alpha}{2} = \frac{1}{2} \left( \cos \alpha - \cos \frac{3\alpha}{2} \right) - \frac{1}{2} \left( \cos \alpha \cos \frac{3\alpha}{2} \right).$$

Then $7\alpha/4 = 90^\circ$, that is $\alpha = 360^\circ / 7$. So the polygon has 7 sides.

**Example 7. (Austria, 1973)** Prove that if the angles of a convex octagon are all equal and the ratio of all pairs of adjacent sides is rational, then each pair of opposite sides has equal length.

**Solution.** Without loss of generality, we may assume the sides of such a polygon $A_1A_2...A_n$ are rational (since the conclusion is the same for octagons similar to such an octagon). Now the sum of all angles of the octagon is $6 \times 180^\circ$. Hence each angle is $45^\circ$.

Let $v_i$ be the vector from $A_i$ to $A_{i+1}$ for $n=1,2,...,8$ (with $A_{n+1}=A_1$). Then the angle between $v_i$ and $v_{i+1}$ at the origin is $45^\circ$. Observe that the sum of these vectors is zero since we start at $A_1$ and traverse the octagon once to return to $A_1$.

Let $i$ and $j$ be a pair of unit vectors perpendicular to each other at the origin. By rotation, we may assume $v_1$ is a vector in the $i$ direction and $v_1$ is in the $j$ direction. Then $v_1 + v_2 = xi$ and $v_3 + v_4 = yj$ for some rational $x$ and $y$. Also,

$$v_1 + v_4 + v_5 + v_6 = r\sqrt{2}i + r\sqrt{2}j$$

for some rational $r$. Then

$$(x + r\sqrt{2})i + (y + r\sqrt{2})j = \sum_{n=1}^{8} v_n = 0.$$

Since, $x$ and $r$ are rational, we must have $x = r = 0$. That is, $v_1 = -v_1$. By rotating the $i, j$ vectors by $45^\circ$, similarly we get $v_6 = -v_6$. Then also $v_7 = -v_7$ and $v_8 = -v_8$. The result follows.

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Problem Corner

We welcome readers to submit their solutions to the problems posed below for publication consideration. The solutions should be preceded by the solver’s name, home (or email) address and school affiliation. Please send submissions to Dr. Kin Y. Li, Department of Mathematics, The Hong Kong University of Science & Technology, Clear Water Bay, Kowloon, Hong Kong. The deadline for sending solutions is April 10, 2015.

Problem 461. Inside rectangle $ABCD$, there is a circle. Points $W, X, Y, Z$ are on the circle such that lines $AW, BX, CY, DZ$ are tangent to the circle. If $AW=3, BX=4, CY=5$, then find $DZ$ with proof.

Problem 462. For all $x_1, x_2, \ldots, x_n \geq 0$, let $x_{n+1} = x_1$, then prove that

$$\sum_{i=1}^{n} \frac{1}{(x_{i}+1)^{2}} + \frac{1}{(x_{i+1}+1)^{2}} \geq \frac{n}{2}.$$  

Problem 463. Let $S$ be a set with 20 elements. $N$ 2-element subsets of $S$ are chosen with no two of these subsets equal. Find the least number $N$ such that among any 3 elements in $S$, there exist 2 of them belong to one of the $N$ chosen subsets.

Problem 464. Determine all positive integers $n$ such that for $n$, there exists an integer $m$ with $2^{m}-1$ divides $m^{2}+289$.

Problem 465. Points $A, E, D, C, F, B$ lie on a circle $\Gamma$ in clockwise order. Rays $AD, BC$, the tangents to $\Gamma$ at $E$ and at $F$ pass through $P$. Chord $EF$ meets chords $AD$ and $BC$ at $M$ and $N$ respectively. Prove that lines $AB, CD, EF$ are concurrent.

*************** Solutions  ***************

Problem 456. Suppose $x_1, x_2, \ldots, x_n$ are non-negative and their sum is 1. Prove that there exists a permutation $\sigma$ of $\{1,2,\ldots,n\}$ such that $x_{\sigma(1)x_{\sigma(2)}x_{\sigma(3)}^{r} \cdots x_{\sigma(n)}x_{\sigma(1)}^{s}} \leq 1/n$.

Solution. CHAN Long Tin (Cambridge University, Year 3), Ioan Viorel CODREANU (Secondary School Satulung, Maramures, Romania), KWOK Man Yi (Baptist Lui Ming Choi Secondary School, S4), Samiron SADHUHKAN (Kendriya Vidyalaya, India) and WONG Yat (G. T. (Ellen Yeung) College).

Assume the contrary is true. Let $\sigma(n+1) = \sigma(1)$ for all permutations $\sigma$. For $1 \leq i < j \leq n$, the terms $x_{\sigma(i)}$ and $x_{\sigma(j)}$ appear a total of $2n(n-2)!$ times in

$$\sum_{i=1}^{n} \sum_{k=1}^{n} x_{\sigma(i)}x_{\sigma(j)}x_{\sigma(k)}.$$  

So, we have

$$\frac{n!}{n} \leq \sum_{i=1}^{n} \sum_{k=1}^{n} x_{\sigma(i)}x_{\sigma(j)}x_{\sigma(k)}$$

$$= 2n(n-2)! \sum_{i=1}^{n} x_{i}$$

$$= n(n-2)! \left( \sum_{i=1}^{n} x_{i} \right)^{2} - \sum_{i=1}^{n} x_{i}^{2}$$

$$= n(n-2)! \left( 1 - \sum_{i=1}^{n} x_{i}^{2} \right)$$

This simplifies to $2n(n-2)! \leq 1/n$. However, by the Cauchy-Schwarz inequality,

$$\sum_{i=1}^{n} x_{i}^{2} \geq \left( \sum_{i=1}^{n} x_{i} \right)^{2} \geq 1,$$

which contradicts (*)

Problem 457. Prove that for each $n = 1,2,3,\ldots$, there exist integers $a, b$ such that if integers $x, y$ are relatively prime, then $\sqrt{(a-x)^{2} + (b-y)^{2}} > n$.

Solution. Samiron SADHUHKAN (Kendriya Vidyalaya, India) and WONG Yat (G. T. (Ellen Yeung) College).

There are $2(n+1)^{2}$ ordered pairs $(r,s)$ of integers satisfying $|r|, |s| \leq n$. Assign a distinct prime number $p_{rs}$ to each such $(r,s)$. By the Chinese remainder theorem, there exist integers $a,b$ such that for all integers $r,s$ satisfying $|r|, |s| \leq n$, we have $a \equiv r \pmod{p_{rs}}$ and $b \equiv s \pmod{p_{rs}}$.

Let integers $x, y$ be relatively prime. Assume $x_{y}$ has distance at most $n$ from $(a,b)$, then $|a-x| \leq n$ and $|b-y| \leq n$. Let $a-x = r$ and $b-y = s$. Then $x = a-r$ and $y = b-s$ are multiples of $p_{rs}$, contradicting $\gcd(x,y) = 1$.

Therefore,

$$\sqrt{(a-x)^{2} + (b-y)^{2}} > n.$$  

Problem 458. Nonempty sets $A_1, A_2, A_3$ form a partition of $\{1,2,\ldots,n\}$. If $x+y=n$ have no solution with $x$ in $A_1$, $y$ in $A_2$, $z$ in $A_3$ and $\{i,j,k\} = \{1,2,3\}$, then prove that $A_1, A_2, A_3$ cannot have the same number of elements.

Solution. Oliver GEUPEL (Brühl, NRW, Germany) and John GLIMMS.

Without loss of generality, say $1 \in A_1$ and the smallest element in $A_2 \cup A_3$ is in $A_2$. Let the elements in $A_2$ be $c_1, c_2, \ldots, c_i$ in increasing order.

Assume $c_i+1 \leq c_{i+1}$ for some $i$. Then take $i$ to be the smallest possible. Since $b \in A_2$, the equations $(c_i-b) + c_{i+1}$ and $(c_{i+1}+1) - c_{i+1}$ imply $c_i-b$ and $c_{i+1}+1$ are both not in $A_1$.

Since $1 \in A_1$ and $(c_i-b)+1 = c_i+1$, so either $c_i-b+1$ and $c_i-b$ both are in $A_2$ or both are in $A_1$. Since $i$ is smallest such that $c_i-b+1 = c_i+1$, so $c_i-b+1$ and $c_i-b$ cannot be in $A_1$. However, $c_i-b+1$ and $c_i-b$ in $A_2$, $b-1$ in $A_1$ (by property of $b$) and $(b+1)+(c_i-b+1) = c_i$ lead to contradiction. So $c_i-c_i \geq 2$ for all $i$.

Finally, since $1+c_i \leq c_i$, we get $c_i-1 \leq c_i$. Hence $c_i-1 \leq c_i$. Then $A_1$ contains $c_1 < c_2 < \ldots < c_i$. Therefore, $A_1$ has more elements than $A_3$.

Problem 459. $H$ is the orthocenter of acute $\Delta ABC, D, E, F$ are midpoints of sides $BC, CA, AB$ respectively. Inside $\Delta ABC$, a circle with center $H$ meets $DE$ at $P, Q, EF$ at $R, S, FD$ at $T, U$. Prove that $CP = CQ = AR = AS = BT = BU$.

Solution. John GLIMMS.

Let lines $AH$ and $FE$ meet at $J$. From $AH \perp BC$ and $BC \parallel FE$, we get $FE$ is perpendicular to $AJ$ and $HJ$. By folding along $DE, EF$ and $FD$, we can make a tetrahedron having $\Delta DEF$ as the base and points $A, B, C$ meet at a point $J$. Then $FE$ is perpendicular to $LI$ and $HJ$. So $FE$ is perpendicular to the plane through $I, H, J$. Then $FE \perp LI$. Similarly, $DE \perp LI$. Then the plane through $DEF$ is perpendicular to $HI$. By Pythagoras’ theorem, $IH^{2} = r^{2} = CP^{2} = CQ^{2} = AR^{2} = AS^{2} = BT^{2} = BU^{2}$, where $r$ is the radius of the circle.

Other commend solvers: Adnan ALI (Atomic Energy Central School 4, Mumbai, India), Andrea FANCHINI.
Problem 460. If \(x, y, z > 0\) and \(x+y+z+2 = xyz\), then prove that
\[
x + y + z + 6 \geq 2\left(\sqrt{yz} + \sqrt{zx} + \sqrt{xy}\right).
\]

Solution. Adnan ALI (Atomic Energy Central School 4, Mumbai, India), CHAN Long Tin (Cambridge University, Year 3), Ioan Viorel CODREANU (Secondary School Satulung, Maramures, Romania), Oliver GEUPEL (Brühl, NRW, Germany), KWOK Man Yi (Baptist Lui Ming Choi Secondary School, S4), Vijaya Prasad NULLARI (Retired Principal, AP Educational Service, India),Nicu ŞLOTA (“Traian Vuia” Technical College, Focşani, Romania) and Titu ZVONARU (Comăneşti, Romania).

Let
\[
a = \frac{1}{1+x}, \quad b = \frac{1}{1+y}, \quad c = \frac{1}{1+z}.
\]
Using \(x+y+z+2 = xyz\), we get \(a+b+c = 1\). If \(x = (1-a)/a = (b+c)/a\) and similarly \(y=(c+a)/b\) and \(z=(a+b)/c\). By the AM-GM inequality, we have
\[
x + y + z + 6 = \sum \frac{b+c}{a} + 6
\]
\[
= \sum \frac{c+a}{b} + \frac{a+b}{c}
\]
\[
\geq \frac{\sum (c+a)(a+b)}{bc}
\]
\[
= 2\left(\sqrt{yz} + \sqrt{zx} + \sqrt{xy}\right).
\]

Other commended solvers: Paolo PERFETTI (Dipartimento di Matematica, Università degli studi di Tor Vergata Roma, via della ricerca scientifica, Roma, Italy), WONG Yat (G.T. ( Ellen Yeung) College).

Olympiad Corner

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Problem 5. On each of the 2014 squares of a 2014×2014-board a light bulb is put. Light bulbs can be either on or off. In the starting situation a number of light bulbs are on. A move consists of choosing a row or column in which at least 1007 light bulbs are on and changing the state of all 2014 light bulbs in this row or column (from on to off or from off to on). Find the smallest non-negative integer \(k\) such that from each starting situation there is a finite sequence of moves to a situation in which at most \(k\) light bulbs are on.

Example 8. (IMO 1997) Equilateral triangles \(ABK, BCL, CDM, DAN\) are constructed inside the square \(ABCD\). Prove that the midpoints of the four segments \(KL, LM, MN, NK\) and the midpoints of the eight segments \(AK, BK, BL, CL, CM, DM, DN, AN\) are the twelve vertices of a regular dodecagon.

Solution.

Let us denote the midpoints of segments \(LM, AN, BL, MN, BK, CM, NK, CL, DN, KL, DM, AK\) by \(P_1, P_2, P_3, P_4, P_5, P_6, P_7, P_8, P_9, P_{10}, P_{11}, P_{12}\), respectively. To prove the dodecagon
\[P_1P_2P_3P_4P_5P_6P_7P_8P_9P_{10}P_{11}P_{12}\]
is regular, we observe that \(BL = BA\) and \(\angle ABL = 30^\circ\). Then \(\angle BAL = 75^\circ\). Similarly \(\angle DAM = 75^\circ\).

\[\angle LAM = \angle BAL + \angle DAM - \angle BAD = 60^\circ\]
Along with \(\angle ALM = \angle AM\), we see triangle \(ALM\) is equilateral.

Looking at triangles \(OLM\) and \(ALN\), we get \(OP_1 = \frac{1}{2}LM, OP_2 = \frac{1}{2}AL\) and \(OP_{12} = AL\). Hence, \(OP_1 = OP_2 = OP_3 = \angle P_1AL = 30^\circ\), \(\angle P_2OM = \angle DAL = 15^\circ\) and \(\angle P_3OP_7 = 2\angle P_1OM = 30^\circ\). By symmetry, we can conclude that the dodecagon is regular.

Example 9. (IMO 1992, Shortlisted Problem from India) Show that in the plane there exists a convex polygon of 1992 sides satisfying the following conditions:

(i) its sides lengths are 1, 2, 3, ..., 1992 in some order;

(ii) the polygon is circumscribable about a circle.

Solution. For a positive number \(r\), let us draw a circle of radius \(r\) and let us draw a polygonal path \(A_1A_2...A_{1992}\) such that for \(i=1\) to 1992, side \(A_iA_{i+1}\) is tangent to the circle at a point \(T_i\), and\(T_iA_iT_{i+1}\) = \(A_1T_1, T_2A_2 = A_2T_2, \ldots, T_{1992}A_{1992} = A_{1992}T_{1992}\).

To achieve condition (i), we need \(A_1A_2, A_2A_3, \ldots, A_{1992}A_{1991}\) to be a permutation of 1, 2, 3, ..., 1992. This can be done as follow:

If \(i\equiv 1\) (mod 4), then let \(AT_i=1/2\).

If \(i\equiv 3\) (mod 4), then let \(AT_i=3/2\).

If \(i\equiv 2\) (mod 4), then let \(AT_i=3/2\).

We can check that the lengths of \(AA_{i+1}\) for \(i=1\) to 1992 are 1, 2, 4, 3, 5, 6, 8, 7, ..., 1989, 1990, 1992, 1991.

To achieve condition (ii), we define a function
\[f(r) = \sum_{i=1}^{1992} \angle A_iOA_{i+1}\]
\[= 2\sum_{i=1}^{1992} \arctan \frac{AT_i}{r}\]
Observe that \(f(r)\) is a continuous function on \((0, \infty)\). As \(r\) tends to 0, \(f(r)\) tends to infinity and as \(r\) tends to infinity, \(f(r)\) tends to 0. By the intermediate value theorem, there exists \(r\) such that \(f(r) = 2\pi\). Then \(A_{1993}A_1\) and \(A_1A_2\ldots A_{1992}\) is a desired polygon.

We remark that if 1992 is replaced by other positive integers of the form 4\(k\), then there are such 4\(k\)-sided polygon.