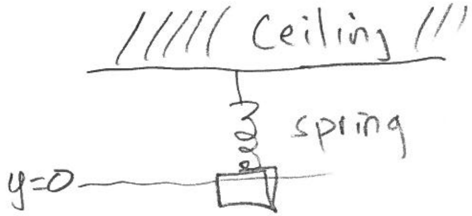
Week 2 Friday L04 11am

Trigonometry review

WW2 #13



t time seconds.

constants

z (cm)

ω $\left(\frac{1}{\text{sec}}\right)$.

Mass is oscillating $y(t) = 6z \cos(10\pi \omega t)$

(a) Furthest distance from $y=0$?

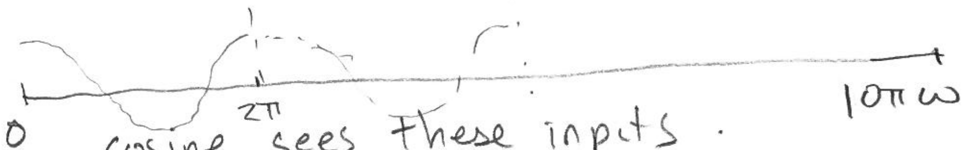
Amplitude = coefficient in front of cos( )

$= 6z$.

(b) How many oscillation are completed in 1 second?

$0 \leq t \leq 1 \rightsquigarrow$ the input $10\pi\omega t$ to cosine varies from 0 to $10\pi\omega$

$t=0$ $\quad$ $t=1$.

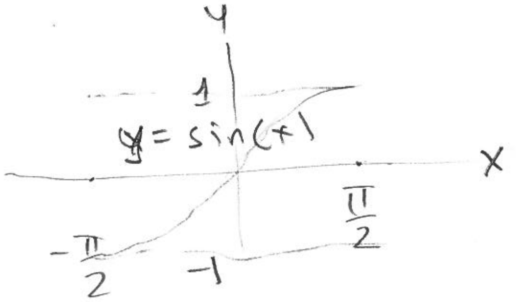


cosine sees these inputs.

$$\# \text{ of oscillations } \frac{10\pi\omega}{2\pi} = 5\omega.$$

Inverse trig functions

$\sin(x+2\pi) = \sin(x)$ $\rightsquigarrow$ for domain $\mathbb{R}$ sine is <u>NOT</u> one-to-one.



If we change domain to $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

then sine becomes one-to-one.

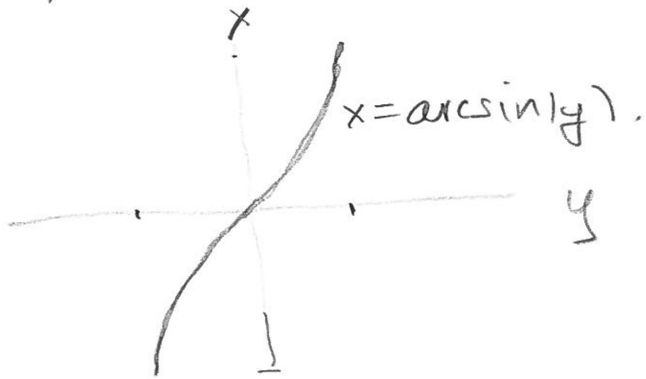
If we also take codomain to be $-1 \leq y \leq 1$

then sine is onto.

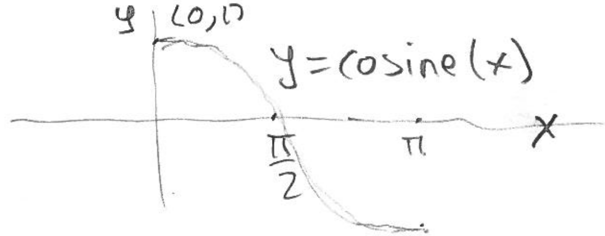
With these two domains and codomains, there is inverse

Call it arcsin (notation arcsin, and $\sin^{-1}$).

Graph of $x = \arcsin(y)$ is



Cosine



$(0,1) \rightsquigarrow (1,0)$

$(\frac{\pi}{2},0) \rightsquigarrow (0,\frac{\pi}{2})$

$(\pi,-1) \rightsquigarrow (-1,\pi)$

For domain $0 \le x \le \pi$

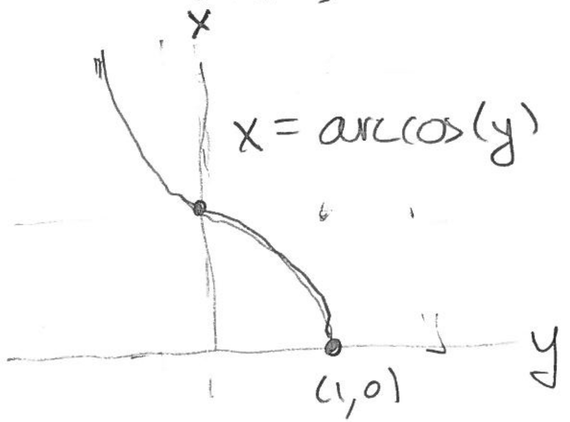
cosine is one-to-one.

If we take codomain

$-1 \le y \le 1$

then cosine is onto

$$\begin{tikzcd} 0 \le x \le \pi \arrow[r, bend left, "\text{cosine}"] & -1 \le y \le 1 \arrow[l, bend left, "\text{arccos}"] \end{tikzcd}$$



WW2 #15

$g(x) = \arcsin(3x+1)$

Find domain and range.

Domain:

arcsin can take inputs between

$-1, 1$

So need $-1 \le 3x+1 \le 1$

$$-2 \le 3x \le 0$$

$$-\frac{2}{3} \le x \le 0.$$

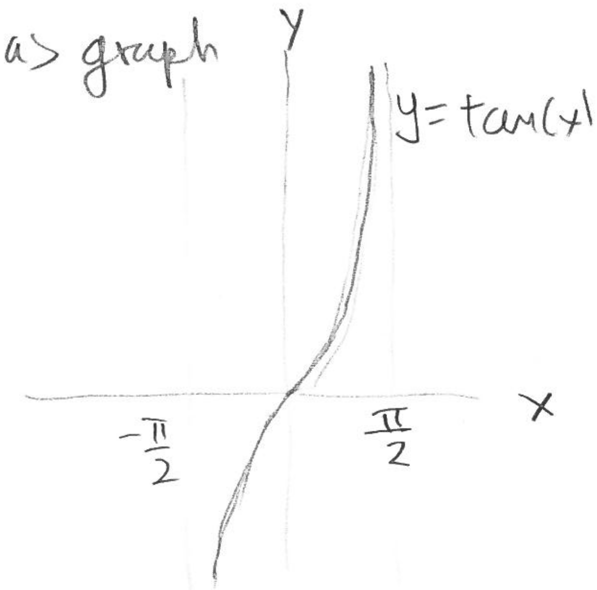
The output values (range) is

As x varies $-\frac{2}{3}$ to 0, the quantity $3x+1$ varies -1 to 1. Input to arcsin.

Output is from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$. Range $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$.

Arctan

tan has graph



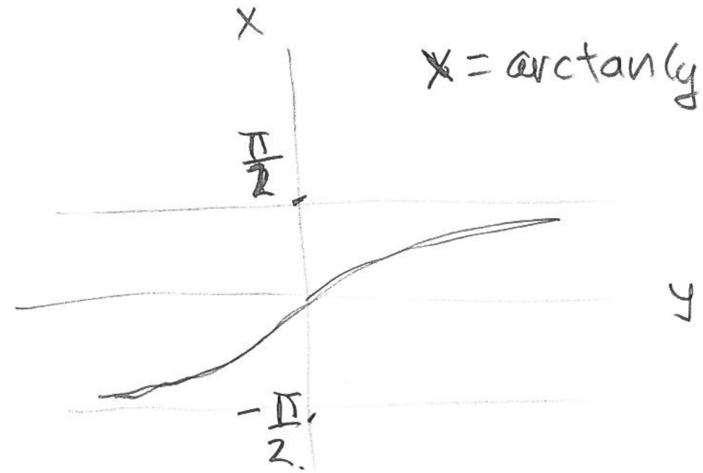
$\tan(x+\pi) = \tan(x)$

Not one-to-one on domain $\{x \in \mathbb{R} \mid x \text{ not odd multiple of } \frac{\pi}{2}\}$.

Restrict / cut down domain to $-\frac{\pi}{2} < x < \frac{\pi}{2}$. Then is one-to-one.

Values of tangent is $-\infty < y < \infty$

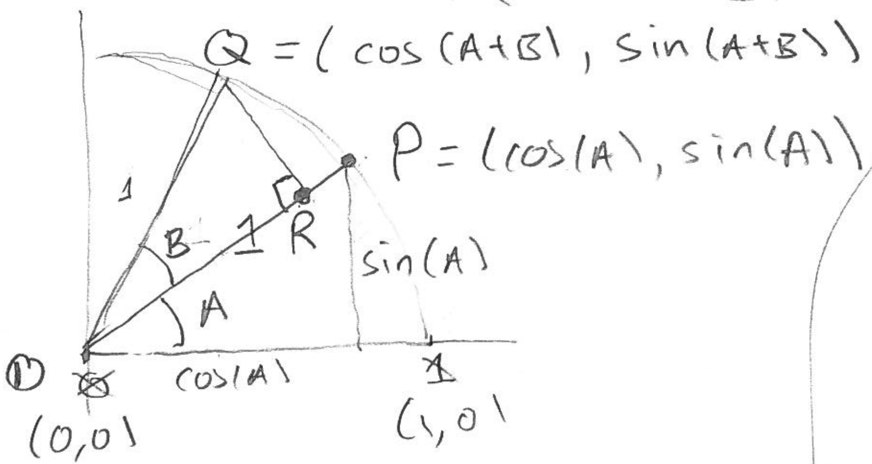
$$\left\{-\frac{\pi}{2} < x < \frac{\pi}{2}\right\} \underset{\arctan}{\overset{\tan}{\rightleftarrows}} \left\{-\infty < y < \infty\right\}$$



Some important trig identities

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B).$$

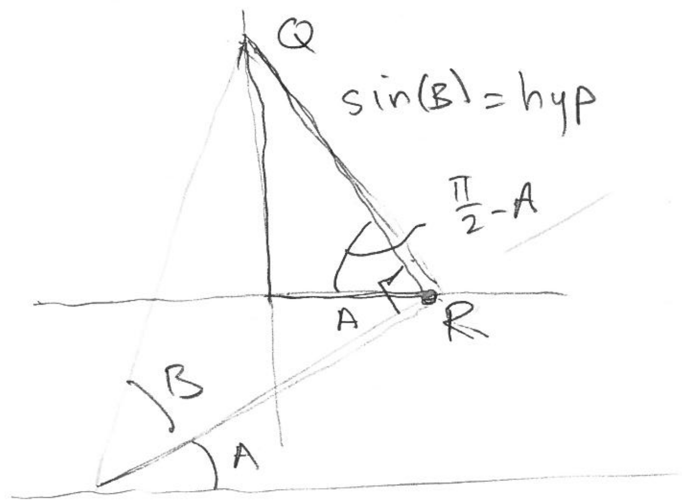


① O = origin (0,0)

length $\overline{OR}$ is $1 \cdot \cos(B)$

$\therefore R = (\cos B) P = \cos B(\cos A, \sin A)$.

②



$x = \text{hyp} \cdot \cos(\frac{\pi}{2} - A) = \sin(B)\sin(A)$

$y = \text{hyp} \cdot \sin(\frac{\pi}{2} - A) = \sin(B)\cos(A)$

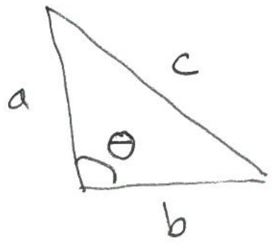
We prove trig identity by

① determining coordinates of R

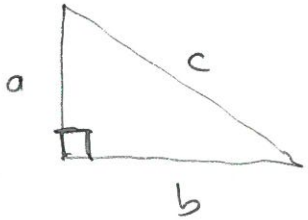
② determine x, y change to go from R to Q.

$$Q = (\underbrace{\cos(A+B)}_{\text{1st identity}}, \underbrace{\sin(A+B)}_{\text{2nd identity}}) = (\cos B\cos A - \sin(B)\sin(A), \cos(B)\sin(A) + \sin(B)\cos(A))$$

Law of cosines:

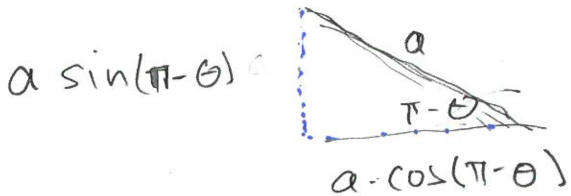


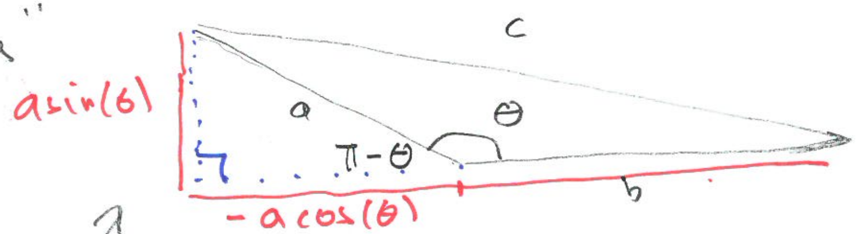
$$c^2 = a^2+b^2-2ab\cos\theta.$$



Pythagorean Thm $a^2+b^2=c^2$.

"Proof of law of cosines"





$$a\sin(\pi-\theta) = a\sin(\theta)$$
$$a\cos(\pi-\theta) = -a\cos(\theta).$$

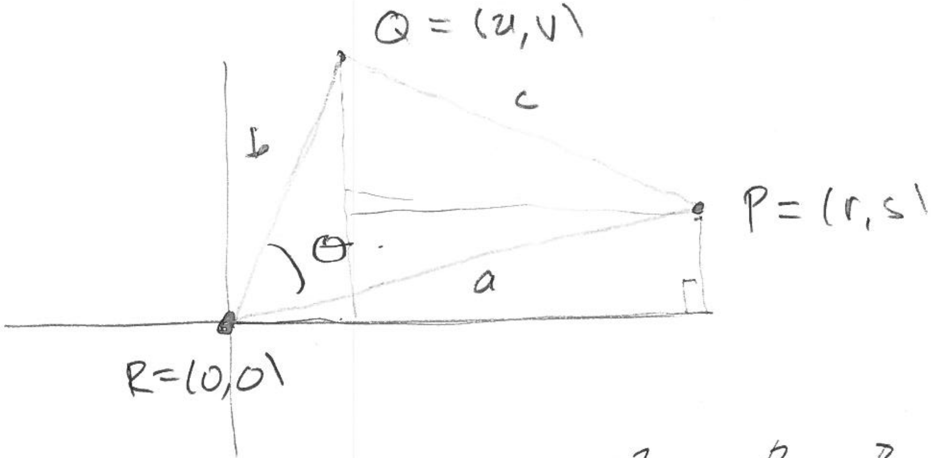
Two sides of above right triangle are $a\sin\theta$, $b-a\cos\theta$.

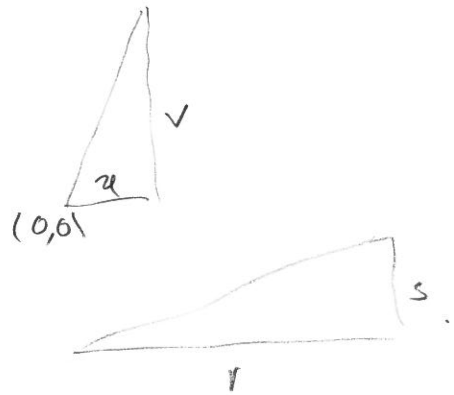
$$c^2 = (a\sin\theta)^2 + (b-a\cos\theta)^2 = a^2(\sin\theta)^2 + b^2 - 2ab\cos\theta + a^2(\cos\theta)^2$$

$$\boxed{c^2 = a^2+b^2-2ab\cos\theta}$$

Law of cosines.

Law of cosine and dot product





a = length $\overline{RP}$, $a^2 = r^2 + s^2$

b = length $\overline{RQ}$, $b^2 = u^2 + v^2$

c = length $\overline{QP}$, $c^2 = (u-r)^2 + (v-s)^2$

$$= \cancel{u^2} - 2ur + \cancel{r^2} + \cancel{v^2} - 2vs + s^2$$

$$-2ab\cos\theta = c^2 - (a^2 + b^2) = -2ur - 2vs$$

$$\cos\theta = \frac{ur + vs}{ab}$$

The quantity $ur + vs$ from (u, v) (r, s)

is called scalar/dot product.

Use to measure angle between $\overline{RQ}$, $\overline{RP}$.