Week 6 W LO3 2pm

1

Explicit VS Implicit functions

Explicit function: Have formula for function

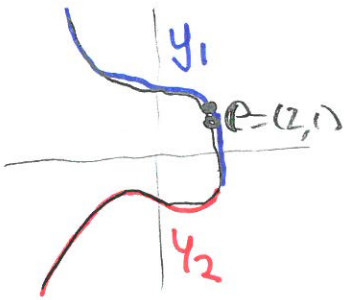
$f(x) = x^2+1$ output can be computed explicitly from input

$g(x) = \sin(x)$

Implicit function Two variables x, y and relationship between them, but NO explicit formula.

WW5 #1 Suppose x, y related by $x^5 + 4xy + y^4 = 41$

Graph of $x^5+4xy+y^4=41$ is a curve in the xy-plane



Verify the point $P=(2,1)$ is on curve.

$$2^5 + 4\cdot 2\cdot 1 + 1^4 \stackrel{?}{=} 41$$

$$32 + 8 + 1 = 41 \checkmark \text{ Okay}$$

Find tangent slope at $(2,1)$ and use it to find tangent line.

We think of y as (implicit) function of x.

Tangent slope is $\frac{dy}{dx}\Big|_{(2,1)}$. We take relation $x^5+4xy+y^4=41$

and take derivative $\frac{d}{dx}$ of both sides

$x \longrightarrow y = y(x) \longrightarrow y^4$

$$\frac{d}{dx}(x^5+4xy+y^4) = \frac{d}{dx}(41) = 0.$$

Use derivative rule on left side.

True on curve.

$$5x^4 + 4(\underbrace{\frac{dx}{dx}}_{1}\cdot y + x\cdot\frac{dy}{dx}) + (4\cdot y^3)\cdot\frac{dy}{dx} = 0.$$

Solve for $\frac{dy}{dx}$:

$$\frac{dy}{dx}(4x + 4y^3) + (5x^4+4y) = 0$$

$$\frac{dy}{dx} = \frac{-(5x^4+4y)}{4(x+y^3)}.$$ on the curve.

Plug in $P=(2,1)$ $x=2,\ y=1$

$$\left(\frac{dy}{dx}\right)\Big|_{(2,1)} = \frac{-(5\cdot 2^4+4\cdot 1)}{4\cdot(2+1^3)} = \frac{-84}{12} = -7$$

tangent slope

tangent line: $\frac{(y-1)}{(x-2)} = -7$ so $y-1 = -7(x-2)$ so $y = -7x+15$.

<u>Rate of change</u> If some quantity Q depends on a variables s

the derivative $\dfrac{dQ}{ds}$ = instantaneous rate of change of Q with respect to variable s

WW5 # 4 The position of a partiple along an axis is given $s(t) = t^3 - 6t^2 + 2t$ $(t \geq 0)$.

(1) Find <u>velocity</u>. Velocity = instantaneous rate of change of position

$= s'(t) = 3t^2 - 6 \cdot 2t + 2$

(2) Find <u>acceleration</u> acceleration = instantaneous rate of change of velocity

$= 3 \cdot 2t - 6 \cdot 2 \cdot 1 + 0$.

#6 The relationship between price (p) and demand (x) of a product is given by

$$2x^2 - 2xp + 50p^2 = 11000$$

Given rate of change of the price is 2 dollars/month when $p = 10$ dollars, find rate of change of demand (x).

Solution ① Determine x when $p = 10$. Plug $p = 10$ into relation and solve for x

$$2x^2 - 20x + 5000 = 11000$$

so $2x^2 - 20x - 6000 = 0$

$x^2 - 10x - 3000 = 0$

$(x-60)(x+50) = 0$

$x = 60$

Point $(x=60, p=10) = (60,10)$ is on the curve.

② Need to find $\frac{dx}{dt}$ at the point $(60,10)$.

Take relation $2x^2 - 2xp + 50p^2 = 11000$ and treat x, and p as functions of t, and take $\frac{d}{dt}(\ )$ of both sides

$$\frac{d}{dt}\left(2x^2 - 2xp + 50p^2\right) = \frac{d}{dt}(11000) = 0$$

Use derivative rules to find Left side

$$2 \cdot 2x \cdot \frac{dx}{dt} - 2\left(\frac{dx}{dt}p + x\frac{dp}{dt}\right) + 50 \cdot 2p \cdot \frac{dp}{dt} = 0$$ on the curve.

Solve for $\frac{dx}{dt}$ in terms of $x, p, \frac{dp}{dt}$

$$\frac{dx}{dt}(4x - 2p) + \frac{dp}{dt}(50 \cdot 2p - 2x) = 0$$

$$\frac{dx}{dt} = -\frac{dp}{dt}\frac{(50p - x)}{(2x - p)}$$

Plug in $x = 60$, $p = 10$, $\frac{dp}{dt} = 2$ dollars/month to get

$$\frac{dx}{dt}_{\left(x=60,\ p=10,\ \frac{dp}{dt}=2\right)} = -2 \cdot \frac{(50 \cdot 10 - 60)}{(2 \cdot 60 - 10)} = -8.$$

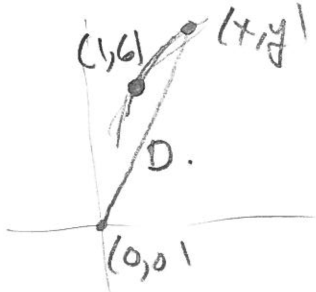
rate of change of demand, when price increasing by 2 dollar/month @ (60, 10) is -8

#7 Particle moving along curve $y = 2\sqrt{4x+5}$.

Verify point $(1,6)$ is on curve.

$$6 \overset{?}{=} 2\sqrt{4\cdot 1+5} = 2\sqrt{9} = 2\cdot 3 \quad \checkmark \text{ okay.}$$

At point $(1,6)$ we have $\frac{dx}{dt} = 5$ unit/sec.



Let $D = \sqrt{x^2+y^2}$ be the distance of the particle to the origin $(0,0)$

Find $\frac{dD}{dt}$ at $(x=1, y=6)$

Solution. Treat x, y as function of t.

① Use relation $y = 2\sqrt{4x+5}$ to find $\frac{dy}{dt}$ in terms $\overset{1,\ 6,}{x, y}$ and $\overset{5}{\frac{dx}{dt}}$.

② Then compute $\frac{dD}{dt}$ by derivative rules to get something in terms of $x, y, \frac{dx}{dt}, \frac{dy}{dt}$.

① $y = 2(4x+5)^{1/2}$ Take $\frac{d}{dt}$ of both sides

$$\frac{dy}{dt} = \cancel{2} \cdot \frac{1}{\cancel{2}}(4x+5)^{-1/2} \cdot \left(4\frac{dx}{dt} + 0\right)$$

so $$\left.\frac{dy}{dt}\right|_{(x=1,\, y=6,\, \frac{dx}{dt}=5} = \frac{1}{\sqrt{4 \cdot 1 + 5}} \cdot (4) \cdot 5 = \frac{20}{3} \text{ units/sec}$$

$x = 1$, $y = 6$, $\frac{dx}{dt} = 5$, $\frac{dy}{dt} = \frac{20}{3}$.

② $D = (x^2+y^2)^{1/2}$, has $\frac{dD}{dt} = \frac{1}{\cancel{2}}(x^2+y^2)^{-\frac{1}{2}} \cdot \left(\cancel{2}x\frac{dx}{dt} + \cancel{2}y\frac{dy}{dt}\right)$

$$\left.\frac{dD}{dt}\right|_{x=1,\, y=6,\, \frac{dx}{dt}=5,\, \frac{dy}{dt}=\frac{20}{3}} = \frac{1}{(1^2+6^2)^{1/2}} \cdot \left(1 \cdot 5 + 6 \cdot \frac{20}{3}\right)$$

$$= \frac{45}{\sqrt{37}}.$$ rate of change of distance to origin $(0,0)$ at $P = (1,6)$ if $\frac{dx}{dt} = 5$