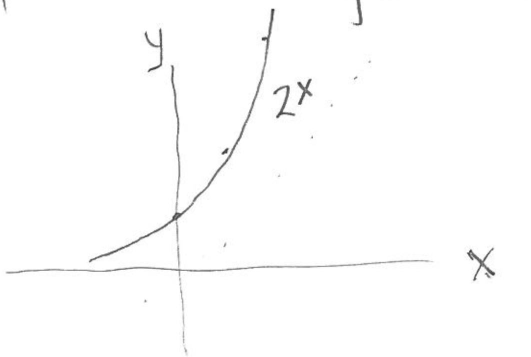
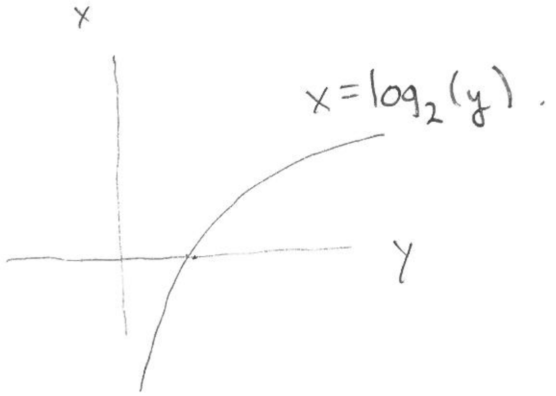
Exponential / logarithm



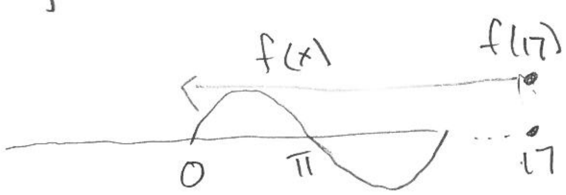


b^x

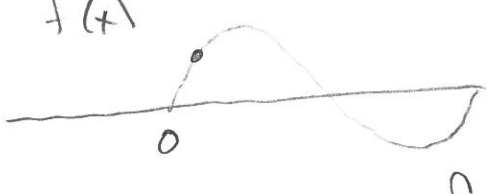
WW2 #7 Translating and scaling graphs

$y = \sin(x)$

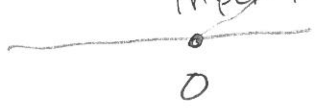
graph of	
$y = \frac{\sin(x)}{17}$	A shift 17 units to RIGHT
$y = \sin(x+17)$	B vertically compressed by factor 17
$y = \sin\left(\frac{x}{17}\right)$	C shifted 17 units to LEFT
$y = \sin(x-17)$	D horizontally stretched by factor 17



$f(x+17)$ for input 0, value $f(17)$



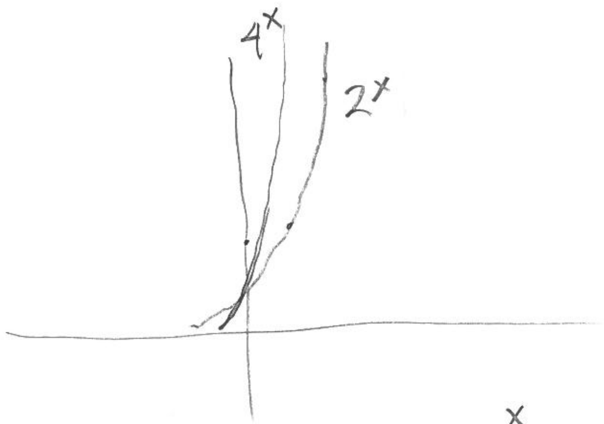
$f\left(\frac{x}{17}\right)$ input $x=0$, value $f(0)$
input $x=17$, value $f(1)$



$f\left(\frac{x}{17}\right)$ has graph stretched by factor of 17

Example $f(x) = 2^x$ vs $g(x) = 4^x = 2^{2x} = f(2x)$

$4 = 2^2$ $\quad f(x)$ vs $f(2x)$



graph is graph of $f(x) = 2^x$ compressed by a factor of 2

Same for b^x vs a^x. Write $a = b^{\log_b(a)}$

So $a^x = \left(b^{\log_b(a)}\right)^x = b^{(\log_b(a))x}$

$= b^{(\log_b(a))x}$

$f(x) = b^x$

$g(x) = a^x = b^{\log_b(a) x}$

$= f(\log_b(a)\, x)$

If we do same thing for logarithms: we get the formula

$$\log_a(x) = \frac{\log_b(x)}{\log_b(a)}$$

WW2 #12 Exponential population growth.

$P(t) = P_0 b^t$

	yr	population
0	19~~80~~	35 mil
10	19~~90~~	60 mil

Two data pts, two unknown constants P_0, b.

$P(0) = 35 = P_0 \cdot b^0 = P_0 \quad \therefore \quad P_0 = 35$

$P(10) = 60 = P_0 \cdot b^{10} = 35\, b^{10}$ solve $b^{10} = \frac{60}{35} = \frac{12}{7}$

$b = \left(\frac{12}{7}\right)^{1/10}$

$$P(t) = 35 \left(\frac{12}{7}\right)^{\left(\frac{t}{10}\right)}$$

(a) Predict population in 2000 $t = 20$.

$$P(20) = 35 \left(\frac{12}{7}\right)^{\frac{20}{10}} = 35 \left(\frac{12}{7}\right)^2 \text{ (millions).}$$

(b) Doubling time

$P(t) = P_0 b^t$. If we have a fixed interval L

$$P(t+L) = P_0\, b^{t+L} = (b^L) \cdot P_0 b^t = b^L P(t)$$

$$P(t+L) = \underbrace{(b^L)}_{\text{fixed factor.}} P(t)$$

$$P(t+2L) = (b^L) P(t+L) = (b^L)(b^L) P(t)$$

The doubling time is the L so that $b^L = 2$

$b = \left(\frac{12}{7}\right)^{1/10}$ $\Bigg\{$ $\left(\frac{12}{7}\right)^{\frac{L}{10}} = 2$ solve for L.

Take $\log_{\left(\frac{12}{7}\right)}$ of both sides $\therefore$ $\frac{L}{10} = \log_{\left(\frac{12}{7}\right)}(2)$

Solve

$$L = 10 \log_{\left(\frac{12}{7}\right)}(2) = 10 \frac{\log_{10}(2)}{\log_{10}(12/7)}.$$

WW2 #14 exercise exp / log.

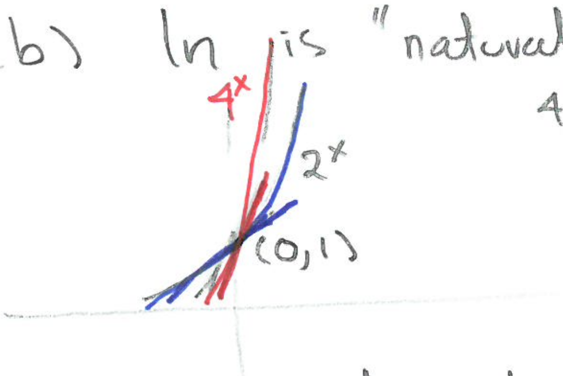
(a) Solve $6^{x-5} = 6$, (b) solve $\ln(x) + \ln(x-1) = 1$

Solution

(a) Exp is one-to-one: $6^{x-5} = 6^1$

$\therefore$ $x - 5 = 1$ so $x = 6$.

(b) ln is "natural logarithm"



$4^x = 2^{2x}$

Tangent slope at (0,1) for graph " , < 1

Tangent slope at (0,1) for " > 1

There is special number $e = 2.71828\ldots$ so tangent slope to e^x at (0,1) is 1

Logarithm for e^x is ln( ).

$\ln(x) + \ln(x-1) = 1$ Solve for x

$\ln(x(x-1)) = 1$

exp both sides $x(x-1) = e^1 = 2.71828\ldots$

$x^2 - x = e$ so $x^2 - x - e = 0$ | $a=1,\ b=-1,\ c=-e$

Solve: $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $\dfrac{--1 \pm \sqrt{1+4e}}{2}$

Possible roots are $\dfrac{1 \pm \sqrt{1+4e}}{2}$

NOT BOTH

ln( ) requires positive inputs

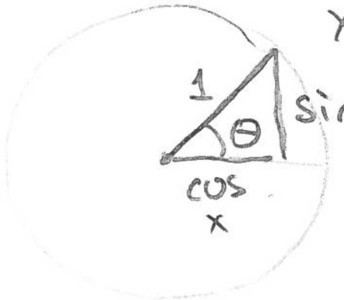
ln(x−1) requires x >

so only $\dfrac{1+\sqrt{1+4e}}{2}$ is solution

other possible solution negative.

Trigonometry review

$x^2+y^2=1$



$(\cos\theta)^2+(\sin\theta)^2=1.$

$\cos(\frac{\pi}{2}-\theta)=\sin(\theta).$

$\sin(\theta)=\frac{\text{opposite}}{\text{hypoth}}$ $\cos(\theta)=\frac{\text{adjacent}}{\text{hypoth}}$

Domain for both is all numbers $\mathbb{R}$.

Identities $\sin(\theta+2\pi)=\sin(\theta)$
$\cos(\theta+2\pi)=\cos(\theta)$

If we use domain $\mathbb{R}$, neither is one-to-one.

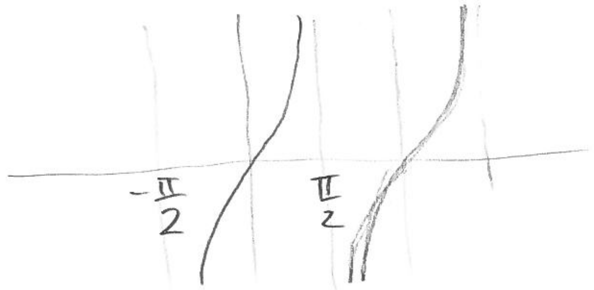
Actual values vary $-1\le y\le 1$. If this is codomain, then both are <u>onto</u>.

Tangent: $\frac{\sin\theta}{\cos\theta}$ $\left(\frac{x}{y}\right)$ slope of line with angle θ.

90 270

Codomain of tangent is $\mathbb{R}$

Domain is $\{x\in\mathbb{R} \mid x\ne$ <u>odd</u> multiple of $\frac{\pi}{2}\}$



adding 180° to angle gives same slope

$\tan(x+\pi)=\tan(x)$

Sum of angles:

$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

Will prove these two identities.