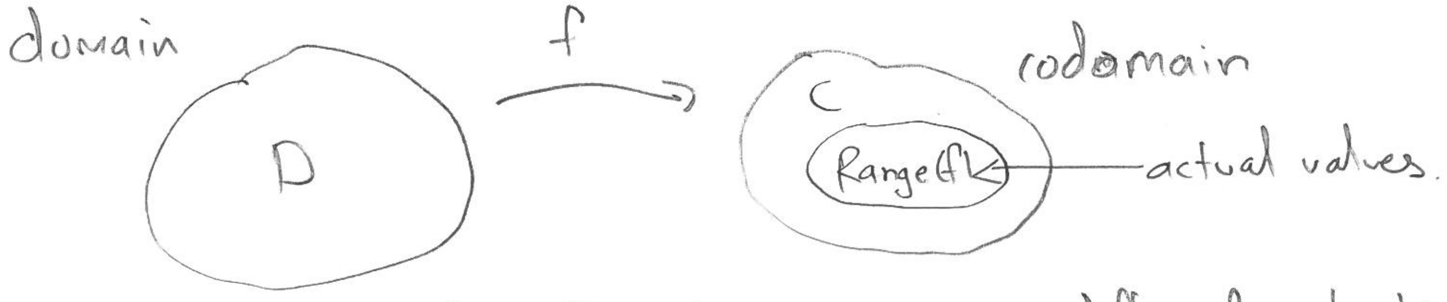
Inverse functions. Use to undo or reverse a function

$1 = \tan(x)$ to solve for x
we must undo tan.

$3 = 10^x$ undo exponential.



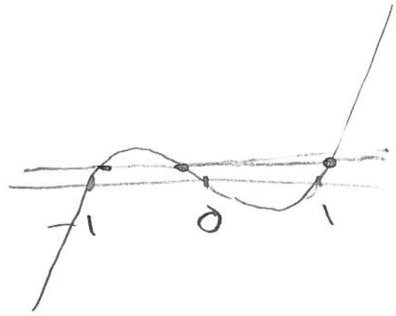
f one-to-one if different inputs give different outputs

f onto

When f is BOTH one-to-one and onto, there is a reverse / inverse function

Two nonexamples

① $f(x) = (x-1)\,x\,(x+1)$



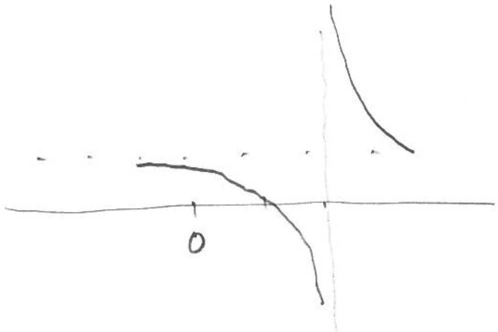
Domain $= \mathbb{R}$

Codomain $= \mathbb{R}$

$\frac{1}{2} = (x-1)\,x\,(x+1)$ 3 different solutions r_1, r_2, r_3

Not one-to-one. But is onto.

② $g(x) = 1 + \frac{1}{x-2}$



Domain $\{x \neq 2\}$

g is one-to-one.

If we take codomain to be $C = \mathbb{R}$, then NOT onto.

g never takes value 1. If we take codomain $= \{y \neq 1\}$

then g is onto

$$\{x \neq 2\} \xrightarrow{g(x) = 1 + \frac{1}{x-2}} \{y \neq 1\}$$

is one-to-one and onto. It has inverse.

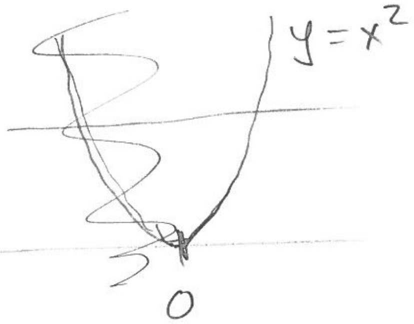
$y = g(x) = 1 + \dfrac{1}{x-2}$ Solve for x in terms of y.

$y - 1 = \dfrac{1}{x-2}$ so $(x-2) = \dfrac{1}{y-1}$

$x = 2 + \dfrac{1}{y-1}$

$$\begin{tikzcd} x \neq 2 \arrow[r, bend left, "g(x) = 1 + \frac{1}{x-2}"] & y \neq 1 \arrow[l, bend left, "x = h(y) = 2 + \frac{1}{y-1}"] \end{tikzcd}$$

Example $f(x) = x^2$



Take domain to be $\mathbb{R}$.

then NOT one-to-one.

To have one-to-one we take domain $\{x \geq 0\}$.

Take codomain to be $\mathbb{R}$. Not onto.

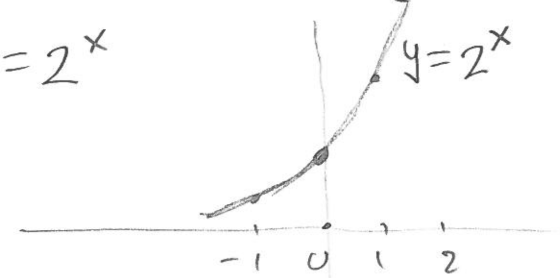
Change codomain to be $\{y \geq 0\}$.

$$\begin{tikzcd} x \geq 0 \arrow[r, bend left, "y = x^2"] & y \geq 0 \arrow[l, bend left, dashed, "x = \sqrt{y}."] \end{tikzcd}$$

one-to-one
onto

Exponentials and logarithms

$f(x) = 2^x$



x	2^x
-1	$1/2$
0	1
1	2
2	4

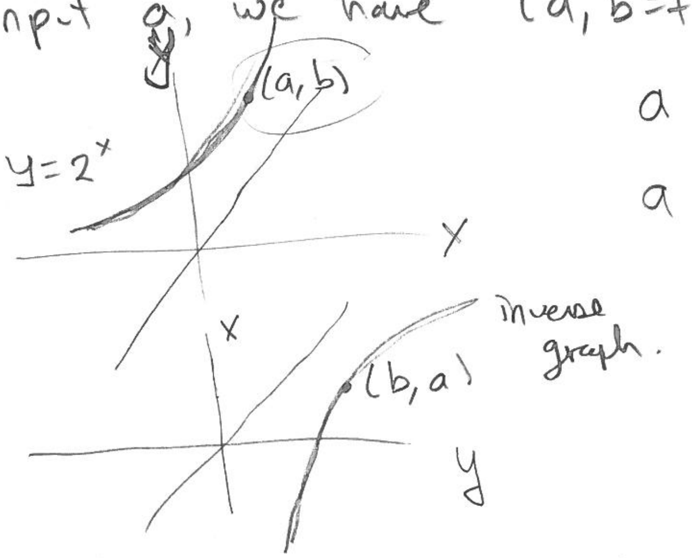
Domain is $\mathbb{R}$. $f(x)=2^x$ is one-to-one.

To have f be onto we need to take codomain $\{y>0\}$

$$\mathbb{R} \underset{x=\log_2(y)}{\overset{y=f(x)=2^x}{\rightleftarrows}} y>0$$

Graph of inverse function

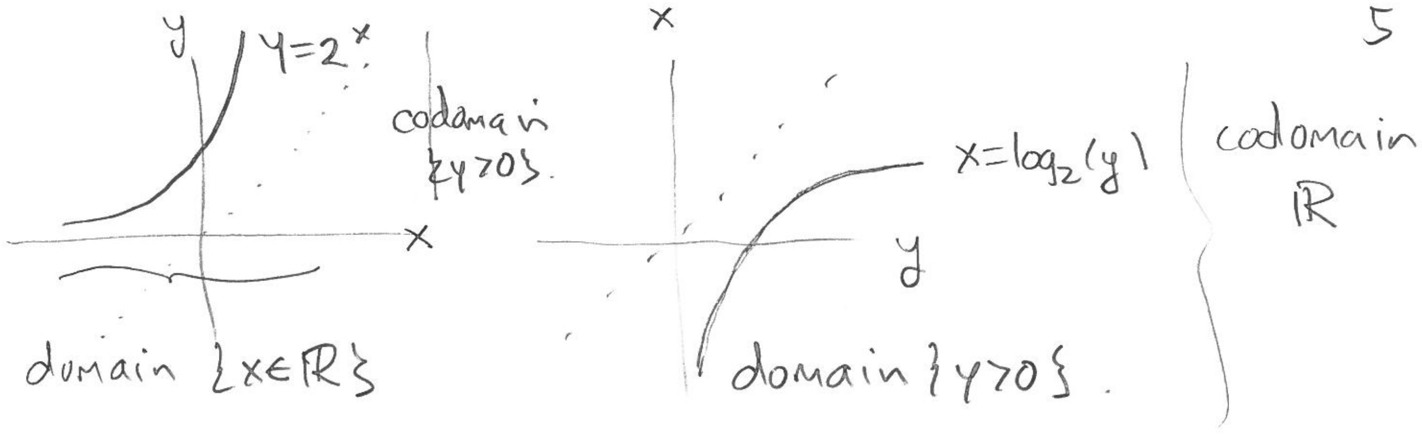
For input a, we have $(a, b=f(a))$ as graph point



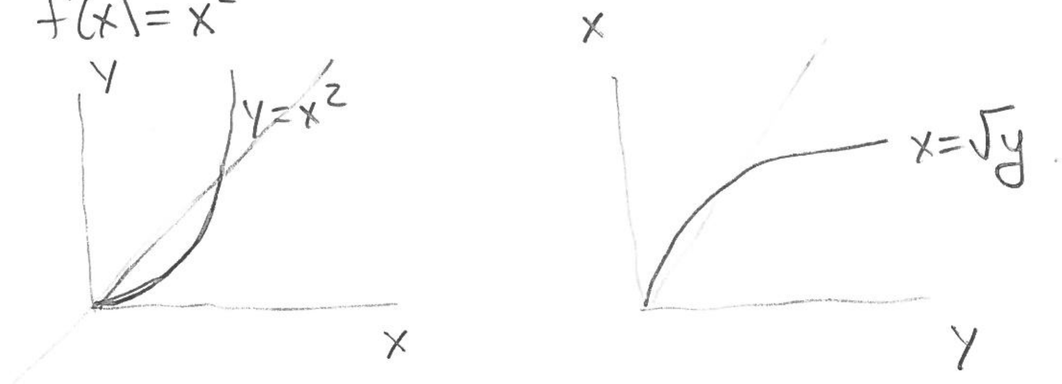
$a \xrightarrow{f} b = f(a)$

$a \xleftarrow{\text{inverse}} b$ so (b,a) is on graph of inverse.

Graph of inverse function obtain by flipping graph of function about $45°$ line, (a,b) becomes (b,a)



Previous $f(x)=x^2$



Important property of exponentials / logarithms.

$$b^{(r+s)} = b^r \cdot b^s$$

exponential of sum $r+s$ equals product of exp of r and exp of s.

Say exponential changes sum to products.

Similar property for logarithm is

$$\log_b(u \cdot v) = \log_b(u) + \log_b(v)$$

Logarithm changes products to sums.

Property $\log(uv) = \log(u) + \log(v)$ is basis for the wonderful mechanical invention called SLIDE RULE

logarithm discovered around 1600's

SLIDE RULES used to calculate for ~1700 – 1975

To multiply $\underbrace{(2.54)}_{u} \times \underbrace{(1.72)}_{v} \doteq 4.38$

$\log(2.54) = 0.406$ $\quad \log(1.72) = 0.236$

$(7.7)^{1/2}$ $\quad \log(7.7)$, $\log(\sqrt{7.7}) = \frac{1}{2}\log(7.7)$.

$=$

$\sqrt{7.7} = 2.77$

$(7.7)^{1/3} \doteq 1.97$

WW Assign 2

#9 $f(x) = \sqrt{3 - e^{2x}}$ $e = 2.718281828\ldots$

Find domain, inverse, domain of inverse
(a) (b) (c)

1a) $\sqrt{\ }$ only allow input ≥ 0, so we must have

$3 - e^{2x} \geq 0$, so $3 \geq e^{2x}$.

$\ln(3) \geq 2x$

$\frac{\ln(3)}{2} \geq x$. domain

(b) $y = \sqrt{3 - e^{2x}}$ solve x in terms of y.

$y^2 = 3 - e^{2x}$

$e^{2x} = 3 - y^2$ so $2x = \ln(3 - y^2)$

$x = \frac{1}{2}\ln(3 - y^2)$.

(c) domain of inverse? $f(x) = \sqrt{3 - e^{2x}}$

output of $\sqrt{\ }$ is numbers ≥ 0,

so output of f is ≥ 0. (so $y \geq 0$).

Input to $\ln(3 - y^2)$ need $3 - y^2 > 0$ so $3 > y^2$

This means $0 \leq y < \sqrt{3}$.