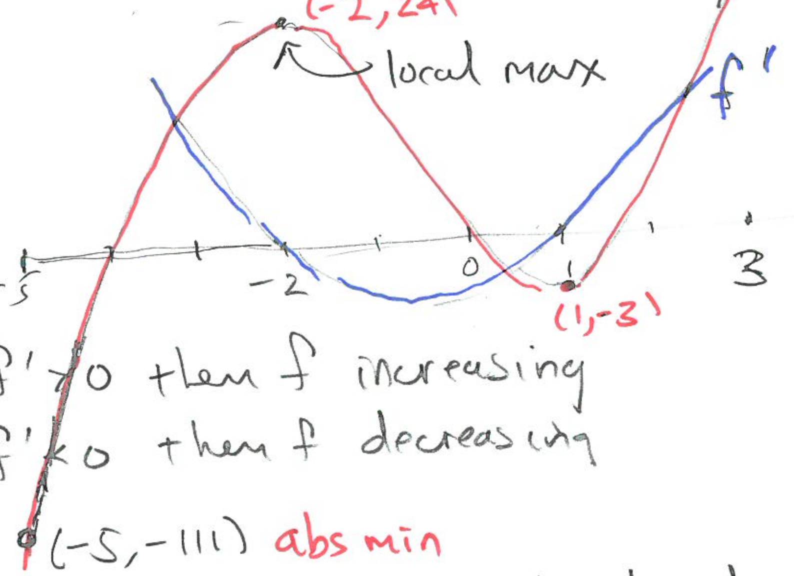
# Use of 1st and 2nd derivative of a function.

Example $f(x) = 2x^3 + 3x^2 - 12x + 4$

$f'(x) = 2\cdot 3x^2 + 3\cdot 2x - 12 = 6(x^2 + x - 2)$

$= 6(x+2)(x-1)$

roots $x = -2,\ x = 1$




| x | f(x) |
|---|---|
| -2 | 24 |
| 1 | -3 |
| 3 | |
| -5 | |

$2(-2)^3 + 3\cdot(-2)^2 + (-12)(-2) + 4$

$\cancel{-16} + \cancel{12} + 24 + 4$

$2\cdot 1^3 + 3\cdot 1^2 - 12\cdot 1 + 4$

$\cancel{2} + 3 - 12 + 4$

$2\cdot 3^3 + 3\cdot 3^2 - 36 + 4$

54, 27, 4, 85, -36, 49

$2\cdot(-5)^3 + 3\cdot(-5)^2 + 60$

$-250 + 75 + 60 + 4$

$= -111$

250, 139, 111, 139

Intuition $f' > 0$ then $f$ increasing
$f' < 0$ then $f$ decreasing

Terminology An input $c$ yields local maximum if value $f(c)$ is larger than values of near by inputs

Similarly for local min.

An absolute max is an input $c$ (if there is one) whose value $f(c)$ is $\geq$ 2 all other values. In above for domain $[-5, 3]$ absolute max at input $c = 3$. Abs min at $c = -5$

**Extreme Value Theorem** If $f$ is a continuous function on closed interval, then it has an absolute max and an absolute min.

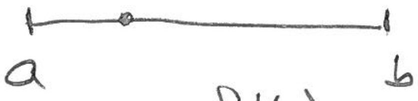
Where/how do you find it?

An absolute max $\Rightarrow$ local max,
absolute min $\Rightarrow$ local min.

If we can find local max, we pick input with largest value.
Similarly for local min.

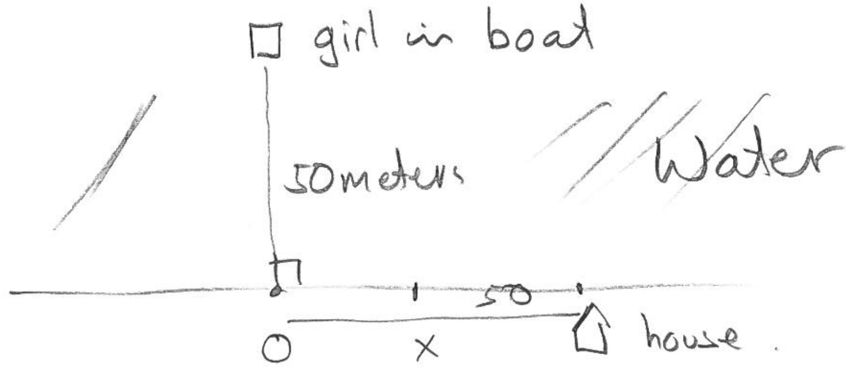
**Local Extreme Value Theorem** Suppose $f$ continuous on closed interval $[a,b]$, and if a local max (or a local min) occurs inside at $c$ (call it interior point), then either

$f'(c) = 0$ **OR** $f'(c)$ DNE




Useful: Need to only look at
1. interior point $c$ so $f'(c)$
2. interior point $c$ so $f'(c)$ DNE
3. endpoints $a$, $b$

## Example

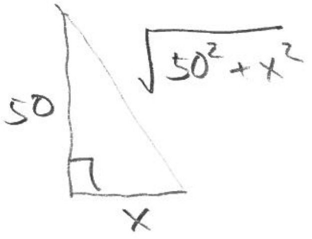



Girl wants to swim/walk to house.

swims at 2m/sec

walks/run at 4m/sec.

Swim




Swim time $\frac{\sqrt{50^2+x^2}}{2}$

Domain $0 \le x \le 50$
closed interval

Walk/run


Walk/run time $\frac{50-x}{4}$

continuous

closed

Total time $T(x) = \frac{\sqrt{50^2+x^2}}{2} + \left(\frac{50-x}{4}\right)$

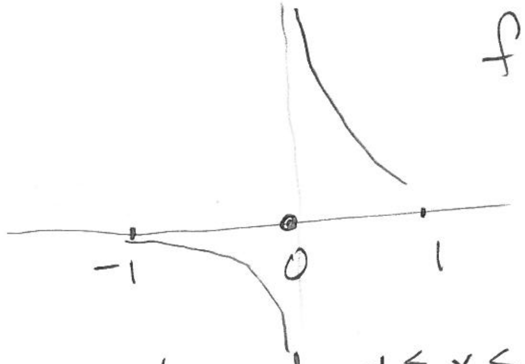
domain $[0, 50]$

T is continuous function, $[0, 50]$ is closed interval

EVT $\Rightarrow$ T does have a absolute max, and a absolute min.

To get to house as quickly as possible, seek absolute min

Note. ① If we do not assume continuous, a function may not have abs max, abs min



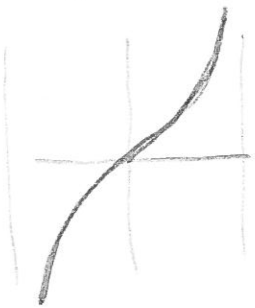

$$f(x) = \begin{cases} 1/x & \text{if } x \neq 0 \\ 0 & \text{if } x = 0 \end{cases}$$

not continuous at $x=0$.

$\lim_{x \to 0^-} f(x) = -\infty$, $\lim_{x \to 0^+} f(x) = +\infty$.

interval $-1 \le x \le 1$

② If we do not assume interval is closed, the function may not have abs max or abs min



$f(x) = \tan(x)$ $\quad -\frac{\pi}{2} < x < \frac{\pi}{2}$ $\quad$ not closed interval

No abs max or abs min.

EVT If $f$ continuous and interval closed $[a,b]$, then $f$ does have abs max, abs min.

Since $T(x) = \dfrac{\sqrt{50^2+x^2}}{2} + \left(\dfrac{50-x}{4}\right)$ is differentiable

LEVT $\Rightarrow$ local max/min occur

① $f'(c) = 0$ ✓
② ~~$f'(c)$ DNE~~
③ endpoints $0, 50$ ✓

$$T'(x) = \frac{1}{2}\,\frac{1}{2}\,(50^2+x^2)^{-1/2}(0+2x) + \left(0-\frac{1}{4}\right)$$

Critical point is interior point $c$ so that $T'(c)=0$

$$0 = \frac{1}{4}(50^2+c^2)^{-1/2}(2c) + \left(-\frac{1}{4}\right)$$

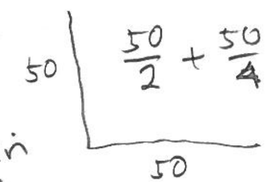
$$1 = \frac{2c}{(50^2+c^2)^{1/2}} \qquad (50^2+c^2)^{1/2} = 2c \qquad c = \left(\frac{50^2}{3}\right)^{1/2}$$

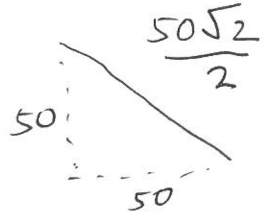
$$50^2+c^2 = 4c^2 \qquad c = \frac{50}{\sqrt{3}}.$$

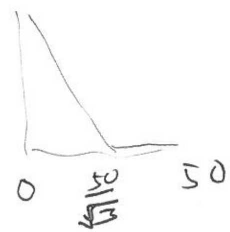
$$50^2 = 3c^2$$

Need to check critical point $\frac{50}{\sqrt{3}}$, and the endpoints $0, 50$

| $x$ | $T(x)$ | |
|---|---|---|
| $0$ | $50\left(\frac{3}{4}\right)$ | abs max |
| $\frac{50}{\sqrt{3}}$ | $34.355$ | abs min |
| $50$ | $\frac{50\sqrt{2}}{2}$ | local max |



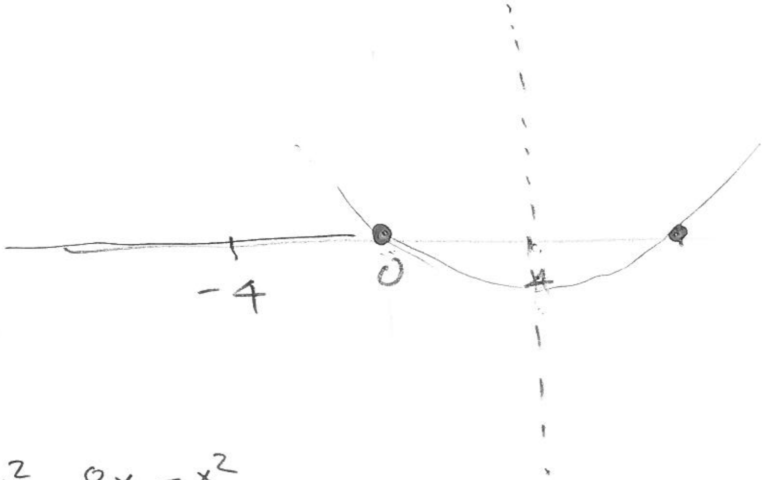



WW6 #9 $f(x) = \dfrac{2x^2}{x-4}$ $x \neq 4$ domain

Find intervals where $f$ increasing $(f' > 0)$
$f$ decreasing $(f' < 0)$

Find local max / local min




$$f'(x) = 2 \cdot \frac{(2x)(x-4) - x^2(1-0)}{(x-4)^2}$$

$2x^2 - 8x - x^2$

$$= 2 \cdot \frac{x^2 - 8x}{(x-4)^2}$$

$$= 2 \frac{x(x-8)}{(x-4)^2}$$

Can ignore $2, \dfrac{1}{(x-4)^2}$ since they are $> 0$.

We focus on parabola $x(x-8)$.
root 0, 8

$f' > 0$ $(-\infty, 0)$ because parabola $x(x-8) > 0$

$f' < 0$ $(0,4) \cup (4,8)$ because $x(x-8) < 0$

$f' > 0$ $(8, \infty)$ because $x(x-8) > 0$.

$f' = 0$ at $x = 0$, $x = 8$

$x = 0$ is <u>local max</u>
$x = 8$ is <u>local min</u>