Look at information page on midterm to find your <u>assigned</u> seat.

#10 Recall <u>derivative</u> is <u>instantaneous rate of change</u>

Need to find where $v'(t)$ is the <u>lowest</u>.

$v'(t)$ is lowest at $t = 15$ Answer (c)

#11 V, S $V = \frac{4}{3}\pi r^3$, $S = 4\pi r^2$ $r = \left(\frac{S}{4\pi}\right)^{1/2}$

Write relationship of V and S.

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(\left(\frac{S}{4\pi}\right)^{1/2}\right)^3 = \frac{\cancel{4}}{3}\cancel{\pi}\frac{S^{3/2}}{\cancel{(4\pi)^{3/2}}} = \frac{S^{3/2}}{3(4\pi)^{1/2}}$$

$$\cancel{4\pi}(4\pi)^{1/2}$$

$$\frac{dV}{dS} = \frac{1}{\cancel{3}(4\pi)^{1/2}}\frac{\cancel{3}}{2}S^{1/2} \qquad \left.\frac{dV}{dS}\right|_{S=36\,cm^2} = \frac{1}{(4\pi)^{1/2}}\frac{1}{2}6\,cm = \frac{3}{2\sqrt{\pi}}$$

Answer (b)

12 (a) Express derivative as limit.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$$

$$f'(a) = \lim_{h\to 0} \frac{f(a+h)-f(a)}{h}.$$

$$f'(a) = \lim_{x\to a} \frac{f(x)-f(a)}{x-a}.$$

(b) Take $f(x) = \frac{1}{\sqrt{x+1}}$, express $f'(x)$, or $f'(a)$ as limit.

$$f'(a) = \lim_{x\to a} \frac{\frac{1}{\sqrt{x+1}} - \frac{1}{\sqrt{a+1}}}{x-a}.$$

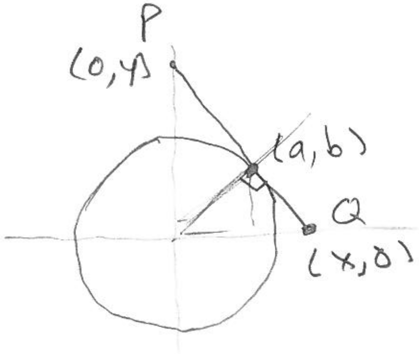
(c) Find the limit of part (b). Manipulate $\dfrac{\frac{1}{\sqrt{x+1}} - \frac{1}{\sqrt{a+1}}}{x-a}$

$$\frac{\frac{\sqrt{a+1}-\sqrt{x+1}}{\sqrt{x+1}\sqrt{a+1}}}{(x-a)} \cdot \frac{\sqrt{a+1}+\sqrt{x+1}}{\sqrt{a+1}+\sqrt{x+1}} = \frac{\frac{\overbrace{(a+1)-(x+1)}^{a-x}}{\sqrt{x+1}\sqrt{a+1}\,(\sqrt{a+1}+\sqrt{x+1})}}{(x-a)}$$

$$= \frac{-1}{\sqrt{x+1}\sqrt{a+1}\,(\sqrt{a+1}+\sqrt{x+1})} \longrightarrow \frac{-1}{\sqrt{a+1}\sqrt{a+1}\,(\sqrt{a+1}+\sqrt{a+1})} = -\frac{1}{2}(a+1)^{-3/2}$$

is $f'(a)$.

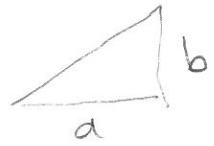
#13



$x^2+y^2=2^2$

$a^2+b^2=4$

Relate x and y.



slope is $\frac{b}{a}$.

so $\perp$ line (from P to Q) has slope $-\frac{a}{b}$.

Equation of line through (a,b) with slope $-\frac{a}{b}$ is

$$y = b + \left(-\frac{a}{b}\right)(x-a)$$

Point P is when $x=0$. $y = b + \left(-\frac{a}{b}\right)(0-a) = b + \frac{a^2}{b} = \frac{b^2+a^2}{b}$

$y = \frac{4}{b}$ $\quad b = \frac{4}{y}$

Point Q is when $y=0$: $0 = b + \left(-\frac{a}{b}\right)(x-a)$, so $\frac{a}{b}(x-a) = b$

$a(x-a) = b^2$

$ax = b^2+a^2 = 4$

$x = \frac{4}{a}$ $\quad a = \frac{4}{x}$.

$4 = a^2+b^2 = \frac{16}{x^2} + \frac{16}{y^2}$ so $1 = \frac{4}{x^2} + \frac{4}{y^2}$.

Find $\frac{dy}{dx}$. Given relation $1 = \frac{4}{x^2} + \frac{4}{y^2}$.

$$x^2y^2 = 4y^2 + 4x^2.$$

To find $\frac{dy}{dx}$, we apply $\frac{d}{dx}$ to both sides

$$\frac{d}{dx}(x^2y^2) = \frac{d}{dx}(4y^2 + 4x^2)$$

$$2x \cdot y^2 + x^2\, 2y \frac{dy}{dx} = 4 \cdot 2y \frac{dy}{dx} + 4 \cdot 2x$$

$$\frac{dy}{dx}(x^2 2y - 4 \cdot 2y) = 4 \cdot 2x - 2xy^2.$$

$$\frac{dy}{dx} = \frac{4 \cdot 2x - 2xy^2}{x^2 2y - 4 \cdot 2y} = \frac{4x - xy^2}{x^2y - 8}$$

$\frac{dy}{dt}$

$$\frac{dy}{dt} = \frac{dy}{dx} \cdot \frac{dx}{dt}$$

$x = 3 + t$

What is $\frac{dy}{dx}$ when $x = 5$. What is y? $1 = \frac{4}{25} + \frac{4}{y^2}$, $\frac{4}{y^2} = 1 - \frac{4}{25} = \frac{21}{25}$

$$\frac{y^2}{4} = \frac{25}{21}$$

$$y = \frac{2 \cdot 5}{\sqrt{21}} = \frac{10}{\sqrt{21}}$$

$$\left.\frac{dy}{dx}\right|_{(x=5,\ y=\frac{10}{\sqrt{21}})} = \left.\left(\frac{4x - xy^2}{x^2y - 8}\right)\right|_{(5,\ \frac{10}{\sqrt{21}})} = -\frac{8\sqrt{21}}{(21)^2}.$$

#14 Graph of function f is shown

Draw rough shape of graph of derivative f'

Key facts. Derivative at input a is the tangent slope at a.

(i) tangent slope $0 \Rightarrow$ derivative 0

(ii) tangent slope $>0 \Rightarrow$ ______ >0

(iii) tangent slope $<0 \Rightarrow$ ______ <0