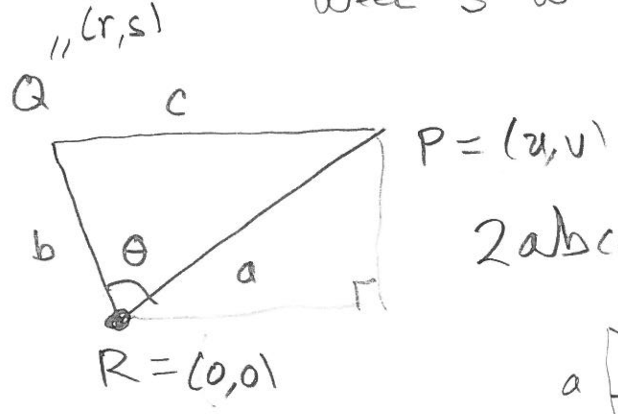
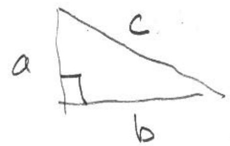
Law of cosines



$$2ab\cos\theta = a^2 + b^2 - c^2$$



$$0 = a^2 + b^2 - c^2$$

If we choose coordinates so

$R = (0,0)$ $\quad a = \overline{RP}$ $\quad a^2 = u^2 + v^2$

$P = (u,v)$ $\quad b = \overline{RQ}$ $\quad b^2 = r^2 + s^s$

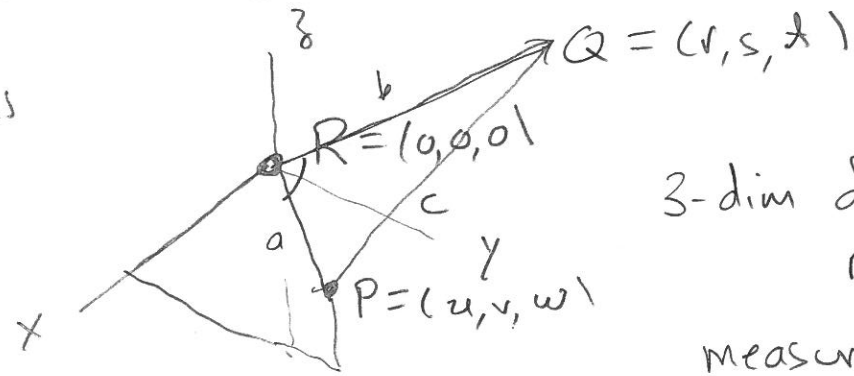
$Q = (r,s)$ $\quad c^2 = (u-r)^2 + (v-s)^2$

substitute to get

$$ab\cos\theta = \tfrac{1}{2}(a^2+b^2-c^2) = \cdots = (ur + vs)$$

quantity called dot product $(u,v), (r,s)$

In 3-dimensions



3-dim dot product is $ru + sv + tw$

measures $ab\cos\theta$.

Example 3-dim

$R = (0,0,0)$

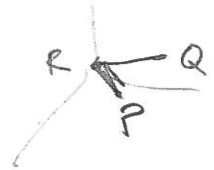
$P = (2,2,1)$

$Q = (1,2,2)$

dot product of $(2,2,1)$ and $(1,2,2)$

$2\cdot 1 + 2\cdot 2 + 1\cdot 2 = 8$

$a = \overline{RP} = \sqrt{2^2+2^2+1^2} = \sqrt{9} = 3.$



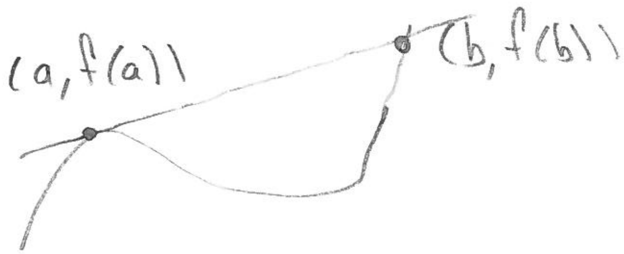
$b = \overline{RQ} = \sqrt{1^2+2^2+2^2} = \sqrt{9} = 3.$

law of cosines $\quad \underset{a}{3}\cdot\underset{b}{3}\cos\theta = 8$, so $\cos\theta = \frac{8}{9}.$

so $\quad \theta = \arccos\left(\frac{8}{9}\right) = 0.4758$ radians $= 0.475 \text{ rad} \, \frac{360^\circ}{2\pi \text{ rad}}$

$= 27.26^\circ.$

Secant line



If (a, f(a)), and (b, f(b)) are distinct points on graph, the line through these two points is secant line.

Things to do with secant line.

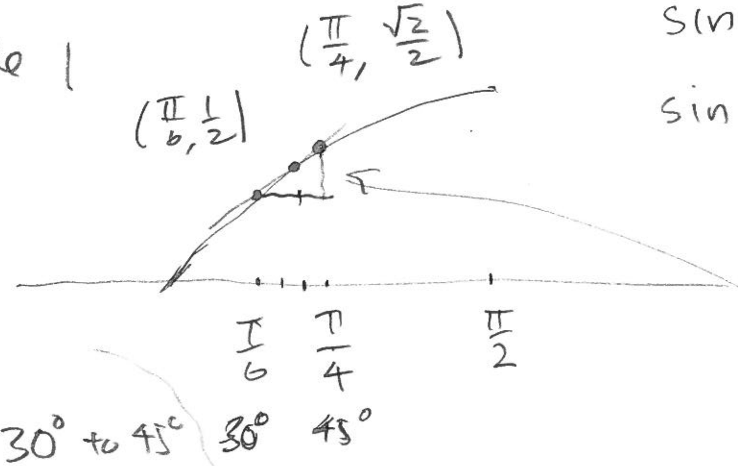
- Interpolation

Example 1

$\sin(45°) = \frac{\sqrt{2}}{2} = 0.707$

$\sin(30°) = \frac{1}{2} = 0.500$

What is good estimate of sin(40°)?



40° is $\frac{2}{3}$ way from 30° to 45°

value of sin(40°) is approximately $\frac{2}{3}$ way from $\frac{1}{2}$ to $\frac{\sqrt{2}}{2}$.

$\sin(40°) \doteq \frac{1}{2} + \frac{2}{3}\left(\frac{\sqrt{2}}{2} - \frac{1}{2}\right) = 0.638$

Actual sin(40°) is 0.643.

Example 2. Find estimate for $\log_{10} 2$.

$$10^{1/2} = \sqrt{10} = 3.1622 \qquad \sqrt{3.1622} = 1.7782$$

$$10^{1/4} = 1.7782.$$

This says $\log_{10}(3.1622) = \frac{1}{2}$, $\log_{10}(1.7782) = \frac{1}{4}$
$= 0.5$ $\qquad = 0.25.$

interval $1.7782 \leq x \leq 3.1622$
has length $3.1622 - 1.7782$

2 is $\left(\dfrac{2 - 1.7782}{3.1622 - 1.7782}\right)$ is of the way
between 1.7782 and 3.1622

$$\text{Estimate for } \log_{10}(2) \doteq 0.25 + \left(\frac{2 - 1.7782}{3.1622 - 1.7782}\right) 0.25 = 0.290.$$

Actual value $\log_{10}(2) = 0.301$

Secant line

$$f(x) = \frac{1}{4}x^2 - x = x\left(\frac{x}{4} - 1\right) \quad \text{roots } 0, 4.$$

The slope of secant line from $(a, f(a))$ to $(b, f(b))$

is $$m = \frac{f(b) - f(a)}{b - a}.$$ We take $a = 3$.

Then $$m = \frac{\left(\frac{1}{4}b^2 - b\right) - \left(\frac{1}{4}3^2 - 3\right)}{b - 3}$$ secant slope.

function of b
domain $= \{ b \in \mathbb{R} \mid b \neq 3 \}$

On domain

$$m = \frac{\frac{1}{4}(b^2 - 3^2) - (b - 3)}{(b - 3)} = \frac{\frac{1}{4}(b+3)(b-3) - (b-3)}{(b-3)}$$

$$m = \frac{1}{4}(b+3) - 1 \quad . \text{ For } b \neq 3.$$

We can "imagine" letting variable b approach 3.
Secant slope approaches $\frac{1}{4}(3+3) - 1 = \frac{3}{2} - 1 = \frac{1}{2}$. This is tangent slope
@ $(3, f(3)) = (3, \frac{9}{4} - 3)$

$f(x) = \frac{1}{4}x^2 - x$

Secant slope from $(a, f(a) = \frac{1}{4}a^2 - a)$, $(b, f(b) = \frac{1}{4}b^2 - b)$

$$m = \frac{f(b) - f(a)}{b - a} = \frac{(\frac{1}{4}b^2 - b) - (\frac{1}{4}a^2 - a)}{b - a}$$

domain as function in b is $\{ b \in \mathbb{R} \mid b \neq a \}$.

$$= \frac{\frac{1}{4}(b^2 - a^2) + (-b - -a)}{b - a}$$

$$= \frac{\frac{1}{4}(b+a)(b-a) - (b-a)}{b-a} = \frac{1}{4}(b+a) - 1$$

Let variable b approach a. The quantity $\frac{1}{4}(b+a) - 1$ approaches $\frac{1}{4}(a+a) - 1 = \frac{a}{2} - 1$. Tangent slope to parabola @ $(a, \frac{1}{4}a^2 - a)$.

Need to answer when secant slopes have "limit" as $b \to a$

Limits $\quad f(x) = x^3$. Find secant slopes and find limits

$\overset{\text{fixed}}{(a, a^3)}$ to $\overset{\text{variable}}{(b, b^3)}$ has secant slope

$$m = \frac{b^3 - a^3}{b - a} \qquad \text{domain is } \{ b \in \mathbb{R} \mid b \neq a \}.$$

$$= \frac{\cancel{(b-a)}(b^2 + ba + a^2)}{\cancel{(b-a)}} = b^2 + ba + a^2$$

As $b \longrightarrow a$, the secant slope $b^2 + ba + a^2 \longrightarrow a^2 + a \cdot a + a^2$
$= 3a^2$

Tangent slope of graph at point (a, a^3) is
$3a^2$.