WW7 # 9 Find $\lim_{t \to \infty} \left(\frac{3^t + 6^t}{3} \right)^{1/t}$ $\quad (\infty)^0$

Take ln of $\left(\frac{3^t + 6^t}{3} \right)^{1/t}$

$$\ln\left(\left(\frac{3^t + 6^t}{3} \right)^{1/t}\right) = \frac{1}{t} \left\{ \ln(3^t + 6^t) - \ln(3) \right\}.$$

The term $\frac{-\ln(3)}{t}$ has limit 0 as $t \to \infty$.

What about $\frac{1}{t}\ln(3^t + 6^t) = \frac{1}{t} \ln\left((6^t) \cdot \left\{ \left(\frac{1}{2}\right)^t + 1 \right\} \right)$

$$= \frac{1}{t} \left\{ \ln(6^t) + \ln\left(\left(\frac{1}{2}\right)^t + 1 \right) \right\}$$

$$= \frac{1}{t} \cdot t \ln 6 + \frac{1}{t} \ln\left(\left(\frac{1}{2}\right)^t + 1 \right)$$

$$= \ln(6) + 0 = \ln(6)$$

So $\lim_{t \to \infty} \ln\left(\left(\frac{3^t + 6^t}{3} \right)^{1/t}\right) = \ln(6)$, so $\lim_{t \to \infty} \left(\frac{3^t + 6^t}{3} \right)^{1/t} = e^{\ln(6)} = 6$.

Derivatives and antiderivatives

A function F is an antiderivative of a function f if $F' = f$

If F is an antiderivative of f, then so too is $F + C$ (constant).

Physical motivation / intuition Suppose $p(t)$ is the position of an object along some axis (t = time).

$p'(t)$ is the velocity $v(t)$ of the object.

So $p(t)$ is antiderivative of velocity.

$v'(t)$ is the acceleration $a(t)$ of the object

So $v(t)$ is antiderivative of acceleration.

WW 8 #5 Race car. Let $p(t)$ = position.

$v(t) = p'(t)$ = velocity

$a(t) = v'(t)$ = acceleration

At $t=0$, $v(0) = 100$ miles/hr

Slam on brakes, so $a(t) = -c$ constant decceleration

Assume $p(0) = 0$, and distance to stop is 491 feet.

Find decceleration c, time t_s to stop.

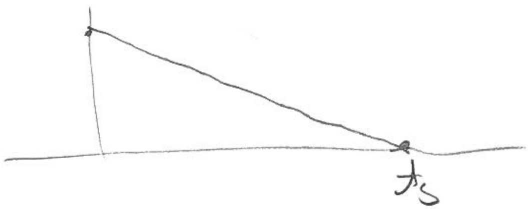
Velocity is antideri $v'(t) = a(t) = -c$, so $v(t) = -ct + B_v$

B_v constant of antiderivative.

100 miles/hr $= v(0) = -c \cdot 0 + B_v$

Change units to feet and seconds $B_v = 100$ miles/hr $= \frac{1\cancel{00}\ 5280 \text{ ft}}{36\cancel{00} \text{ sec}} = \frac{440 \text{ ft}}{3 \text{ secs}}$.

So $v(t) = -ct + \frac{440}{3}$



To find stop time t_s we solve

$$0 = -ct_s + \frac{440}{3}$$

$$t_s = \left(\frac{440}{3}\right)\frac{1}{c}.$$

Position function $p(t)$ is antiderivative of $v(t) = -ct + \frac{440}{3}$

$$p(t) = -c\frac{t^2}{2} + \left(\frac{440}{3}\right)t + B_p \qquad (B_p \text{ constant}).$$

To find constant B_p we set $t=0$ (start to brake).

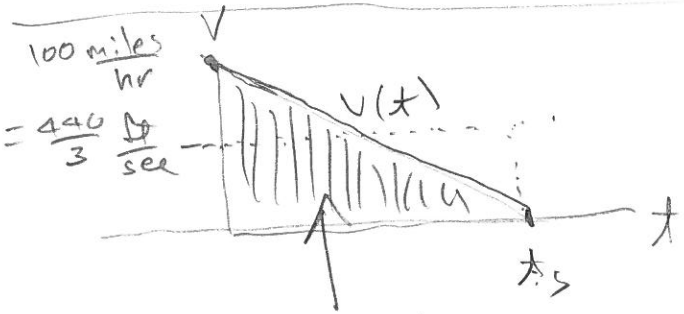
$$0 = p(0) = \cancel{-c\frac{0^2}{2}} + \cancel{\left(\frac{440}{3}\right)\cdot 0} + B_p \quad \text{so} \quad B_p = 0$$

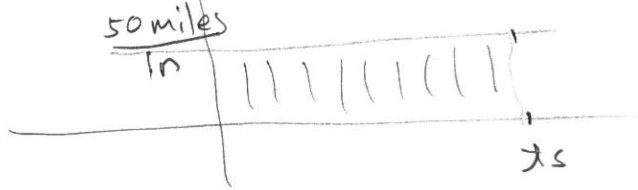
$$\left.\begin{array}{l} \\ \\ \end{array}\right\} t_s = \left(\frac{440}{3}\right)\left(\frac{1}{c}\right)$$

So $$p(t) = -\frac{ct^2}{2} + \left(\frac{440}{3}\right)t.$$

Stopping distance
$$491 = p(t_s)$$
$$= -\frac{c}{2}\left(\frac{440}{3}\right)^2\left(\frac{1}{c}\right)^2 + \left(\frac{440}{3}\right)\cdot\left(\frac{440}{3}\right)\left(\frac{1}{c}\right)$$

$$491 = \frac{1}{2}\left(\frac{440}{3}\right)^2\left(\frac{1}{c}\right) \quad \text{so} \quad c = \frac{1}{2}\left(\frac{440}{3}\right)^2\frac{1}{491}$$
$$(\text{ft}/\text{sec}^2)$$

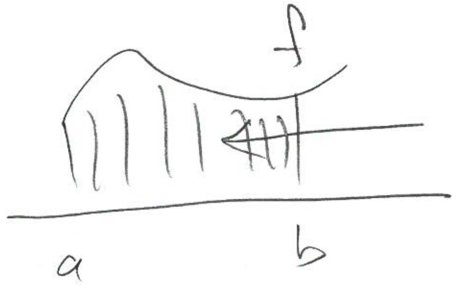




491 is the area under velocity curve

$\left(\frac{50 \text{ miles}}{\text{hr}}\right) \cdot t_s$ = distance travelled

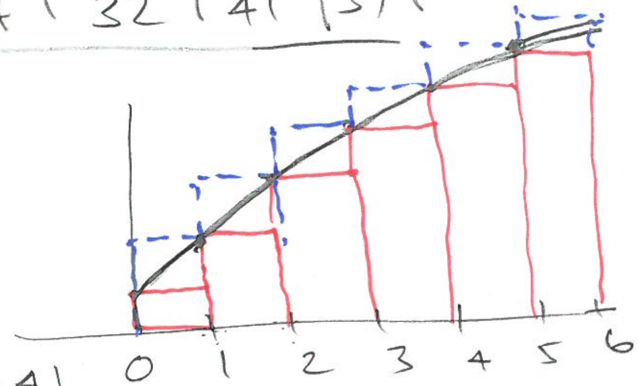
KEY Insight antiderivative of velocity related to area under curve.
function f is related to area under graph of f



This area is related to ~~the~~ antiderivatives of f

#6 Riemann Sums Electric utility pollutes. Rate of pollution is given by table

time month	0	1	2	3	4	5	6
rate of pollution	8	12	18	24	32	41	51



Rate of pollution is increasing.

(a) Find underestimate for amount of pollution for 6 months.

(1 month) · 8 + (1 month) · 12 + ... + (1 month) · 41

1 · 8 + 1 · 12 + 1 · 18 + 1 · 24 + 1 · 32 + 1 · 41

(b) Overestimate is (1 month) · 12 + (1 month) · 18 + 1 · 24 + 1 · 32 + 1 · 41 + 1 · 51

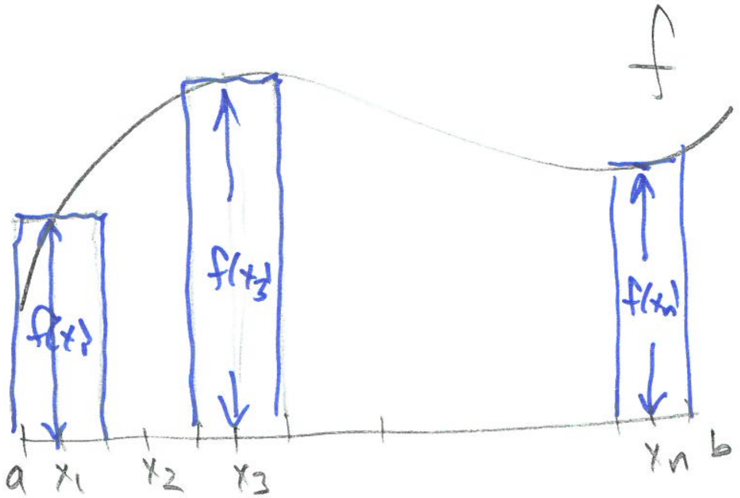
underestimate ≤ true amount of pollution ≤ overestimate.

is area under pollution rate graph.

Riemann Sums Suppose f is a function on an interval $a \le x \le b$. Take some (positive) integer n.

Divide $[a,b]$ into n equal length subintervals

$$\Delta x = \frac{b-a}{n}.$$



pick x_i from i subinterval

Total area of rectangles is $\Delta x f(x_1) + \Delta x f(x_2) + \dots + \Delta x f(x_n)$ is called a Riemann Sum. This is an approximation for area.