Mathematical Excalibur

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Olympiad Corner

The 2002 Canadian Mathematical Olympiad

Problem 1. Let S be a subset of $\{1, 2, \ldots, 9\}$, such that the sums formed by adding each unordered pair of distinct numbers from S are all different. For example, the subset $\{1, 2, 3, 5\}$ has this property, but $\{1, 2, 3, 4, 5\}$ does not, since the pairs $\{1, 4\}$ and $\{2, 3\}$ have the same sum, namely 5. What is the maximum number of elements that S can contain?

Problem 2. Call a positive integer n **practical** if every positive integer less than or equal to n can be written as the sum of distinct divisors of n.

For example, the divisors of 6 are **1**, **2**, **3**, and **6**. Since

$$1 = \mathbf{1},\quad 2 = \mathbf{2},\quad 3 = \mathbf{3},\quad 4 = \mathbf{1} + \mathbf{3},\quad 5 = \mathbf{2} + \mathbf{3},\quad 6 = \mathbf{6}$$

we see that 6 is practical.

Prove that the product of two practical numbers is also practical.

(continued on page 4)

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On-line: http://www.math.ust.hk/mathematical_excalibur/

The editors welcome contributions from all teachers and students. With your submission, please include your name, address, school, email, telephone and fax numbers (if available). Electronic submissions, especially in MS Word, are encouraged. The deadline for receiving material for the next issue is ***December 15, 2002***.

For individual subscription for the next five issues for the 01-02 academic year, send us five stamped self-addressed envelopes. Send all correspondence to:

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簡介費馬數

梁達榮

考慮形狀如 2^m+1 的正整數，如果它是質數，則 m 一定是 2 的正次冪。否則的話，設 $m = 2^n s$，其中 s 是 3 或以上的奇數，我們有 $2^m+1 = 2^{2^n s}+1 = (2^{2^n})^s+1 = (2^{2^n}+1)((2^{2^n})^{s-1} - (2^{2^n})^{s-2} + \cdots \pm 1)$，容易看到 2^m+1 分解成兩個正因子的積。 業餘數學家費馬(1601-1665)曾經考慮過以下這些"費馬"整數，設 $F_n = 2^{2^n}+1$，$n=0,1,2,\ldots$，費馬看到 $F_0 = 2^{2^0}+1=3$，$F_1=2^{2^1}+1 = 5$，$F_2=2^{2^2}+1=17$，$F_3=2^{2^3}+1=257$，$F_4=2^{2^4}+1=65537$，都是質數，(最後一個是質數，需要花些功夫證明)，他據此而猜想，所有形如 $2^{2^n}+1$ 的正整數都是質數。

不幸的是，大概一百年後，歐拉$(1707 - 1783)$發現，F_5 不是質數，事實上，直到現在，已知的 F_n，$n\ge 5$，都不是質數。對於 F_5 不是質數，有一個簡單的證明。事實上 $641 = 5^4+2^4 = 5\times2^7+1$，因此 641 整除 $(5^4+2^4)2^{28} = 5^4\times2^{28}+2^{32}$。另一方面，由於 $641 = 5\times2^7+1$，因此 641 也整除 $(5\times2^7+1)(5\times2^7-1) = 5^2\times 2^{14}-1$，由此，得到 641 整除 $(5^2\times2^{14}-1)(5^2\times2^{14}+1) = 5^4\times 2^{28}-1$。最後 641 整除 $5^4\times2^{28}+2^{32}$ 和 $5^4\times2^{28}-1$ 之差，即是 $2^{32}+1=F_5$。

這個證明很簡潔，但並不自然，首先，如何知道一個可能的因子是 641，其二，641 能夠寫成兩種和式，實有點幸運。或許可以探究一下，歐拉是怎樣發現 F_5 不是質數。我們相信大概的過程是這樣的，歐拉觀察到，如果 p 是 $F_n = 2^{2^n}+1$ 的質因子，則 p 一定是 $k\cdot2^{n+1}+1$ 的形式。用模算術的言語，如果 p 整除 $2^{2^n}+1$，則 $2^{2^n}\equiv -1 \pmod p$，取平方，得出 $2^{2^{n+1}}\equiv 1\pmod p$。另外，用小費馬定理，(歐拉時已經存在)，知 $2^{p-1}\equiv 1\pmod p$。如果 d 是最小的正整數，使得 $2^d\equiv 1\pmod p$，可以證明(請自証)，d 整除 $p-1$，也整除 2^{n+1}，但 d 不整除 2^n，(因為 $2^{2^n}\equiv -1\pmod p$)，所以 $d = 2^{n+1}$，再因 d 整除 $p-1$，所以 $p-1 = k\cdot 2^{n+1}$，或者 $p = k\cdot2^{n+1}+1$。(如果用到所謂的二次互反律，還可以證明，p 實際上是 $k\cdot2^{n+2}+1$ 的形式。)

例如考慮 F_4，它的質因子一定是 $32k+1$ 的形式，取 $k = 1, 2, \ldots$，等，得可能的因子是 97, 193，(小於 $\sqrt{65537}$，以 $32k+1$ 形式出現的質數)。但 97 和 193 都不整除 65537，所以 65537 是質數。另外，F_5 的質因子一定是 $64k+1$ 的形式，取 $k = 1, 2, \ldots$，等，得可能的因子是 193, 257, 449, 577, 641, …，經幾次嘗試，得出 $2^{2^5}+1 = 4294967297 = 641\times 6700417$，這樣快就找出 F_5 的一個質因子，也算幸運，事實上第二個因子也是質數，不過要證明就比較麻煩。

但是如果試圖用這樣的方法找尋其他費馬數的因子，很快就遇上問題。舉例說，$F_6 = 2^{2^6}+1$ 是一個二十位數，它的平方根是一個十位數 $(\approx 4.29\times10^9)$，其中形狀如 $k\cdot2^7+1 = 128k+1$ 的數有三百多萬個，要從中找尋 F_6 的因子，可不是易事。讀者可以想像一下，F_5 的完全分解歐拉在 1732 年已找到，而在一百年後 Landry 和 Le Lasseur (1880) 才找到 F_6 的完全分解，再過約一百年， Morrison 和 Brillhart (1970) 發現 F_7 的完全分解，因此找尋費馬數的因子分解肯定不是易事。另一方面由於找尋費馬數不是

易事，Pepin在1877年找到費馬數是否質數的一個判斷：$N>3$是一個形如 $2^{2^n}+1$ 的費馬數，則N是質數的一個充分必須條件是 $3^{\frac{N-1}{2}} \equiv -1 \pmod N$。 考慮到 $\frac{N-1}{2}=2^{2^n-1}$ ，因此是對3不斷取平方，然後求對 N 的摸。近代對求費馬數是否一個質數上，許多都以此為起點。也因此，曾經有一段長時間，已經知道 F_7 不是質數，但它的任一因子都不知道。

再簡述一下近代的結果，現在已知由 F_5 至 F_{11} ，都是合數，並且已完全分解。 F_{12},F_{13},F_{15} 至 F_{19} 是合數，並且知道部分因子。但 F_{14} , F_{20} , F_{22} 等，知道是合數，但一個因子也不知道。最大的費馬合數，並且找到一個因子的是 F_{382447} ，讀者可想像一下，如果以十進制形式寫下這個數，它是多少個位數。另外如 F_{33},F_{34},F_{35} 等，究竟是合數或質數，一點也不知道。有興趣的話，可參考網頁 http://www.fermatsearch.org/status.htm。

由於費馬數和相關的數有特定的形式，而且具備很多有趣的性質,因此也常在競賽中出現。舉例如下：

例一： 給定費馬數 $F_0,F_1,\ldots,F_n$，有以下的關係 $F_0F_1\cdots F_{n-1}+2=F_n$。

證明： 事實上 $F_n=2^{2^n}+1=2^{2^n}-1+2$ $=2^{2^{n-1}2}-1+2=(2^{2^{n-1}}+1)(2^{2^{n-1}}-1)+2$ $=(2^{2^{n-1}}-1)F_{n-1}+2$。

對於 $2^{2^{n-1}}-1$ ，可以再分解下去，就可以得到要求的結果。當然嚴格證明可以用歸納法。

例二： 給定費馬數 $F_m,F_n,\ m>n$，則 F_m,F_n 是互質的。

證明： 因為 $F_m=F_{m-1}\cdots F_n\cdots F_0+2$。設 d 整除 F_m 和 F_n，則 d 也 整除2，所以$d=1$或2。但 $d\neq 2$ ，因為 F_m,F_n 都是奇數，因此 $d=1$ ，即 F_m,F_n 互質。

(因此知道， $F_0,F_1,F_2,\ldots$，是互質的，即他們包括無限多數個質因子，引申是有無限多個質數。)

例三： 有無限多個 n，使得 F_n+2 不是質數。

證明： 只要嘗試幾次就可以觀察到 $F_1+2=7,\ F_3+2=259$, 都是7的倍數。事實上，對於$n=0,1,2,\ldots,\ 2^{2^n}\equiv 2,4,2,4,\ldots \pmod 7$。因此對於奇數$n$, $F_n+2\equiv 2^{2^n}+1+2\equiv 4+1+2\equiv 0 \pmod 7$，所以不是質數。

另一個容易看到的事實是：

例四： 對於 $n>1,\ F_n$ 最尾的數字是 7。

證明: 對於 $n>1,\ 2^n$ 是 4 的倍數，設 $2^n=4k$, 得 $F_n=2^{2^n}+1=2^{4k}+1=(2^4)^k+1\equiv 1^k+1\equiv 2 \pmod 5$。因此 F_n 最尾的數字是 2 或 7，它不可以是 2，因為 F_n 不是偶數。

例五： 證明存在一個正整數 k，使得對任何正整數 n， $k\cdot 2^n+1$ 都不是質數。

（如果n固定，但容許k在正整數中變動，由一個重要的定理 (Dirichlet) 知道在序列中存在無限多個質數。但若果k固定,而n變動,在序列中究竟有多少個質數，是否無限多個，一般都不大清楚。事實上，反可以找到一個k，對於任何正整數n， $k\cdot 2^n+1$ 都不是質數。這原是波蘭數學家Sierpinski (1882-1969)的一個結果，後來演變成美國數學奧林匹克(1982)的一個題目，直到現在，基本是只有一種證明方法，並且與費馬數有關。）

(續於第四頁)

The 2002 Hong Kong IMO team at the Hong Kong Chek Lap Kok Airport taken on August 1, 2002. From left to right, *Chau Suk Ling*, *Chao Khek Lun*, *Cheng Kei Tsi*, *Chiang Kin Nam* (Deputy Leader), *Yu Hok Pun*, *Ip Chi Ho*, *Leung Wai Ying*, *Li Kin Yin* (Leader).

Problem Corner

We welcome readers to submit their solutions to the problems posed below for publication consideration. Solutions should be preceded by the solver's name, home (or **email**) address and school affiliation. Please send submissions to *Dr. Kin Y. Li, Department of Mathematics, The Hong Kong University of Science & Technology, Clear Water Bay, Kowloon.* The deadline for submitting solutions is ***December 15, 2002***.

Problem 161. Around a circle are written all of the positive integers from 1 to N, $N \geq 2$, in such a way that any two adjacent integers have at least one common digit in their base 10 representations. Find the smallest N for which this is possible.

Problem 162. A set of positive integers is chosen so that among any 1999 consecutive positive integers, there is a chosen number. Show that there exist two chosen numbers, one of which divides the other.

Problem 163. Let a and n be integers. Let p be a prime number such that $p > |a| + 1$. Prove that the polynomial $f(x) = x^n + ax + p$ cannot be a product of two nonconstant polynomials with integer coefficients.

Problem 164. Let O be the center of the excircle of triangle ABC opposite A. Let M be the midpoint of AC and let P be the intersection of lines MO and BC. Prove that if $\angle BAC = 2\angle ACB$, then $AB = BP$.

Problem 165. For a positive integer n, let $S(n)$ denote the sum of its digits. Prove that there exist distinct positive integers $n_1, n_2, \ldots, n_{50}$ such that

$$n_1 + S(n_1) = n_2 + S(n_2) = \cdots = n_{50} + S(n_{50}).$$

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Solutions

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Problem 156. If $a, b, c > 0$ and $a^2 + b^2 + c^2 = 3$, then prove that

$$\frac{1}{1+ab} + \frac{1}{1+bc} + \frac{1}{1+ca} \geq \frac{3}{2}.$$

(*Source: 1999 Belarussian Math Olympiad*)

Solution. **SIU Tsz Hang** (STFA Leung Kau Kui College, Form 7) and **WONG Wing Hong** (La Salle College, Form 5).

By the AM-GM and AM-HM inequalities, we have

$$\frac{1}{1+ab} + \frac{1}{1+bc} + \frac{1}{1+ca}$$

$$\geq \frac{1}{1+\frac{a^2+b^2}{2}} + \frac{1}{1+\frac{b^2+c^2}{2}} + \frac{1}{1+\frac{c^2+a^2}{2}}$$

$$\geq \frac{9}{3+a^2+b^2+c^2} = \frac{3}{2}.$$

Other commended solvers: **CHAN Wai Hong** (STFA Leung Kau Kui College, Form 7), **CHAN Yat Fei** (STFA Leung Kau Kui College, Form 6), **CHEUNG Yun Kuen** (Hong Kong Chinese Women's Club College, Form 5), **CHU Tsz Ying** (St. Joseph's Anglo-Chinese School, Form 7), **CHUNG Ho Yin** (STFA Leung Kau Kui College, Form 6), **KWOK Tik Chun** (STFA Leung Kau Kui College, Form 5), **LAM Ho Yin** (South Tuen Mun Government Secondary School, Form 6), **LAM Wai Pui** (STFA Leung Kau Kui College, Form 6), **LEE Man Fui** (STFA Leung Kau Kui College, Form 6),**Antonio LEI** (Colchester Royal Grammar School, UK, Year 12), **LO Chi Fai** (STFA Leung Kau Kui College, Form 7), **POON Ming Fung** (STFA Leung Kau Kui College, Form 5), **TAM Choi Nang Julian** (SKH Lam Kau Mow Secondary School, teacher), **TANG Ming Tak** (STFA Leung Kau Kui College, Form 6), **TANG Sze Ming** (STFA Leung Kau Kui College, Form 5), **YAU Chun Biu** and **YIP Wai Kiu** (Jockey Club Ti-I College, Form 5) and **Richard YEUNG Wing Fung** (STFA Leung Kau Kui College, Form 5).

Problem 157. In base 10, the sum of the digits of a positive integer n is 100 and of $44n$ is 800. What is the sum of the digits of $3n$? (*Source: 1999 Russian Math Olympiad*)

Solution. **CHAN Wai Hong** (STFA Leung Kau Kui College, Form 7), **CHAN Yat Fei** (STFA Leung Kau Kui College, Form 6), **Antonio LEI** (Colchester Royal Grammar School, UK, Year 12), **LO Chi Fai** (STFA Leung Kau Kui College, Form 7), **POON Ming Fung** (STFA Leung Kau Kui College, Form 5), **SIU Tsz Hang** (STFA Leung Kau Kui College, Form 7), **TANG Ming Tak** (STFA Leung Kau Kui College, Form 6), and **WONG Wing Hong** (La Salle College, Form 5).

Let $S(x)$ be the sum of the digits of x in base 10. For digits a and b, if $a + b > 9$, then $S(a + b) = S(a) + S(b) - 9$. Hence, if we have to carry in adding x and y, then $S(x + y) < S(x) + S(y)$. So in general, $S(x + y) \leq S(x) + S(y)$. By induction, we have $S(kx) \leq kS(x)$ for every positive integer k. Now

$$\begin{aligned} 800 &= S(44n) = S(40n + n) \\ &\leq S(40n) + S(4n) = 2S(4n) \\ &\leq 8S(n) = 800. \end{aligned}$$

Hence equality must hold throughout and there can be no carry in computing $4n = n + n + n + n$. So there is no carry in $3n = n + n + n$ and $S(3n) = 300$.

Other commended solvers: **CHU Tsz Ying** (St. Joseph's Anglo-Chinese School, Form 7).

Problem 158. Let ABC be an isosceles triangle with $AB = AC$. Let D be a point on BC such that $BD = 2DC$ and let P be a point on AD such that $\angle BAC = \angle BPD$. Prove that

$$\angle BAC = 2\angle DPC.$$

(*Source: 1999 Turkish Math Olympiad*)

Solution. **LAM Wai Pui** (STFA Leung Kau Kui College, Form 6), **POON Ming Fung** (STFA Leung Kau Kui College, Form 5), **SIU Tsz Hang** (STFA Leung Kau Kui College, Form 7) , **WONG Wing Hong** (La Salle College, Form 5) and **Richard YEUNG Wing Fung** (STFA Leung Kau Kui College, Form 5).

Let E be a point on AD extended so that $PE=PB$. Since $\angle CAB=\angle EPB$ and $CA/AB=1=EP/PB$, triangles CAB and EPB are similar. Then $\angle ACB=\angle PEB$, which implies A, C, E, B are concyclic. So $\angle AEC=\angle ABC=\angle AEB$. Therefore, AE bisects $\angle CEB$.

Let M be the midpoint of BE. By the angle bisector theorem, $CE/EB = CD/DB = 1/2$. So $CE = ½EB = ME$. Also, $PE = PE$ and PE bisects $\angle CEM$. It follows triangles CEP and MEP are congruent. Then $\angle BAC = \angle BPE = 2\angle MPE = 2\angle CPE = 2\angle DPC$.

Other commended solvers: **CHAN Yat Fei** (STFA Leung Kau Kui College, Form 6), **CHEUNG Yun Kuen** (Hong Kong Chinese Women's Club College, Form 5) and **Antonio LEI** (Colchester Royal Grammar School, UK, Year 13).

Problem 159. Find all triples (x, k, n) of positive integers such that

$$3^k - 1 = x^n.$$

(*Source: 1999 Italian Math Olympiad*)

Solution. (*Official Solution*)
For $n = 1$, the solutions are $(x, k, n) = (3^k - 1, k, 1)$, where k is for any positive integer.

For $n > 1$, if n is even, then $x^n + 1 \equiv 1$ or 2 (mod 3) and hence cannot be $3^k \equiv 0$ (mod 3). So n must be odd. Now $x^n + 1$ can be factored as

$$(x+1)(x^{n-1} - x^{n-2} + \cdots + 1).$$

If $3^k = x^n + 1$, then both of these factors are powers of 3, say they are 3^s, 3^t, respectively. Since

$$x + 1 \le x^{n-1} - x^{n-2} + \cdots + 1,$$

so $s \le t$. Then

$$0 \equiv 3^t \equiv (-1)^{n-1} - (-1)^{n-2} + \cdots + 1$$
$$= n \pmod{x+1}$$

implying n is divisible by $x + 1$ (and hence also by 3). Let $y = x^{n/3}$. Then

$$3^k = y^3 + 1 = (y + 1)(y^2 - y + 1).$$

So $y + 1$ is also a power of 3, say it is 3^r. If $r = 1$, then $y = 2$ and $(x, k, n) = (2, 2, 3)$ is a solution. Otherwise, $r > 1$ and

$$3^k = y^3 + 1 = 3^{3r} - 3^{2r+1} + 3^{r+1}$$

is strictly between 3^{3r-1} and 3^{3r}, a contradiction.

Other commended solvers: **LEE Pui Chung** (Wah Yan College, Kowloon, Form 7), **LEUNG Chi Man** (Cheung Sha Wan Catholic Secondary School, Form 6), **POON Ming Fung** (STFA Leung Kau Kui College, Form 5) and **SIU Tsz Hang** (STFA Leung Kau Kui College, Form 7).

Problem 160. We are given 40 balloons, the air pressure inside each of which is unknown and may differ from balloon to balloon. It is permitted to choose up to k of the balloons and equalize the pressure in them (to the arithmetic mean of their respective pressures.) What is the smallest k for which it is always possible to equalize the pressures in all of the balloons?
(*Source:1999 Russian Math Olympiad*)

Solution. **CHEUNG Yun Kuen** (Hong Kong Chinese Women's Club College, Form 5) and **Antonio LEI** (Colchester Royal Grammar School, UK, Year 13).

For $k = 5$, it is always possible. We equalize balloons 1 to 5, then 6 to 10, and so on (five at time). Now take one balloon from each of these 8 groups. We have eight balloons, say a, b, c, d, e, f, g, h. We can equalize a, b, c, d, then e, f, g, h, followed by a, b, e, f and finally c, d, g, h. This will equalize all 8 balloons. Repeat getting one balloon from each of the 8 groups for 4 more times, then equalize them similarly. This will make all 40 balloons having the same pressure.

For $k < 5$, it is not always possible. If the i-th balloon has initial pressure $p_i = \pi^i$, then after equalizing operations, their pressures will always have the form $c_1 p_1 + \cdots + c_{40} p_{40}$ for some rational numbers $c_1, \ldots, c_{40}$. The least common multiple of the denominators of these rational numbers will always be of the form $2^r 3^s$ as $k = 1, 2, 3$ or 4 implies we can only change the denominators by a factor of 2, 3 or 4 after an operation. So $c_1, \ldots, c_{40}$ can never all be equal to 1/40.

Olympiad Corner

(continued from page 1)

Problem 3. Prove that for all positive real numbers a, b, and c,

$$\frac{a^3}{bc} + \frac{b^3}{ca} + \frac{c^3}{ab} \ge a + b + c$$

and determine when equality occurs.

Problem 4. Let Γ be a circle with radius r. Let A and B be distinct points on Γ such that $AB < \sqrt{3}r$. Let the circle with center B and radius AB meet Γ again at C. Let P be the point inside Γ such that triangle ABP is equilateral. Finally, let the line CP meet Γ again at Q. Prove that $PQ = r$.

Problem 5. Let $N = \{0, 1, 2, \ldots\}$. Determine all functions $f: N \to N$ such that

$$xf(y) + yf(x) = (x + y) f(x^2 + y^2)$$

for all x and y in N.

簡 介 費 馬 數

(續第二頁)

證明：（證明的起點是中國餘式定理，設 $m_1, m_2, \ldots, m_r$ 是互質的正整數，$a_1, a_2, \ldots, a_r$ 是任意整數，則方程組 $x \equiv a_1 \pmod{m_1}, x \equiv a_2 \pmod{m_2}, \ldots, x \equiv a_r \pmod{m_r}$，有解。並且其解對於模 $m = m_1 m_2 \cdots m_r$ 唯一。現在考慮到任意正整數 n，都可以寫成 $2^h q$ 的形式，其中 q 是奇數。如果能夠選擇 k，使得 $k > 1, k \equiv 1 \pmod{2^{2^h} + 1}$，則 $k \cdot 2^n + 1 = k \cdot 2^{2^h q} + 1 \equiv (1)(2^{2^h})^q + 1 \equiv (1)(-1)^q + 1 \equiv (-1) + 1 \equiv 0 \pmod{2^{2^h} + 1}$，所以 $k2^n + 1$ 不是質數。留意到這裡用到 q 是奇數的性質。不過，如果這樣做的話，h 會因 n 而變，而 k 隨 h 而變，這是不容許的，k 要在起先之前決定，而不受 n 影響。）

解決的方法是這樣的，我們可以先選擇 k，使得 $k > 1,\ k \equiv 1 \pmod{2^{2^h} + 1}$，其中 $h = 0, 1, 2, 3, 4$。這是可能的，因為我們知道 F_0, F_1, F_2, F_3 和 F_4 是不同的質數。這樣的話，可以證明對於所有 $n = 2^h q$，其中 $h < 5$，q 是奇數，$k \cdot 2^n + 1$ 都不是質數。對於 $n = 2^h q$，$h \ge 5$，又可以怎樣處理呢。留意到所有這樣的數，都可以寫成 $n = 2^h q = 2^5 m$ 的形式，其中 m 可以是奇數，也可以是偶數。另一方面，我們知道 $F_5 = 2^{2^5} + 1 = (641) \times (6700417)$，其中 $P = 641$，$Q = 6700417$ 是不同的質數。如果我們選擇 k，使得 $k > 1$，和 $k \equiv -1 \pmod{P}$，$k \equiv 1 \pmod{Q}$，則 $k2^n + 1 = k2^{2^5 m} + 1 \equiv (-1)(2^{2^5})^m + 1 \equiv (-1)(-1)^m + 1 \equiv (-1)^{m+1} + 1 \pmod{P}$，另一方面 $k2^n + 1 = k2^{2^5 m} + 1 \equiv (1)(2^{2^5})^m + 1 \equiv (-1)^m + 1 \pmod{Q}$。如果 m 是偶數，則 $k2^n + 1$ 是 P 的倍數，如果 m 是奇數，則 $k2^n + 1$ 是 Q 的倍數，因此都不是質數。歸納言之，選擇 k，使得 $k \equiv 1 \pmod{x}$，$x = 3, 5, 17, 257, 65537, 6700417$，$k \equiv -1 \pmod{641}$，則對於所有形如 $k2^n + 1$ 的數，都不是質數。（最後要留意的是，這方程組的最小正整數解不可能是 1，因此所有的 $k2^n + 1$，都不是質數。）