Explicit vs Implicit functions

Explicit function Have formula for the function

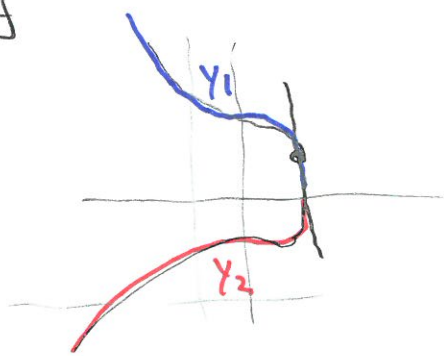
$$f(x) = x^2 + 1, \quad g(x) = \sin(x)$$

Implicit function Only relationship between two variables y and x.

Ex WW5 #1 Suppose x, y related by

$$x^5 + 4xy + y^4 = 41$$

No explicit formula to find y given x



Verify $P = (2,1)$ is on the curve and find tangent slope and tangent line there.

Plug in $(2,1)$ into relation $\quad 2^5 + 4 \cdot 2 \cdot 1 + 1^4 \overset{?}{=} 41$ $\quad$ Point $P = (2,1)$ is on curve.

$$32 + 8 + 1 = 41 \checkmark$$

Want to find $\frac{dy}{dx}$ at point $P=(2,1)$.

Start with $x^5 + 4xy + y^4 = 41$.

Take $\frac{d}{dx}$ of both sides $\frac{d}{dx}(x^5 + 4xy + y^4) = \frac{d}{dx}(41) = 0$

$$5x^4 + 4\cdot\left(1\cdot y + x\cdot\frac{dy}{dx}\right) + 4\cdot y^3\frac{dy}{dx} = 0.$$

$$\frac{d}{dx}(x^5)$$

$$\frac{d}{dx}(xy) = \frac{dx}{dx}\cdot y + x\cdot\frac{dy}{dx} = 1\cdot y + x\cdot\frac{dy}{dx}$$

$$\frac{d}{dx}(y^4) = 4\cdot y^3\cdot\frac{dy}{dx}$$

Now solve for $\frac{dy}{dx}$.

$$(5x^4 + 4y) + (4x + 4y^3)\frac{dy}{dx} = 0$$

$$\frac{dy}{dx} = \frac{-(5x^4 + 4y)}{4(x + y^3)}$$ For (x,y) on the curve.

For point $P=(2,1)$ we get $\left.\frac{dy}{dx}\right|_{P=(2,1)} = \frac{-(5\cdot 2^4 + 4\cdot 1)}{4\cdot(2+1^3)} = \frac{-84}{12} = -7$.

Tangent line is $\frac{(y-1)}{(x-2)} = -7$ which is $y-1 = -7(x-2)$, so $y = -7x + 15$

Recall rate of change. If we have one quantity Q as a function of a variable r, then derivative

$$Q'(r) = \text{instantaneous rate of change}$$

WW5 #4. A particle moves as $s(t) = t^3 - 6t^2 + 2t \quad (t \geq 0)$ position.

(1) velocity = instantaneous rate of change of position

$$= s'(t) = 3t^2 - 6\cdot 2t + 2$$

(2) acceleration = instantaneous rate of change of velocity

$$= v'(t) = 3\cdot 2t - 6\cdot 2\cdot 1 + 0.$$

(3) when is particle at rest? Rest is when velocity $= 0$.

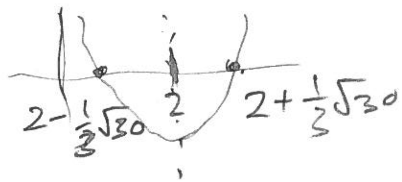
144
-24

So solve $0 = 3t^2 - 12t + 2$ Use quadratic equation

roots $\dfrac{-(-12) \pm \sqrt{144 - 4\cdot 3\cdot 2}}{2\cdot 3} = \dfrac{12 \pm \sqrt{120}}{6} = 2 \pm \frac{1}{3}\sqrt{30}$

(4) when is velocity positive $t \geq 0$

$[0, 2 - \frac{1}{3}\sqrt{30})$ and $(2 + \frac{1}{3}\sqrt{30}, \infty)$.



Implicit differentiation and rate of change.

WW5 #6 The price (p) and the demand (x) for a product is related by $2x^2 - 2xp + 50p^2 = 11000$.

When p = 10, the rate of change of p with respect to time is $\frac{dp}{dt} = 2$ dollar/month. Find rate of change of demand x.

Find $\frac{dx}{dt}$. ① 1st plug p = 10 into relation to find corresponding x.

$$2x^2 - 20x + 50 \cdot 100 = 11000$$

$$\begin{array}{r} 11000 \\ -5000 \\ \hline 6000 \end{array}$$

$$2x^2 - 20x - 6000 = 0$$

$$x^2 - 10x - 3000 = 0$$

$$(x-60)(x+50)$$

Conclude $x = 60$ when $p = 10$

To find $\frac{dx}{dt}$ we derivatie $\frac{d}{dt}$ of the relation

$$\frac{d}{dt}\left(2x^2 - 2xp + 50p^2\right) = \frac{d}{dt}(11000) = 0 \text{ . on curve.}$$

We view x, p as function of t and use derivative rules

$$2 \cdot 2x \cdot \frac{dx}{dt} - 2\left(\frac{dx}{dt} \cdot p + x \cdot \frac{dp}{dt}\right) + 50\, 2p \cdot \frac{dp}{dt} = 0 .$$

Now solve for $\frac{dx}{dt}$ in terms of $x, p, \frac{dp}{dt}$ (which are $x = 60$, $p = 10$, $\frac{dp}{dt} = 2$ dollars/month)

$$\frac{dx}{dt}(4x - 2p) + (-2x + 50 \cdot 2p)\frac{dp}{dt} = 0$$

$$\frac{dx}{dt} = \frac{(2x - 50 \cdot 2p)}{(4x - 2p)} \frac{dp}{dt} = \frac{(x - 50p)}{(2x - p)} \cdot \frac{dp}{dt}$$

$$\left.\frac{dx}{dt}\right|_{(60,10)} = \frac{(60 - 50 \cdot 10)}{(2 \cdot 60 - 10)} \cdot 2 = -8$$

\#7 Particle moving along curve $y = 2\sqrt{4x+5}$

As particle passes through point $P = (1, 6)$

x-coordinate is increasing rate of 5 units/sec

This means $\left.\frac{dx}{dt}\right|_{(1,6)} = 5$ units/sec.

Find rate of change of particle from the ORIGIN $(0,0)$.

Distance to origin is $D = \sqrt{x^2+y^2}$

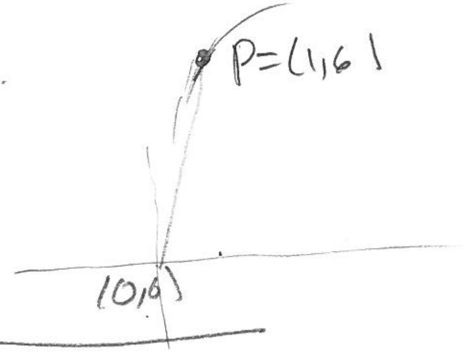
Find $\frac{dD}{dt}$.

Verify $(1,6)$

$6 = 2\sqrt{4 \cdot 1 + 5}$

$= 2 \cdot 3$

okay.



Solution ① Use relation $y = 2\sqrt{4x+5}$ to get $\frac{dy}{dt}$ in terms of x, y, and $\frac{dx}{dt}$ $\left(x = 1,\ y = 6,\ \frac{dx}{dt} = 5\right)$

② To find $\frac{dD}{dt} = \frac{d}{dt}\left((x^2+y^2)^{1/2}\right)$ which write in terms of $x, y, \frac{dx}{dt}, \frac{dy}{dt}$

$$= \frac{1}{2}(x^2+y^2)^{-1/2} \cdot \left(2x\frac{dx}{dt} + 2y\frac{dy}{dt}\right)$$

From $y = 2(4x+5)^{1/2}$ we get

$$\frac{dy}{dt} = \cancel{2} \cdot \frac{1}{\cancel{2}}(4x+5)^{-1/2} \cdot \left(4\frac{dx}{dt} + 0\right)$$

$$\left.\frac{dx}{dt}\right|_{(1,6)} = 5$$

So $\left.\frac{dy}{dt}\right|_{(1,6)} = 1(4\cdot 1+5)^{-1/2} \cdot (4\cdot 5) = \frac{4\cdot 5}{3}.$

So $x=1,\ y=6$

$\frac{dx}{dt} = 5,\ \frac{dy}{dt} = \frac{20}{3}.$

Plug into $\left.\frac{dD}{dt}\right|_{(1,6)} = (x^2+y^2)^{-1/2}\left(x\frac{dx}{dt} + y\cdot\frac{dy}{dt}\right)\Bigg|_{(1,6)}$

$$= (1+36)^{-1/2}\left(1\cdot 5 + 6\cdot\frac{20}{3}\right)$$

$5 + 40$

$$\frac{dD}{dt} = \frac{45}{\sqrt{37}}.$$