Week 2 W L04 11am.

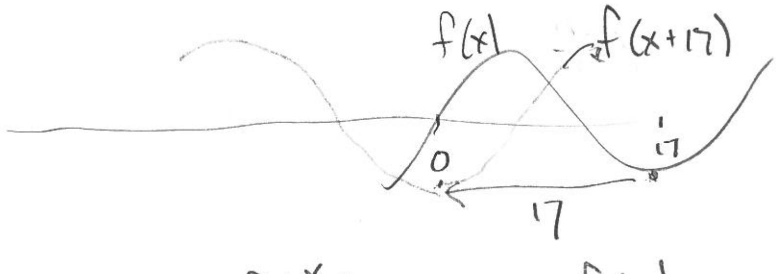
"Scaling and translations" in graphs.

ww2 #7.

Graph f(x) vs	
$y = \frac{f(x)}{17}$	A graph shift 17 units right
$y = f(x+17)$	B compressed VERTICALLY by factor of 17
$y = f(\frac{x}{17})$	C shifted 17 unit left
$y = f(x-17)$	D stretched horizontally by factor of 17.

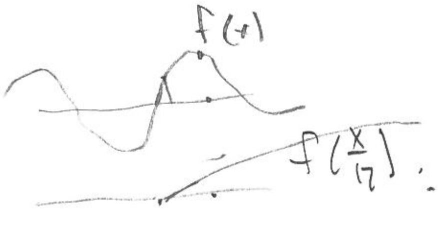
value of $f(x+17)$ at input $x=0$ is $f(0+17) = f(17)$

$f(x+17)$ at input $x=1$ is $f(1+17) = f(18)$



input $x=0$ into $f(\frac{x}{17})$ gives $f(0)$

$x=1$ into $f(\frac{x}{17})$ gives $f(\frac{1}{17})$



Comparing graphs of exponentials

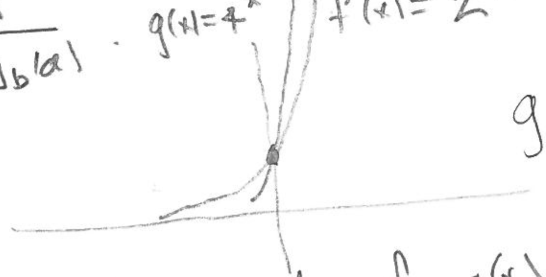
$f(x) = b^x \quad (b > 1)$. $\quad g(x) = a^x \quad (a > 1)$.

$a = b^{\log_b(a)}$ and if we substitute into $g(x) = a^x$

we get $\quad g(x) = a^x = (b^{\log_b(a)})^x = b^{\log_b(a)x}$

$$g(x) = f(\log_b(a)\, x)$$

$\therefore$ graph of a^x is graph of b^x stretched by a factor of $\frac{1}{\log_b(a)}$.



$g(x) = 4^x = (2^2)^x$
$= 2^{2x}$

Since $\log_2(4) = 2$ The graph of $g(x) = 4^x$ is graph of 2^x stretched by factor of $\frac{1}{2}$

In same way for logarithms we have

$$\log_a x = \frac{\log_b x}{\log_b a}$$

WW2 #12 Population growth

Population growing exponentially

$$P(t) = P_0 b^t$$

	t	$P(t)$
0	1980	35 million
10	1990	60

Determine P_0 and b.

$$35 = P(0) = P_0 b^0 = P_0$$

$$60 = P(10) = \underset{35}{P_0} b^{10}$$ solve for b so $b^{10} = \left(\frac{60}{35}\right)$

$$b = \left(\frac{60}{35}\right)^{1/10}.$$

$$P(t) = 35\left(\frac{60}{35}\right)^{\frac{t}{10}}$$

(a) Predict $P(20)$: $P(20) = 35\left(\frac{60}{35}\right)^{\frac{20}{10}} = 35\left(\frac{60}{35}\right)^2$

yr 2000

Fact about exponentials. $P(t) = P_0 b^t$

$$P(t+L) = P_0 b^{(t+L)} = b^L (P_0 b^t) = b^L P(t)$$

After addition time L, the quantity is changed by a factor of $b^L = C$. This means $P(t+2L) = C^2 P(t)$

$$P(t+3L) = C^3 P(t)$$

$$\vdots$$

Doubling time: If $b^L = 2$, then each addition time of L doubles our quantity.

Since we have $P(t) = 35\left(\frac{60}{35}\right)^{\frac{t}{10}}$ $b = \left(\frac{60}{35}\right)^{1/10}$.

Doubling time is L so that $\left(\left(\frac{60}{35}\right)^{1/10}\right)^L = 2$

$$\left(\frac{12}{7}\right)^{\frac{L}{10}} = 2 \qquad b = \left(\frac{12}{7}\right)$$

$$\log_b\left(\left(\quad\right)^{L/10}\right) = \log_b(2)$$

$$\frac{L}{10} = \log_b(2)$$

$$L = 10\log_b(2) = 10\cdot\frac{\log_{10}(2)}{\log_{10}\left(\frac{12}{7}\right)}.$$

WW2 # 14 Solve

(a) $6^{x-5} = 6^1$, (b) $\ln(x) + \ln(x-1) = 1$.

<u>Solution</u> (a) Exponentials one-to-one

Since input $x-5$ and input 1 are suppose to give same output we have

$x - 5 = 1$ so $x = 6$.

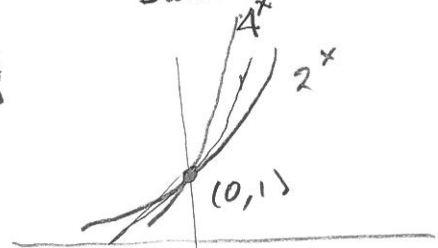
(b) $\ln(x) + \ln(x-1) = 1$ $\quad$ ln = "natural log"

$\ln(x(x-1)) = 1$ $\quad$ base is $e = 2.71828\ldots$

Take both sides to $e^{(\ )}$ to get

$x(x-1) = e^1$

$x^2 - x - e = 0$



tangent slope of 4^x @ $(0,1) > 1$

tangent slope of 2^x @ $(0,1) < 1$

tangent slope of e^x @ $(0,1)$ equals 1

Solve with quadratic formula $\dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1$

$b = -1$

$c = -e$

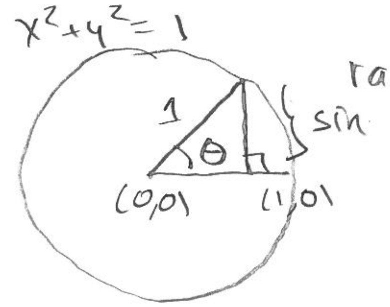
$$\frac{--1 \pm \sqrt{1 - 4\cdot 1(-e)}}{2\cdot 1} = \frac{1 \pm \sqrt{1+4e}}{2}$$ POSSIBLE root

Because $\ln(x)$ requires $x > 0$

$\ln(x-1)$ require $x > 1$

only $\dfrac{1 + \sqrt{1+4e}}{2}$ is > 1.

Review of trigonometry



radius 1 circle (center @ (0,0))

For right triangle sine of angle θ is $\frac{\text{opposite}}{\text{hypothenuse}}$

cosine (θ) = $\frac{\text{adjacent}}{\text{hypoth}}$.

Very basic identity of sin / cos is

$$(\cos(\theta))^2 + (\sin(\theta))^2 = 1$$

Notation $\cos^2\theta$ means $(\cos\theta)^2$, $\sin^2\theta$ means $(\sin\theta)^2$

tangent: $\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}}$ (slope of angle θ).

Domain: sin/cos all numbers $\mathbb{R}$ are allowed as inputs.

$$\sin(x+2\pi) = \sin(x), \quad \cos(x+2\pi) = \cos(x)$$

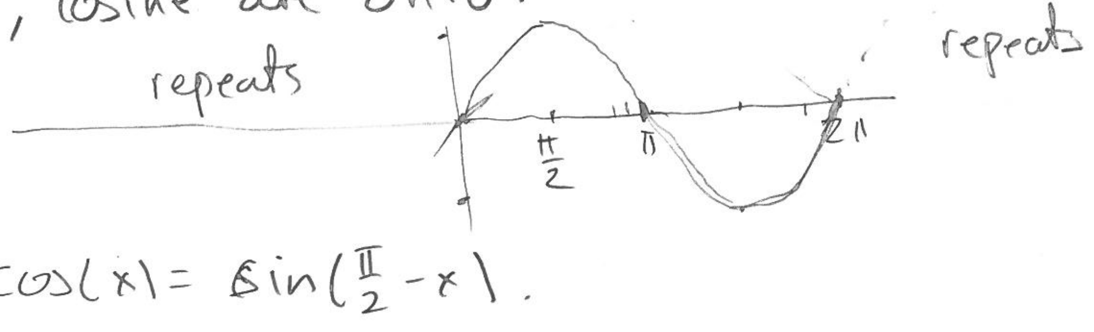
These are "periodic"

With domain $\mathbb{R}$, neither is one-to-one.

Actual set of values is $-1 \le y \le 1$.

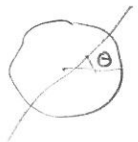
If we take codomain to be $-1 \le y \le 1$, then sine, cosine are onto.

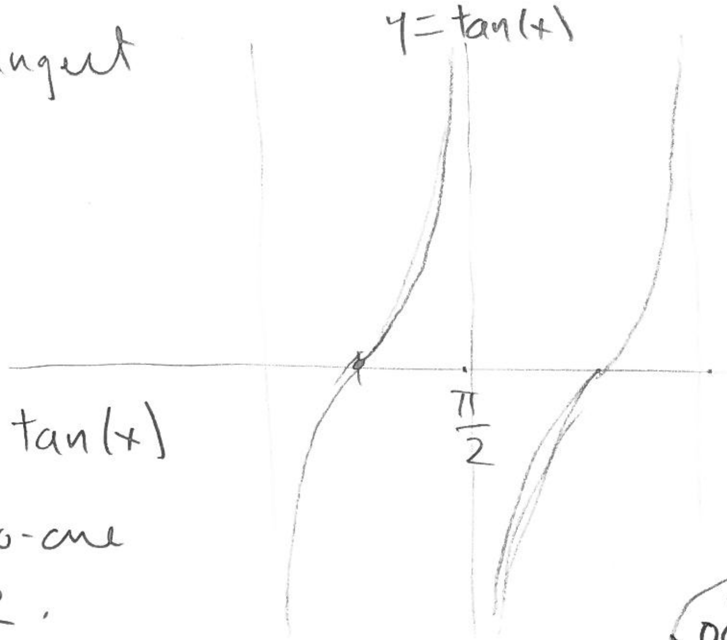
ta



$$\cos(x) = \sin\left(\frac{\pi}{2} - x\right).$$

Graph of tangent





$\tan(x+\pi) = \tan(x)$

so Not one-to-one on domain $\mathbb{R}$.

Domain of tangent $\{ x \in \mathbb{R} \mid x \neq \text{odd multiple of } \frac{\pi}{2} \}$.

Set of values of tangent $-\infty < y < \infty$ all of $\mathbb{R}$.

A very important trig identity

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B).$$

$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$

Friday: We will "prove" this identity.