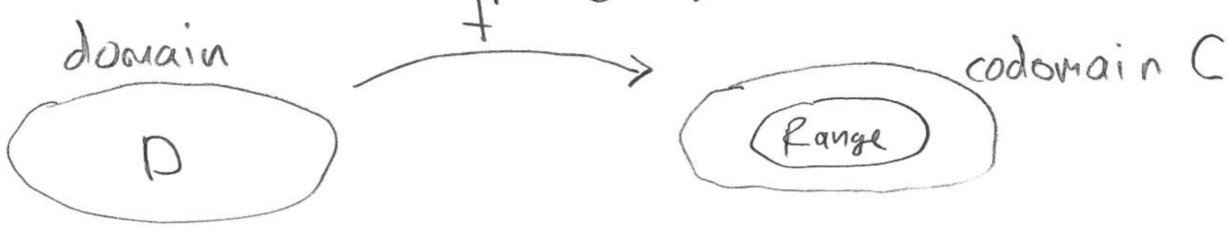
Inverse functions.

Solve equations $\underline{1} = \tan(x)$ Use inverse function.

$3 = 10^x$



Under suitable assumptions we can "undo" the function with inverse function

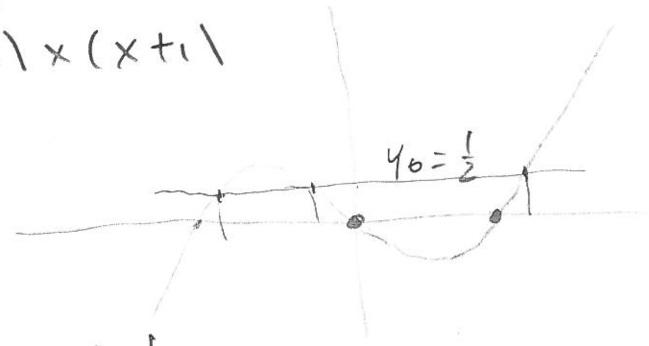
If a function is BOTH one-to-one AND onto, then there is a "reverse/inverse" function

Examples. (Non examples)

① $f(x) = (x-1) \times (x+1)$

Domain $= \mathbb{R}$

Codomain $= \mathbb{R}$



graph of f.

Not one-to-one

Is onto. f does not have inverse.

$\frac{1}{2} = (x-1)(x)(x+1)$. 3 roots r_1, r_2, r_3

(2) $g(x) = 1 + \frac{1}{x-2}$

Domain $\neq \mathbb{R}$

Domain $= \{x \neq 2\}$.

g is one-to-one

Codomain: If $C = \mathbb{R}$, then NOT onto.

Range (actual values) $= \{y \neq 1\}$.

If we change codomain to $\{y \neq 1\}$, then

$g(x) = 1 + \frac{1}{x-2}$ has inverse

$$\{x \neq 2\} \xrightarrow{g} \{y \neq 1\}$$

domain | codomain = range

one-to-one and onto

There is inverse: Take $y = 1 + \frac{1}{x-2}$ and solve for x in terms of y.

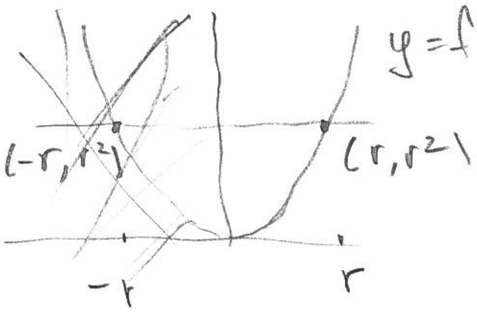
$$y = 1 + \frac{1}{x-2} \rightsquigarrow y - 1 = \frac{1}{x-2}$$

$$x - 2 = \frac{1}{y-1}$$

$$x = 2 + \frac{1}{y-1}$$

$$\begin{tikzcd} x \neq 2 \arrow[r, bend left, "y = 1 + \frac{1}{x-2}"] & y \neq 1 \arrow[l, bend left, "x = 2 + \frac{1}{y-1}"] \end{tikzcd}$$

Examples



$y = f(x) = x^2$ is not one-to-one when domain is $\mathbb{R}$

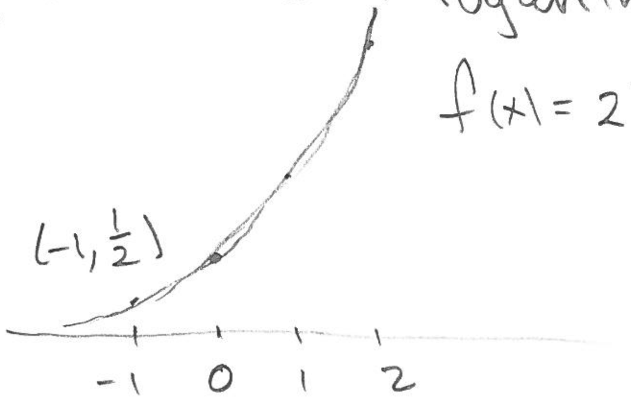
To make things one-to-one, we take $D = \{x \geq 0\}$, then one-to-one. If we take codomain $C = \mathbb{R}$ then <u>not</u> onto. If we take codomain $\{y \geq 0\}$ then onto, and there is inverse

$$\begin{tikzcd} x \geq 0 \arrow[r, bend left, "y = f(x) = x^2"] & y \geq 0 \arrow[l, bend left, "x = \sqrt{y}"] \end{tikzcd}$$

new domain　　　　new codomain

Exponentials and logarithms

$f(x) = 2^x$



x	2^x
-1	1/2
0	1
1	2
2	4

Domain $= \mathbb{R}$ and one-to-one.

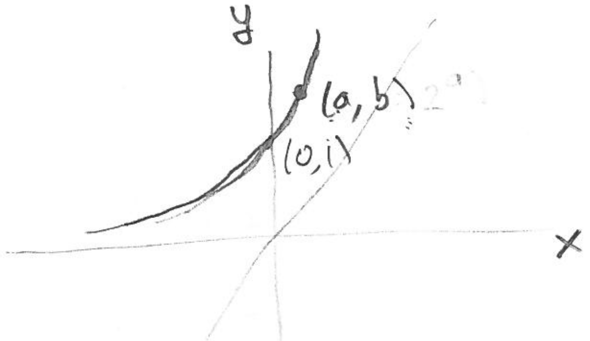
If we take codomain $= \{y > 0\}$ then onto.

$$\mathbb{R} \xrightarrow{y = 2^x} y > 0$$

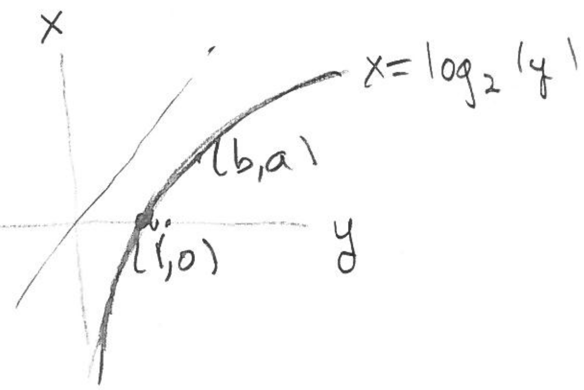
$$\mathbb{R} \xleftarrow{x = \log_2(y)} y > 0$$

$$a \xrightarrow{3^x} b$$

$$a \xleftarrow{\log} b$$

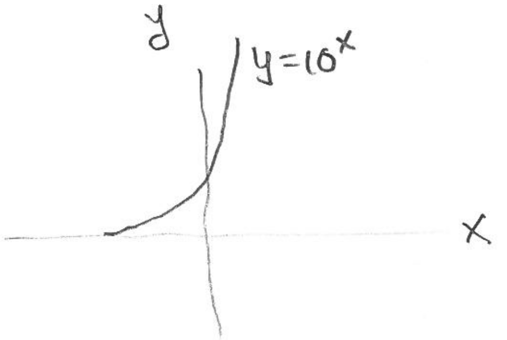


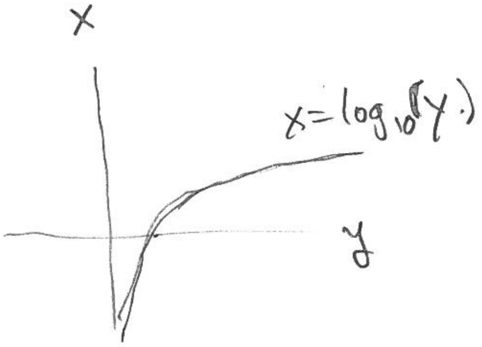
flip graph across 45° degree lin



$\cancel{f(x) = 1^x}$ $\quad f(x) = b^x$ $(b > 0,\ b \neq 1)$, then one-to-one

domain can be $\mathbb{R}$. onto $\{y > 0\}$.





$b^{(r+s)} = b^r \cdot b^s$.

exponential of a sum of $r+s$ equals product of exponentials of r and of s.

Say exponential change addition to multiplication.

For logarithm $\log_b$. this becomes

$$\log_b(u \cdot v) = \log_b(u) + \log_b(v).$$

Say logarithm change multiplication to addition.

This property of logarithm is basis of SLIDE RULES.

$(1.8) \times (2.7) \doteq 4.85$

$\log_{10}(1.8) \doteq .256 \qquad \log(2.7) \doteq .432$

$\log_{10}(1.8)$

$\sqrt{7.3} \qquad r = \log(7.3)$

$\log(\sqrt{7.3}) = \frac{1}{2}$

$\sqrt{7.3} \doteq 2.7$

$(7)^{1/3} = 1.925$

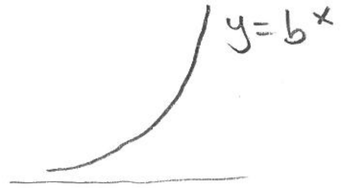
WW Assign 2

9 $f(x) = \sqrt{3 - e^{2x}}$

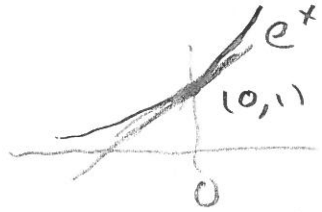
(a) Domain of f.

(b) Inverse

(c) Domain of inverse



For the special number
$e = 2.718281828\ldots$



For e, the tangent slope at $(0,1)$ is $m = \underline{1}$.

(a) f is composition.
$\sqrt{\ }$ requires inputs ≥ 0 so $3 - e^{2x} \geq 0$.

$3 \geq e^{2x}$ and $\log_e(3) \geq 2x$ $\quad \frac{\log_e(3)}{2} \geq x$.

Output of $\sqrt{\ }$ is ≥ 0,

(b) $y = \sqrt{3 - e^{2x}}$ solve for x in terms of y.

$y^2 = 3 - e^{2x}$ so $e^{2x} = 3 - y^2$

$2x = \log_e(3 - y^2)$

$x = \frac{1}{2}\log_e(3 - y^2)$

(c) domain of inverse

Need $y \geq 0$ and $3 - y^2$ (input to $\log_e$) to be > 0

$3 - y^2 > 0$ so $3 > y^2$ $\quad \sqrt{3} > y$

$0 \leq y < \sqrt{3}$ domain of inverse