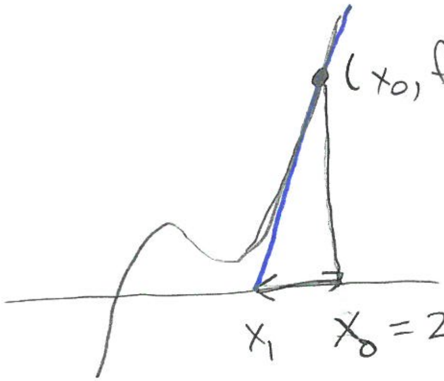
Week 8 Wed L03 2pm

WW6 #6 Newton's method Approximate root of equation $\cos(x^2+2) = x^3$.

Take $f(x) = x^3 - \cos(x^2+2)$. Solution of $f(x)=0$.

Graph of f



$$f(2) = 2^3 - \cos(2^2+2) = 8 - \cos(6) = 7.04$$

horizontal $= x_0 - x_1$

vertical $= f(x_0)$.

$$\frac{\text{vertical}}{\text{horizontal}} = \frac{f(x_0)}{x_0 - x_1} = f'(x_0) \text{ tangent slope}$$

Solve for x_1: $\quad x_0 - x_1 = \dfrac{f(x_0)}{f'(x_0)}$ so $\boxed{x_1 = x_0 - \dfrac{f(x_0)}{f'(x_0)}}$

$$f'(x) = 3x^2 - -\sin(x^2+2) \cdot 2x = 3x^2 + 2x\sin(x^2+2)$$

$$x_1 = 2 - \frac{8-\cos(6)}{3\cdot 2^2 + 2\cdot 2\sin(6)} = 1.353.$$

BAD initial guess $x_0 = 2$.

x_2 = same process $= 0.5$, $x_3 = 0$,

$f'(x_3) = 0$

#2 $\lim_{x \to 2} \left(\frac{x^2+2x-8}{x^2-2x} \right)$

1st try entering 2.

top $= 2^2 + 2 \cdot 2 - 8 = 0$

bottom $= 2^2 - 2 \cdot 2 = 0$

$\frac{0}{0}$. (indeterminant).

$$\frac{x^2+2x-8}{x^2-2x} = \frac{\cancel{(x-2)}(x+4)}{x\cancel{(x-2)}} = \frac{x+4}{x}$$

Now try entering $x=2$.

$$\lim_{x \to 2} \frac{(x+4)}{x} = \frac{2+4}{2} = \frac{6}{2} = 3.$$

Answer (c)

#3 $\lim_{x \to \infty} \left(\sqrt{x^2+3x+1} - \sqrt{x^2+x+2} \right)$

1st try entering $x = \infty$

$\sqrt{\infty^2 + 3\infty + 1} = \infty$

$\sqrt{\infty^2 + \infty + 2} = \infty$

$\infty - \infty$ indeterminant

$$\left(\sqrt{x^2+3x+1} - \sqrt{x^2+x+2} \right) \cdot \frac{\left(\sqrt{x^2+3x+1} + \sqrt{x^2+x+2} \right)}{\left(\sqrt{x^2+3x+1} + \sqrt{x^2+x+2} \right)}$$

$$\frac{(\cancel{x^2}+3x+1) - (\cancel{x^2}+x+2)}{\sqrt{x^2+3x+1} + \sqrt{x^2+x+2}} = \frac{2x-1}{\sqrt{x^2+3x+1} + \sqrt{x^2+x+2}} \frac{\left(\frac{1}{x}\right)}{\left(\frac{1}{x}\right)}$$

$$\frac{2\infty - 1}{\infty + \infty} \quad \frac{\infty}{\infty}$$

$$= \frac{2 - \left(\frac{1}{x}\right)}{\sqrt{1 + \frac{3}{x} + \frac{1}{x^2}} + \sqrt{1 + \frac{1}{x} + \frac{2}{x^2}}}$$

$$\frac{1}{x} = \frac{1}{\sqrt{x^2}}$$

$$= \frac{2 - \frac{1}{\infty}}{\sqrt{1 + \frac{3}{\infty} + \frac{1}{\infty^2}} + \sqrt{1 + \frac{1}{\infty} + \frac{2}{\infty^2}}} = \frac{2-0}{\sqrt{1} + \sqrt{1}} = \frac{2}{2} = 1.$$

Answer (b).

#4 $\lim\limits_{x \to -2^+} f(f(x))$ composite function

1st find limit of inside function

$$\lim_{x \to -2^+} f(x) = -1^+$$

Next find $\lim\limits_{y \to -1^+} f(y) = 1^+$, so $\lim\limits_{x \to -2^+} f(f(x)) = \underline{1^+}$.

Answer (b)

#5 Make $g(x) = \begin{cases} 4x - a + 3 & x < 1 \\ ax^2 + 3x & x \geq 1 \end{cases}$

continuous. Part for $x < 1$ is continuous (polynomial)

——— $x \geq 1$ ——————— (polynomial)

For g to be continuous, we need to insure no jump at $\underline{1}$.

$g(1) = a \cdot 1^2 + 3 \cdot 1 = a + 3$.

To be continuous $\lim_{x \to 1^-} g(x) = a + 3$.

But $\lim_{x \to 1^-} g(x) = \lim_{x \to 1^-} (4x - a + 3) = 4 \cdot 1 - a + 3 = 7 - a$.

So we set $a + 3 = 7 - a$ $\quad 2a = 4 \quad a = 2$ Answer (b)

#6 Tangent slope of $y = \frac{4}{x^2}$ at graph point $(2,1)$.

$y = 4x^{-2}$, so $\frac{dy}{dx} = 4(-2)x^{-3} = -8x^{-3}$

$$\left.\frac{dy}{dx}\right|_2 = -8(2)^{-3} = -1.$$ Answer (b)

#7 For $f(x) = \frac{x\sin x}{\cos x + 1}$ find $f'(2\pi)$

$(x\sin x)' = 1\cdot\sin x + x\cos x$

$(\cos x + 1)' = -\sin x + 0$

Use derivative rules

$$f'(x) = \frac{(1\sin x + x\cos x)\cdot(\cos x + 1) - (x\sin x)(-\sin x)}{(\cos x + 1)^2}$$

Input 2π.

$$f'(2\pi) = \frac{\left(\overset{0}{\cancel{\sin(2\pi)}} + (2\pi)\overset{1}{\cancel{\cos(2\pi)}}\right)\left(\overset{1}{\cancel{\cos 2\pi}} + 1\right) - \overset{\text{zero}}{\cancel{(2\pi\sin 2\pi)(-\sin 2\pi)}}}{(\cos 2\pi + 1)^2}$$

$$= \frac{(0 + 2\pi)(2)}{(1+1)^2} = \frac{4\pi}{4} = \pi$$ Answer (a)

#8 Find derivative $(f \circ g)'(1)$ composition of g inside
f outside

~~$f \cdot g$ product~~

~~$f \circ g$ composition~~ Chain rule $(f \circ g)'(1) = f'(g(1))\, g'(1)$.

From the two graphs find $g(1)$, $g'(1)$, $f(g(1))$, $f'(g(1))$

$$g(1) = 0, \quad g'(1) = \frac{2}{3}, \quad f(0) = \frac{1}{2}, \quad f'(g(1)) = \frac{3}{2}.$$

So $(f \circ g)'(1) = \frac{3}{2} \cdot \frac{2}{3} = 1$. Answer (b)