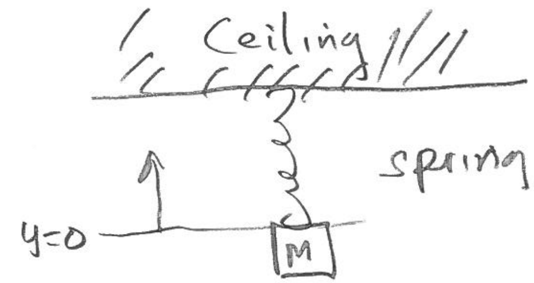
Week 2 Friday L03 2pm

Trignometry review.

WW2 #13 Mass is oscillating



$$y(t) = 6\delta \cos(10\pi\omega t)$$

δ, ω constants.

(a) What is amplitude? Furthest it swing up or down

amplitude is 6δ. $\quad -1 \le \cos(\ ) \le 1$

(b) How many oscillations in one second?
In one second, time t goes from 0 to 1 $\Leftrightarrow 0 \le t \le 1$.

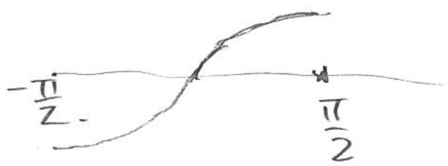
Input to cosine is $10\pi\omega t$, varies

Number oscillations is

$$\frac{10\pi\omega}{2\pi} = 5\omega.$$

Inverse trig functions

$\sin(x+2\pi) = \sin(x)$ so sine NOT one-to-one when domain is $\mathbb{R}$



If limit domain to $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, then sine function becomes one-to-one.

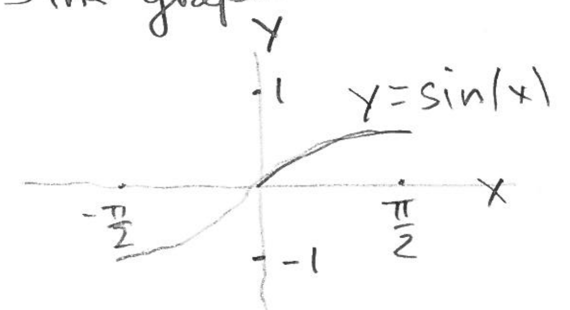
The output of sine is $-1 \le y \le 1$, so if we take this to be codomain, sine function is onto

$$-\frac{\pi}{2} \le x \le \frac{\pi}{2} \xrightarrow{\sin(x)} -1 \le y \le 1$$

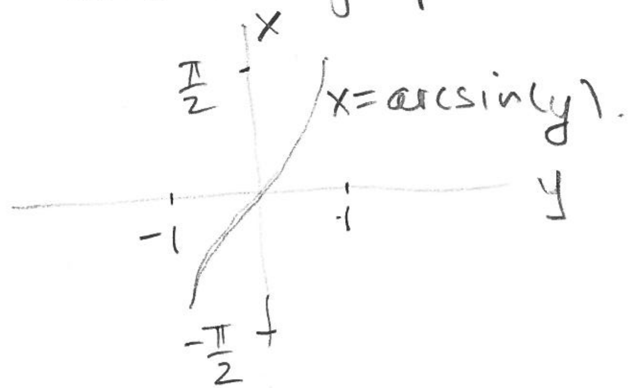
one-to-one
onto

Inverse function is called arcsin, (also written as $\sin^{-1}$)

Sine graph



arcsin graph (flip sin graph across 45° line



WW2 #15 Find domain, range of $g(x) = \arcsin(3x+1)$.

Since arcsin requires inputs be in interval $[-1,1]$, we must have

$-1 \le 3x+1 \le 1$. $\rightsquigarrow$ gives $-2 \le 3x \le 0$

$-\frac{2}{3} \le x \le 0$.

(a) Domain of g is $-\frac{2}{3} \le x \le 0$.

(b) Output of g is. As x varies over $[-\frac{2}{3}, 0]$, then

$3x+1$ varies over $[-1,1]$, so

output of $\arcsin(3x+1)$ varies. $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

Range is $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

<u>Arctan</u>: For domain $\{x \in \mathbb{R} \mid x \text{ not odd multiple of } \frac{\pi}{2}\}$.

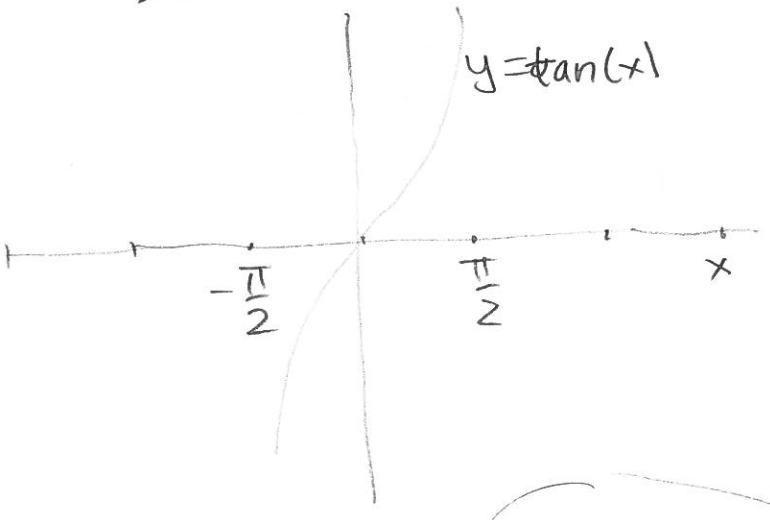
tangent satisfies

$$\tan(x+\pi) = \tan(x)$$

so not one-to-one. We <u>cut down domain</u> to

$$-\frac{\pi}{2} < x < \frac{\pi}{2}$$

to make tangent one-to-one



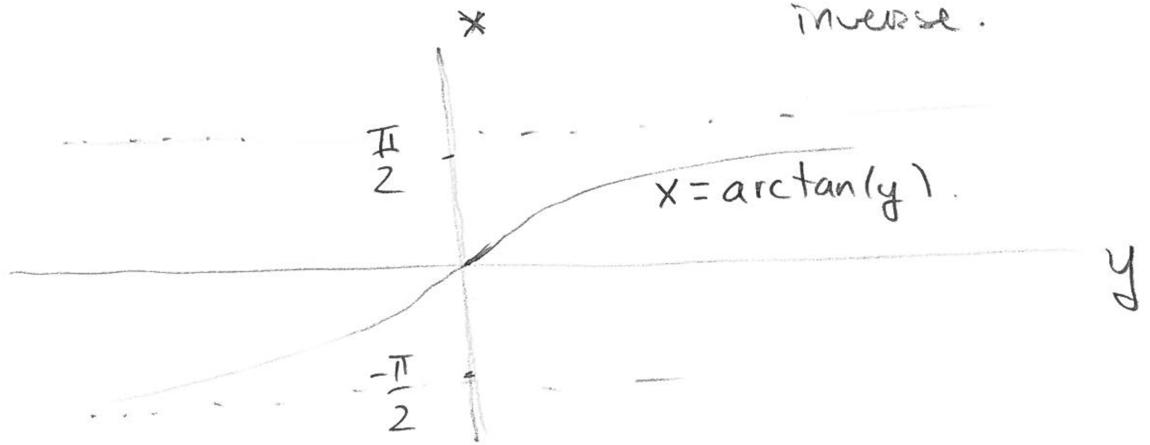
Then outputs are $-\infty < y < \infty$

If we take this as codomain.

$$-\frac{\pi}{2} < x < \frac{\pi}{2} \xrightarrow{\tan} -\infty < y < \infty$$

tan is one-to-one onto

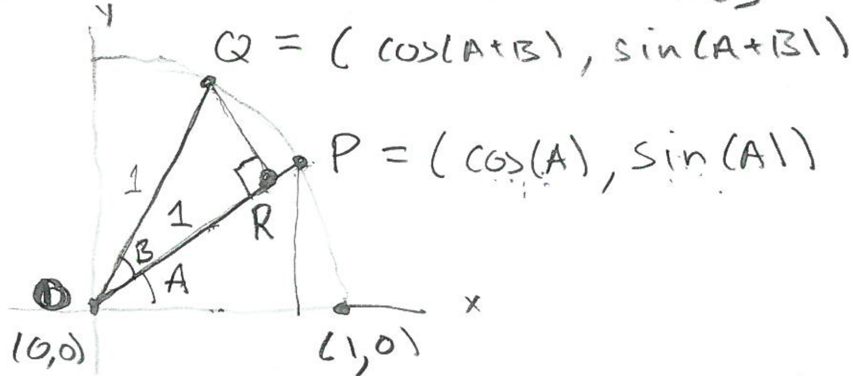
arctan is inverse.



Important trig identities

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$
$$\sin(A+B) = \cos(A)\sin(B) + \sin(A)\cos(B).$$

"Proof" of these two identities



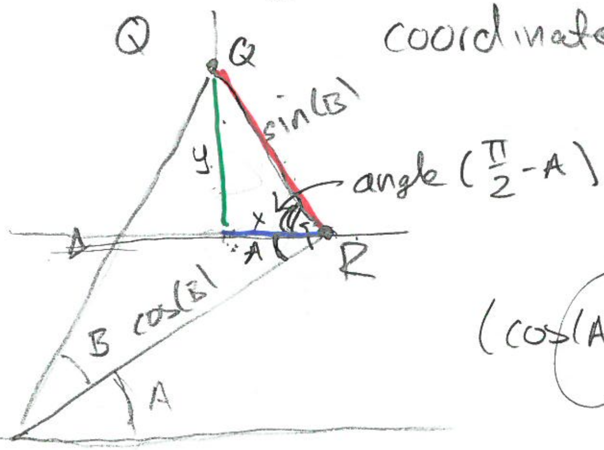
O = origin $= (0,0)$.

$\overline{OP}$ has length 1

$\overline{OR}$ has length $1 \cdot \cos(B)$

Two steps: ① Find coordinates of R $\therefore$ $R = \cos(B) \cdot P = (\cos(B)\cos(A), \cos(B)\sin(A))$

② Relate coordinates of R to coordinates of Q.



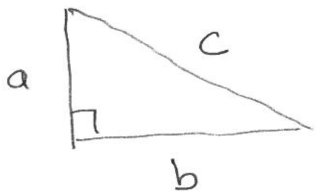
so $x = \text{hypo} \cdot \cos(\frac{\pi}{2} - A) = \sin(B)\sin(A)$

$y = \text{hypo} \cdot \sin(\frac{\pi}{2} - A) = \sin(B)\cos(A).$

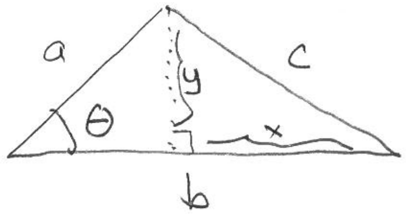
$$(\cos(A+B), \sin(A+B)) = Q = (\cos(B)\cos(A) - \sin(B)\sin(A), \cos(B)\sin(A) + \sin(B)\cos(A))$$

equals

Law of cosines:

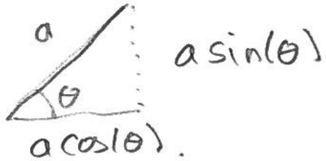


Pythagorean Theorem $c^2 = a^2 + b^2$.



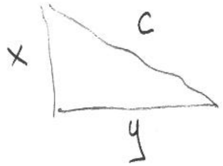
Law of cosines: $c^2 = a^2 + b^2 - 2ab\cos(\theta)$

"Proof". From picture



$y = a\sin(\theta)$

$x = b - a\cos(\theta)$

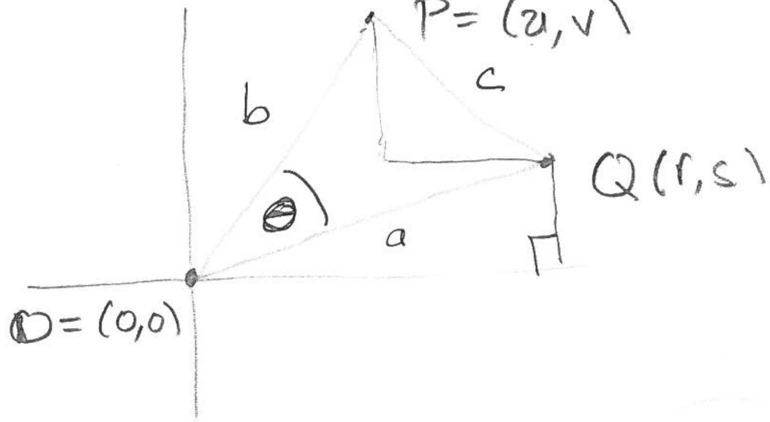


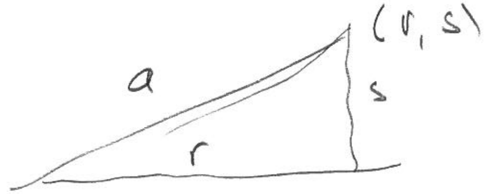
$$c^2 = x^2 + y^2 = (b - a\cos\theta)^2 + (a\sin(\theta))^2$$

$$= b^2 - 2ab\cos\theta + a^2(\cos\theta)^2 + a^2(\sin\theta)^2$$

$$c^2 = b^2 + a^2 - 2ab\cos\theta$$

Dot product.





How to calculate angle θ? Law of cosines

$$c^2 = a^2 + b^2 - 2ab\cos(\theta)$$

$$2ab\cos(\theta) = a^2 + b^2 - c^2$$

$$2ab\cos(\theta) = 2ru + 2sv$$

$$\boxed{ab\cos(\theta) = ru + sv}$$

$$a^2 = r^2 + s^2$$

$$b^2 = u^2 + v^2$$

$$c^2 = (r-u)^2 + (s-v)^2$$

$$= r^2 - 2ru + u^2 + s^2 - 2sv + v^2$$

This quantity is called dot product of vectors (r, s) and (u, v)

$$\cos(\theta) = \frac{ru + sv}{a \cdot b} = \frac{\text{dot product}}{a \cdot b}.$$