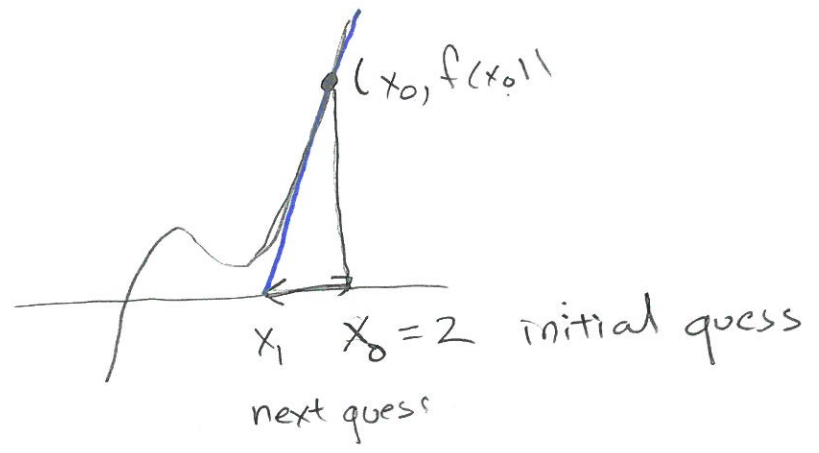


WW6 #6 Newton's method Approximate root of equation $\cos(x^2 + 2) = x^3$.

Take $f(x) = x^3 - \cos(x^2 + 2)$. Solution of $f(x) = 0$.

Graph of f



$$f(2) = 2^3 - \cos(2^2 + 2) = 8 - \cos(6) = 7.04$$

horizontal = $x_0 - x_1$
vertical = $f(x_0)$.

$$\frac{\text{vertical}}{\text{horizontal}} = \frac{f(x_0)}{x_0 - x_1} = f'(x_0) \text{ tangent slope.}$$

Solve for x_1 :

$$x_0 - x_1 = \frac{f(x_0)}{f'(x_0)} \text{ so } \boxed{x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}}$$

$$f'(x) = 3x^2 - \sin(x^2 + 2) \cdot 2x = 3x^2 + 2x \sin(x^2 + 2)$$

$$x_1 = 2 - \frac{8 - \cos(6)}{3 \cdot 2^2 + 2 \cdot 2 \sin(6)} = 1.353$$

BAD initial guess $x_0 = 2$.

$x_2 = \text{same process} = 0.5$, $x_3 = 0$,
 $f'(x_3) = 0$

#2

$$\lim_{x \rightarrow 2} \left(\frac{x^2 + 2x - 8}{x^2 - 2x} \right)$$

1st try entering 2.

$$\text{top} = 2^2 + 2 \cdot 2 - 8 = 0$$

$$\text{bottom} = 2^2 - 2 \cdot 2 = 0$$

 $\frac{0}{0}$ (indeterminate).

$$\frac{x^2 + 2x - 8}{x^2 - 2x} = \frac{(x-2)(x+4)}{x(x-2)} = \frac{x+4}{x}$$

Now try entering $x=2$.

$$\lim_{x \rightarrow 2} \frac{(x+4)}{x} = \frac{2+4}{2} = \frac{6}{2} = 3$$

Answer (c)

$$\#3 \lim_{x \rightarrow \infty} (\sqrt{x^2+3x+1} - \sqrt{x^2+x+2})$$

1st try entering $x = \infty$

$$\sqrt{\infty^2 + 3\infty + 1} = \infty$$

$$\sqrt{\infty^2 + \infty + 2} = \infty$$

$\infty - \infty$ indeterminate

$$(\sqrt{x^2+3x+1} - \sqrt{x^2+x+2}) \cdot \frac{(\sqrt{x^2+3x+1} + \sqrt{x^2+x+2})}{(\sqrt{x^2+3x+1} + \sqrt{x^2+x+2})}$$

$$\frac{(\cancel{x^2} + 3x + 1) - (\cancel{x^2} + x + 2)}{\sqrt{x^2+3x+1} + \sqrt{x^2+x+2}}$$

$$= \frac{2x - 1}{\sqrt{x^2+3x+1} + \sqrt{x^2+x+2}}$$

$$\left(\frac{1}{x}\right)$$

$$\frac{2\infty - 1}{\infty + \infty}$$

$$\frac{\infty}{\infty}$$

$$\left(\frac{1}{x}\right)$$

$$\infty + \infty$$

$$\frac{\infty}{\infty}$$

$$\frac{1}{x} = \frac{1}{\sqrt{x^2}}$$

$$= \frac{2 - \left(\frac{1}{x}\right)}{\sqrt{1 + \frac{3}{x} + \frac{1}{x^2}} + \sqrt{1 + \frac{1}{x} + \frac{2}{x^2}}}$$

$$= \frac{2 - \frac{1}{\infty}}{\sqrt{1 + \frac{3}{\infty} + \frac{1}{\infty^2}} + \sqrt{1 + \frac{1}{\infty} + \frac{2}{\infty^2}}} = \frac{2 - 0}{\sqrt{1} + \sqrt{1}} = \frac{2}{2} = 1$$

Answer (b).

$$\#4 \quad \lim_{x \rightarrow -2^+} f(f(x))$$

composite function

1st find limit of inside function

$$\lim_{x \rightarrow -2^+} f(x) = -1^+$$



$$\text{Next find } \lim_{y \rightarrow -1^+} f(y) = 1^+, \text{ so } \lim_{x \rightarrow -2^+} f(f(x)) = 1^+.$$

Answer (b)

#5 Make $g(x) = \begin{cases} 4x - a + 3 & x < 1 \\ ax^2 + 3x & x \geq 1 \end{cases}$

continuous. Part for $x < 1$ is continuous (polynomial)
 $\underbrace{\hspace{10em}}_{x \geq 1}$ (polynomial)

For g to be continuous, we need to insure no jump at 1.

$$g(1) = a \cdot 1^2 + 3 \cdot 1 = a + 3.$$

To be continuous $\lim_{x \rightarrow 1^-} g(x) = a + 3.$

$$\text{But } \lim_{x \rightarrow 1^-} g(x) = \lim_{x \rightarrow 1^-} (4x - a + 3) = 4 \cdot 1 - a + 3 = 7 - a.$$

$$\text{So we set } a + 3 = 7 - a \quad 2a = 4 \quad a = 2 \quad \text{Answer (b)}$$

#6 Tangent slope of $y = \frac{4}{x^2}$ at graph point (2, 1).

$$y = 4x^{-2}, \text{ so } \frac{dy}{dx} = 4(-2)x^{-3} = -8x^{-3}$$

$$\frac{dy}{dx} \Big|_2 = -8(2)^{-3} = -1. \text{ Answer (b)}$$

#7 For $f(x) = \frac{x \sin x}{\cos x + 1}$ find $f'(2\pi)$.

$$(x \sin x)' = 1 \cdot \sin x + x \cos x$$
$$(\cos x + 1)' = -\sin x + 0$$

Use derivative rules

$$f'(x) = \frac{(1 \sin x + x \cos x) \cdot (\cos x + 1) - (x \sin x) (-\sin x)}{(\cos x + 1)^2}$$

Input 2π .

$$f'(2\pi) = \frac{(\overset{0}{\sin(2\pi)} + (2\pi) \overset{1}{\cos(2\pi)}) (\overset{1}{\cos(2\pi)} + 1) - \cancel{(2\pi \sin(2\pi)) (-\sin(2\pi))}}{(\cos(2\pi) + 1)^2}$$

$$= \frac{(0 + 2\pi)(2)}{(1+1)^2} = \frac{4\pi}{4} = \pi \text{ Answer (a)}$$

#8 Find derivative $(f \circ g)'(1)$ composition of g inside
 f outside

~~$f \cdot g$ product~~

~~$f \circ g$ composition~~

Chain rule $(f \circ g)'(1) = f'(g(1)) g'(1)$.

From the two graphs find $g(1)$, $g'(1)$, $f(g(1))$, $f'(g(1))$

$$g(1) = 0, \quad g'(1) = \frac{2}{3}, \quad f(0) = \frac{1}{2}, \quad f'(g(1)) = \frac{3}{2}.$$

So $(f \circ g)'(1) = \frac{3}{2} \cdot \frac{2}{3} = 1$. Answer (b).