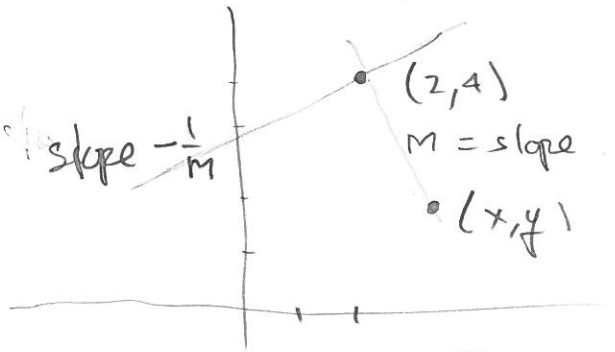
Parallel and perpendicular lines $y = mx + b$

m slope

b y-intercept

#3 (WW Assign 2)



slope -7

Find line parallel to line $y = -7x + 4$

Find line $\perp$ to $y = -7x + 4$

$$\frac{(y-4)}{(x-2)} = -7 \text{ so } (y-4) = -7(x-2) = -7x + 14$$

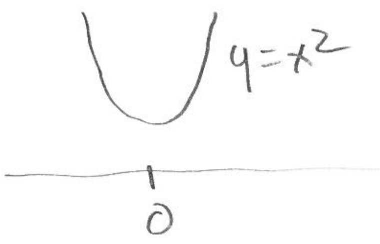
$$y = -7x + 18.$$

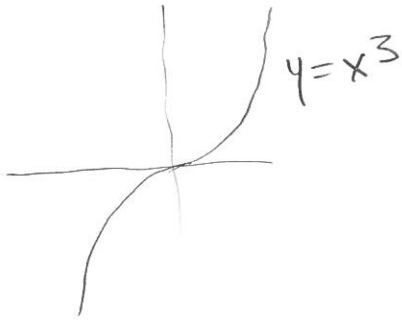
$\perp$ line

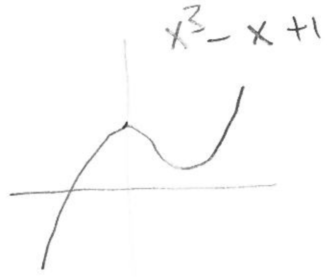
$$\frac{(y-4)}{(x-2)} = \frac{1}{7} \text{ so } (y-4) = \frac{1}{7}(x-2) = \frac{1}{7}x - \frac{2}{7}$$

$$y = \frac{1}{7}x + \frac{26}{7}$$

#6 Shapes of graphs of polynomials.







$f(x) = x^4/4 - x^2/3 - 1$ has shape 

$f(x) = f(-x)$ even function. $f(0) = y$-intercept $= -1$

graph A

$f(x) = x(3-x)^2/3 = \frac{x(x^2-6x+9)}{3} = \frac{x^3}{3} + \cdots$

$f(x) = 0$ when $x = 0$, OR $x = 3$.

graph B

$f(x) = -x(3-x)^2/3$ graph D.

$f(x) = x^3/3 - \frac{x^2}{3} - 3x + \frac{1}{2}$ graph C.

$\frac{1}{2} = f(0) = y$-intercept

One-to-one function

Example. Not one-to-one.

$x \longrightarrow S(x)$ = surname/last name of student x.

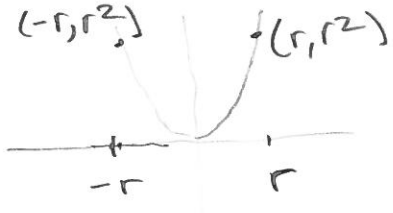
Many students with last name WONG

One-to-one function

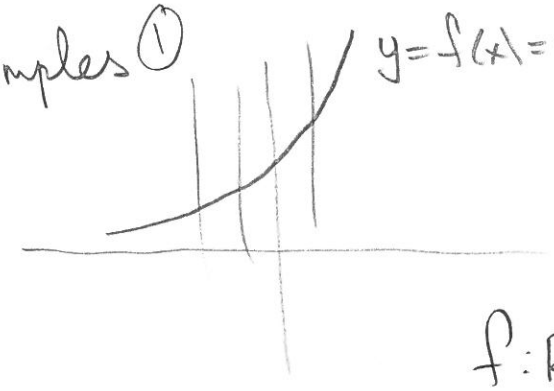
$x \neq y \longrightarrow ID(x) \neq ID(y)$.

In general $f: D \longrightarrow C$ is one-to-one if different inputs give different outputs.

Numerical example. $f(x) = x^2$ is NOT one-to-one.



Examples ①



Domain is all numbers.

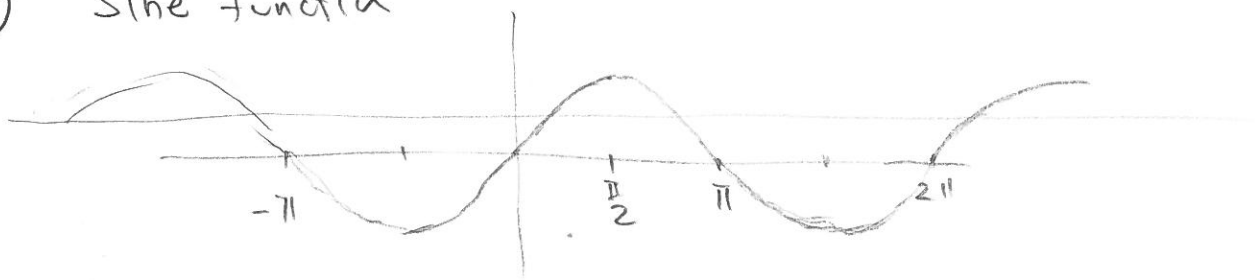
$f: \mathbb{R} \to \mathbb{R}$ is <u>not</u> onto
values never negative

$f: \mathbb{R} \to \{y > 0\}$ is onto.

$$\mathbb{R} \underset{g(y)=\log_2(y)}{\overset{f(x)=2^x}{\rightleftarrows}} \{y>0\}$$ onto, and one-to-one

Inverse function of 2^x is logarithm function $\log_2(y)$.

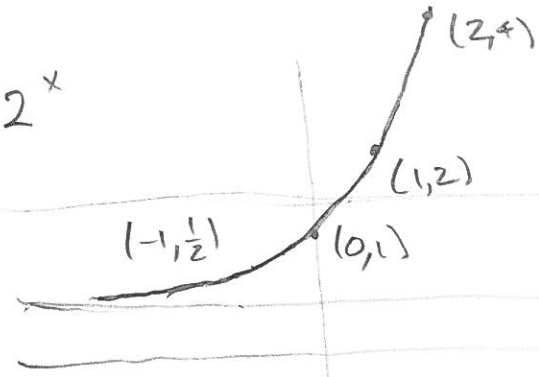
② Sine function



$\mathbb{R} \xrightarrow{\sin} \mathbb{R}$ Range(sine) $= \{-1 \le y \le 1\}$.

If we take <u>codomain to be $\mathbb{R}$</u>, then sin NOT onto.
If we take codomain to be $[-1, 1]$ then sin is onto.

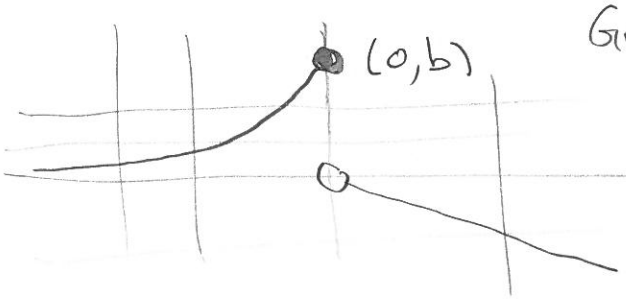
$f(x) = 2^x$ one-to-one



When we have numerical functions, the graph tells us if function is one-to-one.

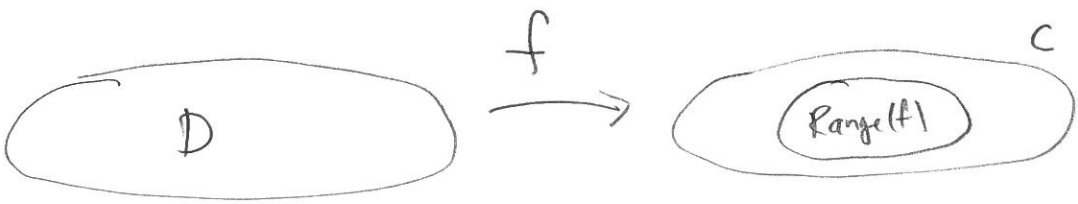
Horizontal Line Test f is one-to-one precisely when each horizontal line intersects graph in at most one point.

Example



Graph is of a one-to-one function.

Onto



The range (Range(f)) is the set of values of the function. We say f is onto the codomain C if $C = \text{Range}(f)$

Examples ① $f(x) = x^2$ Domain all numbers $\mathbb{R}$ then Range(f) $= \{y \geq 0\}$.

If we take codomain to be $\mathbb{R}$, then clearly $\{y \geq 0\} \neq \mathbb{R}$ so NOT onto.

$f(x) = x^2$ $\quad D =$ all numbers $\mathbb{R}$

$C = \{y \geq 0\}$.

Here f is <u>onto</u>

#8 For the function $f(x) = \frac{4x-1}{2x+3}$.

Determine domain, and find a codomain so function is onto.

<u>Domain</u> Need $2x+3 \neq 0$ so $\{x \neq -\frac{3}{2}\}$.

<u>Codomain</u> $y = \frac{4x-1}{2x+3} = \frac{4x+6}{2x+3} + \frac{-7}{2x+3} = 2 - \frac{7}{2x+3}$.

see that we <u>never</u> get value 2.

If we take codomain to be $\{y \neq 2\}$, then f is onto.

$$\{x \neq -\tfrac{3}{2}\} \underset{g}{\overset{f}{\rightleftarrows}} \{y \neq 2\} \text{ onto}$$

x $\quad$ $y = f(x)$

Can also check f is one-to-one.

That f is <u>one-to-one</u> AND <u>onto</u>, there is reverse/inverse function which "undoes" f.

Solve for x in terms of y.

$y = \frac{4x-1}{2x+3}$, so $y(2x+3) = (4x-1)$

$3y+1 = 4x - y2x = x(4-2y)$

$x = \frac{3y+1}{4-2y}$. $\quad g(y) = \frac{3y+1}{4-2y}$.

inverse function

$$x \xrightarrow{f} f(x) \xrightarrow{g} g(f(x)) = x$$

$$y \xrightarrow{g} g(y) \xrightarrow{f} f(g(y)) = y$$