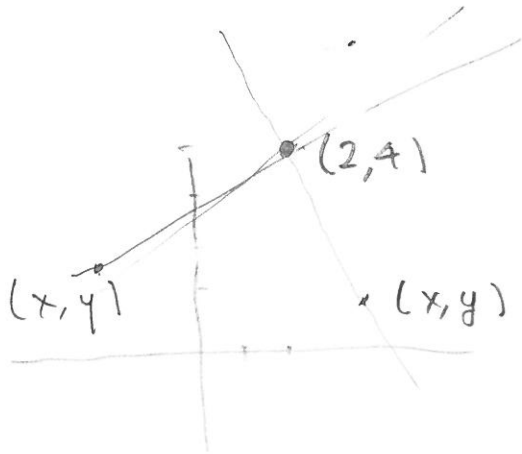
Week 1 Friday L04 11am

Parallel and perpendicular lines



WW Assign 2

#3 Point (2,4)

parallel and $\perp$ to

$$y = -7x + 4.$$

slope is -7.

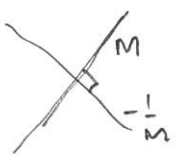
Parallel line is $\therefore$

$$\frac{(y-4)}{(x-2)} = -7, \text{ so } (y-4) = -7(x-2)$$

$$y = -7x + 14 + 4$$

$$y = -7x + 18$$

Perpendicular slope is $-\left(\frac{1}{-7}\right) = \frac{1}{7}$.

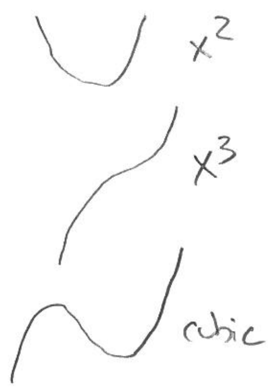


$$\frac{(y-4)}{(x-2)} = \frac{1}{7} \text{ so } (y-4) = \frac{1}{7}(x-2)$$

$$y = \frac{1}{7}x - \frac{2}{7} + 4$$

$$= \frac{1}{7}x + 3\tfrac{5}{7}.$$

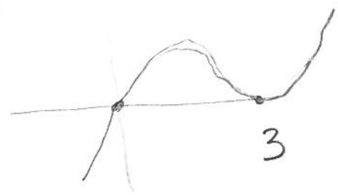
Shapes of graphs of polynomials.



#6 $\qquad \dfrac{x(3-x)^2}{3}$ cubic. $\qquad$ General shape

$= \dfrac{x(x^2-6x+9)}{3} = \dfrac{x^3}{3} + \dots$

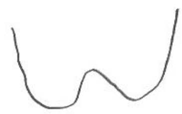
Roots (where value of function is zero).
are $\quad x=0, \quad x=3$.



Graph B.

Graph of $-\dfrac{x(3-x)^2}{3}$ is graph D

$\dfrac{x^4}{4} - \dfrac{x^2}{3} - 1$ quartic



graph A

One-to-one functions

$$D \xrightarrow{f} C$$

Function is called one-to-one if different inputs give different outputs.

Example <u>NOT one-to-one</u>

$D =$ set of UST students $\longrightarrow$ $C =$ set of last names

$s \longrightarrow L(s) =$ last name of student

One-to-one

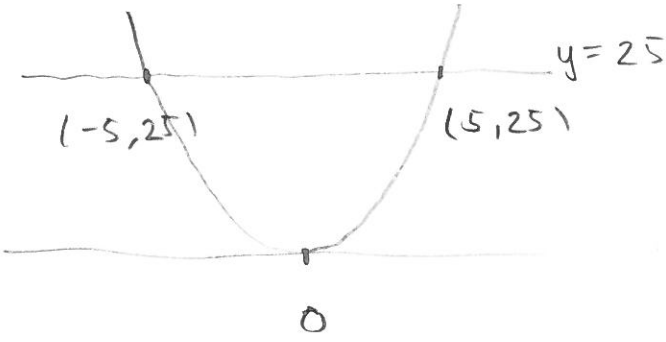
$D =$ set of UST students $\qquad C =$ student ID numbers

$s \longrightarrow ID(s) =$ ID number of s

When domain and codomain of a function is sets of numbers. We have criterion in terms of graph for function to be one-to-one

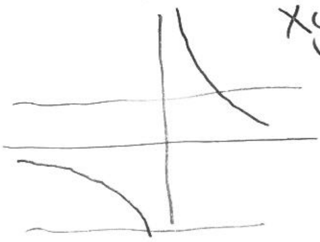
Examples

$y = f(x) = x^2$ <u>Not one-to-one</u>.

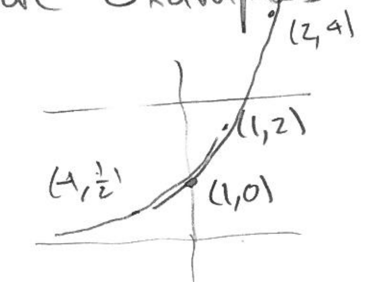


<u>Horizontal Line Test</u> A function f: is one-to-one if every horizontal line intersects graph in at most one point.

$xy = 1 \quad y = \frac{1}{x}$

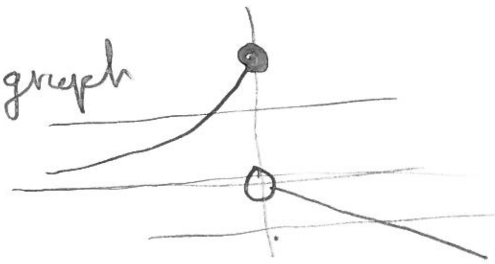


More examples of one-to-one functions

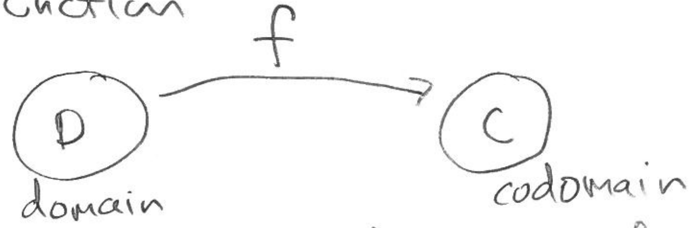


$y = 2^x$ is one-to-one.

The function with graph



Onto A function



The range of f is the set of values of f.

When range is ALL of codomain we say function is onto.

Ex

$xy = 1$. $y = \frac{1}{x}$. Domain $\{x \neq 0\}$.

Codomain $= \mathbb{R}$

$\text{Range}(\frac{1}{x}) = \{y \neq 0\}$.

The function $y = \frac{1}{x}$ is NOT onto the set $\mathbb{R}$.
Because 0 never occus as value.

Examples

(i) $f(x) = x^2$ Domain $= \mathbb{R}$, Codomain $= \mathbb{R}$.

$\text{Range}(f) = \{y \geq 0\}$

f is NOT onto Codomain $\mathbb{R}$.

(ii) $f(x) = x^2$, Domain $= \mathbb{R}$, Codomain $\{y \geq 0\}$.

$\qquad \quad \Vert$

Range(f).

(iii) $f(x) = \sin(x)$. Domain $= \mathbb{R}$.

If codomain is $\mathbb{R}$, then f is NOT onto.

If codomain is $[-1, 1]$, then f is onto.

#8 $f(x) = \dfrac{4x-1}{2x+3} = \dfrac{4x+6}{2x+3} - \dfrac{7}{2x+3} = 2 - \dfrac{7}{2x+3}$.

Determine domain. Cannot have $2x+3=0$.

So $x \neq -\frac{3}{2}$

$D = \{ x \neq -\frac{3}{2} \}$.

1st take codomain to be $\mathbb{R}$. Do we get all numbers as values of f?

Range$(f) = \{ y \neq 2 \}$. which is not $\mathbb{R}$.

If codomain is $\mathbb{R}$, then f is NOT onto.

If codomain is taken to be $\{y \neq 2\}$, then f is onto.

$f(x) = \dfrac{4x-1}{2x+3}$

$$\begin{tikzcd} x = \frac{-3}{2} \arrow[r, bend left, "f(x)=\frac{4x-1}{2x+3}"] & y \neq 2 \arrow[l, bend left, "g \text{ inverse function}"] \end{tikzcd}$$

onto.

f also one-to-one.

$y = \dfrac{4x-1}{2x+3}$ $\quad y(2x+3) = 4x-1$

Inverse function. Solve for x in terms of y

$3y+1 = 4x - y2x = x(4-2y)$

so $x = \dfrac{3y+1}{4-2y}$. My input should not be 2

$x \xrightarrow{f} y = f(x) \xrightarrow{g} g(f(x)) = x$

$y \xrightarrow{g} g(y) \xrightarrow{f} f(g(y)) = y$.