WW6 #6 Use Newton's method to approximate root of equation $\cos(x^2+2) = x^3$.

Take $f(x) = x^3 - \cos(x^2+2)$.

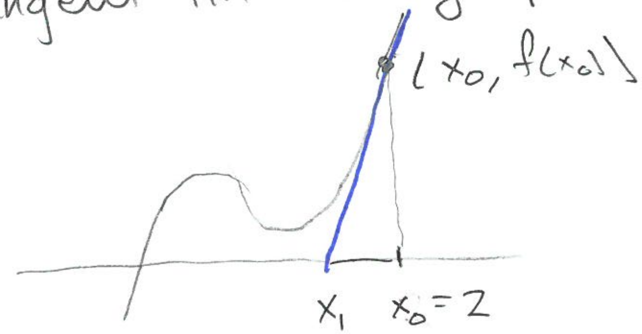
Want to find root $f(x) = 0$.

$f(2) = 2^3 - \cos(2^2+2)$
$= 8 - \cos(6)$

$f'(x) = 3x^2 - -(\sin(x^2+2))\,2x$

Newton's method Take initial guess. $x_0 = 2$

Construct tangent line to graph at $(x_0, f(x_0))$



$x_0 - x_1$ = horizontal
$f(x_0)$ = vertical

$\frac{f(x_0)}{x_0 - x_1}$ = slope $m = f'(x_0)$

where tangent line to $(x_0, f(x_0))$ crosses x-axis is new guess x_1

Solve for x_1 to get $x_0 - x_1 = \frac{f(x_0)}{f'(x_0)}$, so $\boxed{x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}}$

$$x_1 = 2 - \frac{8 - \cos(6)}{3 \cdot 2^2 + (\sin(6))2 \cdot 2} \doteq 1.353$$

Sample midterm

#2 $\lim_{x \to 2} \left(\dfrac{x^2+2x-8}{x^2-2x} \right)$

1st try to input $x=2$

bottom denominator $= 0$

top numerator $= 0$

situation $\frac{0}{0}$.

Try to factor and cancel whatever is making bottom 0, with something on top

$$\frac{\text{top}}{\text{bottom}} = \frac{\cancel{(x-2)}(x+4)}{x\cancel{(x-2)}} = \frac{x+4}{x} \quad (\text{for } x \neq 2).$$

Now we can input $x=2$ without problems $\frac{2+4}{2} = \frac{6}{2} = 3$.

Answer (E)

#3 $\lim_{x \to \infty} \left(\sqrt{x^2+3x+1} - \sqrt{x^2+x+2} \right)$

Can we ienter $x=\infty$? $\sqrt{\infty^2+3\infty+1}$ $\sqrt{\infty^2+\infty+2}$

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$\infty - \infty$. (we need to be more carefull.

$$\left(\sqrt{x^2+3x+1} - \sqrt{x^2+x+2}\right) \cdot \frac{\left(\sqrt{x^2+3x+1} + \sqrt{x^2+x+2}\right)}{\left(\sqrt{x^2+3x+1} + \sqrt{x^2+x+2}\right)}$$

$$= \frac{(\cancel{x^2}+3x+1) - (\cancel{x^2}+x+2)}{\left(\sqrt{x^2+3x+1} + \sqrt{x^2+x+2}\right)} = \frac{(2x-1)\left(\frac{1}{x}\right)}{\left(\sqrt{x^2+3x+1} + \sqrt{x^2+x+2}\right)\left(\frac{1}{x}\right)}$$

$\left(\frac{1}{\sqrt{x^2}}\right)$

try $x=\infty$

$$\frac{2\infty - 1}{\sqrt{\infty^2} + \sqrt{\infty^2}}$$

$\frac{\infty}{\infty}$

still need to be carefull.

$$= \frac{\left(2 - \frac{1}{x}\right)}{\sqrt{\frac{x^2+3x+1}{x^2}} + \sqrt{\frac{x^2+x+2}{x^2}}}$$

$\left(2 - \frac{1}{x}\right) \longrightarrow 2 - 0$ when we put $x=\infty$.

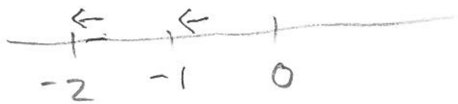
$$= \frac{\left(2 - \frac{1}{x}\right)}{\sqrt{1+\frac{3}{x}+\frac{1}{x^2}} + \sqrt{1+\frac{1}{x}+\frac{2}{x^2}}} \longrightarrow \frac{(2-0)}{\sqrt{1+0+0}+\sqrt{1+0+0}} = \frac{2}{2} = 1.$$

Answer (b)

#4 $\lim\limits_{x \to -2^+} f(f(x))$

First $\lim\limits_{x \to -2^+} f(x) = -1^+$

$\lim\limits_{y \to -1^+} f(y) = 1$



So $\lim\limits_{x \to -2^+} f(f(x)) = \lim\limits_{y \to -1^+} f(y) = 1$

Answer (b)

#5 Make $g(x) = \begin{cases} 4x - a + 3 & \text{when } x < 1 \\ \underline{ax^2 + 3x} & \text{when } x \geq 1 \end{cases}$

continuous. $\lim\limits_{x \to b} g(x) = g(b)$. Each part is continuous. Need to adjust a to have the two graphs joined together.

$g(1) = a \cdot 1^2 + 3 \cdot 1 = a + 3$.

$\lim\limits_{x \to 1^-} g(x) = \lim\limits_{x \to 1^-} (4x - a + 3) = 4 - a + 3 = 7 - a$

Continuous means $\lim\limits_{x \to 1^-} g(x) = g(1)$ so $7 - a = a + 3$ $\quad 4 = 2a$ $\quad a = 2$. Answer b.

#6 Tangent slope to $y = \frac{4}{x^2} = 4x^{-2}$ at point $(2,1)$

$\frac{dy}{dx} = 4(-2)x^{-3}$ when $x = 2$ we get $\left.\frac{dy}{dx}\right|_{x=2} = 4(-2)\frac{1}{2^3} = -1$

Answer (b).

#7 Find $f'(2\pi)$ of $f(x) = \frac{x\sin x}{\cos x + 1}$

$(x\sin x)' = 1\sin x + x\cos x$

$(\cos x + 1)' = -\sin x$

Use derivative rules.

$$f'(x) = \frac{(\sin x + x\cos x)(\cos x + 1) - (x\sin x)(-\sin x)}{(\cos x + 1)^2}$$

$$f'(2\pi) = \frac{(\overset{0}{\cancel{\sin 2\pi}} + 2\pi\cos 2\pi)(\cos 2\pi + 1) - \cancel{(2\pi \cdot \overset{0}{\sin 2\pi})(-\overset{0}{\sin(2\pi)})}}{(\cos 2\pi + 1)^2}$$

$$= \frac{(2\pi \cdot 1)(1+1)}{(1+1)^2} = \pi. \quad \text{Answer (a).}$$

8 Find derivative $(f \circ g)'(1)$ composite function

Chain rule

$$(f \circ g)'(1) = f'(g(1)) \cdot g'(1).$$

$$= \frac{3}{2} \cdot \frac{2}{3} = 1.$$

Answer (b)

$g(1)$, $g'(1)$

$f'(g(1))$.

$g(1) = 0$,

$g'(1) = \frac{2\text{up}}{3\text{across}} = \frac{2}{3}$

$f(g(1)) = f(0) = \frac{1}{2}$

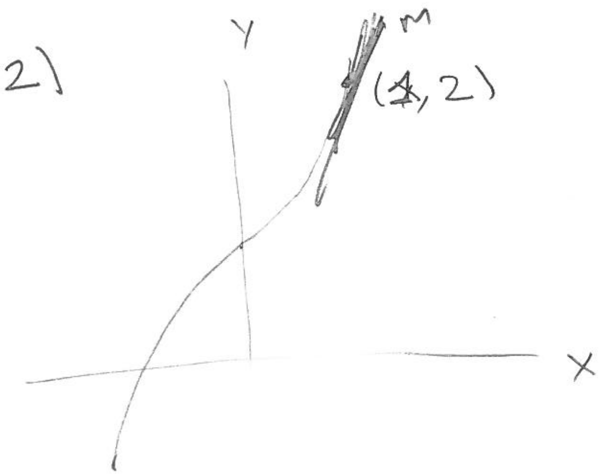
$f'(g(1)) = f'(0) =$

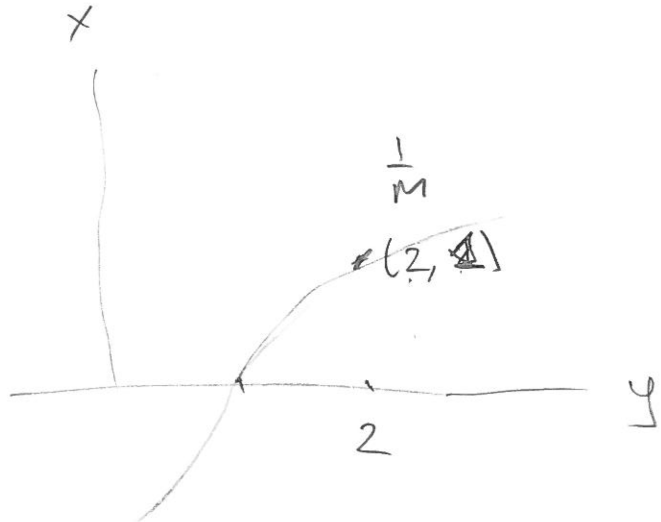
$\frac{3\text{up}}{2\text{across}} = \frac{3}{2}$

\#9 $y = f(x) = x^3 + 1$ is one-to-one function
(there is inverse function)

h = inverse function

Find $h'(2)$





$x^3 + 1 = 2$, $x^3 = 1$
$x = 1$

$$\frac{dy}{dx} = 3x^2 + 0, \quad \left.\frac{dy}{dx}\right|_{x=1} = 3.$$

$$\frac{dx}{dy} = \frac{1}{3}$$

$h'(2) = \frac{1}{3}$. Answer (a)