Derivatives and antiderivatives

$$F' = f$$

we say f is derivative of F, and F is an antiderivative of f

If F is antiderivative of f, then F+C (C constant) is also antiderivative.

Physical intuition $p(t)$ = position function of a moving object.

Then $p'(t)$ = rate of change of position
velocity / speed of object.
$v(t)$

So position function is an antiderivative of velocity.

Also $v'(t)$ = acceleration of object ($a(t)$)

So velocity function is an antiderivative of acceleration

WW 8 #5 Fast car accelerates to 100 miles/hr

Slam on brakes to get CONSTANT rate of decceleration.

Takes 491 feet to stop. Find the constant of decceleration.

$p(t)$

$v(t)$

$a(t) = -c$ (c the constant of deceleration)

$t = 0$ when brakes are applied, and assume $\boxed{p(0) = 0}$.

What is $\underline{v(t)}$? $v(t)$ is antiderivative of $a(t)$

$v(t) = -ct + B_v$ $\quad B_v$ is constant (for antiderivative)

What is $\underline{p(t)}$? $p(t)$ is antiderivative of $v(t)$

$p(t) = -c\frac{t^2}{2} + B_v t + B_p$ $\quad B_p$ constant for antiderivative

Set $t=0$ to see $0 = p(t) = 0^2 + 0 + B_p$ so $B_p = 0$. and

$p(t) = -c\frac{t^2}{2} + B_v t$. Let t_s be stopping time.

$0 = v(t_s) = -ct_s + B_v$ so $\underline{t_s = \frac{B_v}{c}}$.

Plug stopping time into $p(t)$ to get stopping distance

$$491\,ft = p(t_s) = -c\frac{t_s^2}{2} + B_v t_s$$

$$= -\frac{c}{2}\left(\frac{B_v}{c}\right)^2 + B_v\left(\frac{B_v}{c}\right) = \frac{-B_v^2}{2c} + \frac{B_v^2}{c} = \frac{B_v^2}{2c}.$$

~~Forgot~~: Initial speed 100 miles/hr $= v(0) = -c\cdot 0 + B_v$, $B_v = 100$ miles/hr

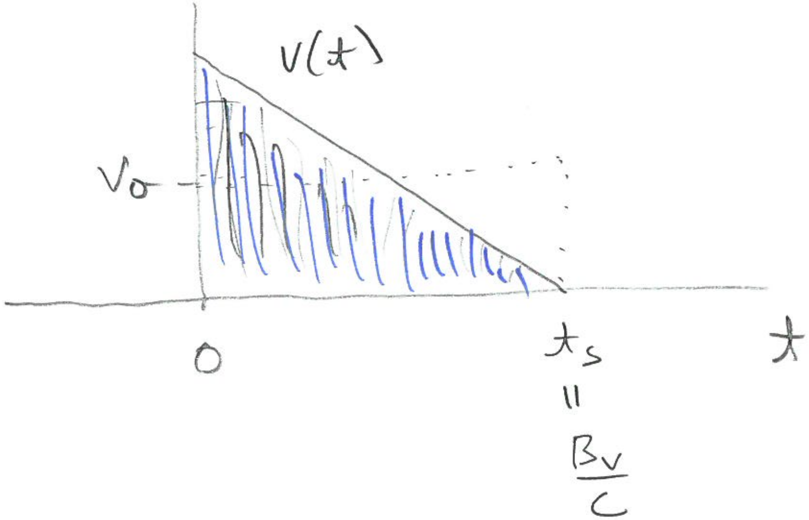
Units: Velocity miles/hr $\rightsquigarrow$ 5280 ft/3600 sec, distance ft.

Now every unit is feet or seconds.

$$B_v = 100\text{ mile/hr} = 1\cancel{0}\cancel{0}\cdot 5280\text{ ft}/36\cancel{0}\cancel{0}\text{ sec} = \frac{5280\text{ ft}}{36\text{ sec}} = \frac{440\text{ ft}}{3\text{ secs}}.$$

$$491\text{ ft} = \frac{B_v^2}{2c} \text{ so } c = \frac{B_v^2}{491\text{ ft}} = \frac{\left(\frac{440}{3}\right)^2}{2\,491}\frac{ft^2/sec^2}{ft} = \frac{\left(\frac{440}{3}\right)^2}{2\,491}\text{ ft}/sec^2$$

$$c = \frac{\left(\frac{440}{3}\right)}{2\cdot 491}\text{ ft}/sec^2$$



$$v(t) = -ct + B_v$$

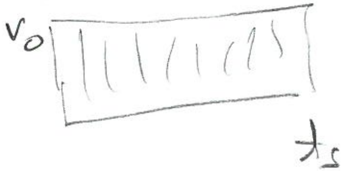
$$B_v = \frac{440}{3} \text{ ft/sec}$$

$$c = \frac{\left(\frac{440}{3}\right)^2}{2 \cdot 491} \text{ ft/sec}^2$$

How is stopping distance related to velocity $v(t)$?

If velocity were constant v_0, then stopping distance is

time · velocity $= t_s \cdot v_0$.



The AREA under velocity $v(t)$ between $0 \le t \le t_s$,

$0 \le v \le v(t)$.

The area under curve of velocity is related to position $p(t)$

which is the antiderivative of velocity.

The <u>area</u> under velocity is related to <u>antiderivative</u> of velocity

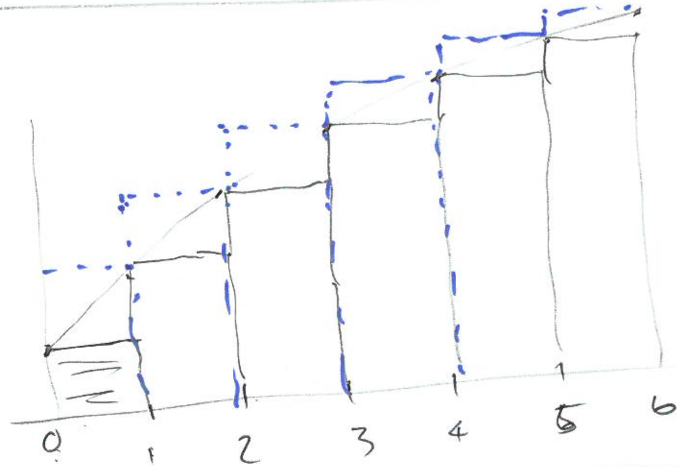
<u>Key Insight</u> Antiderivative of a function is related to area under its curve.

Ano

Another example WW8 # 6

Pollution caused by utility plan increases over time

time months	0	1	2	3	4	5	6
pollution rate	8	12	18	24	32	41	51



Underestimate for pollution $1 \cdot 8 + 1 \cdot 12 + 1 \cdot 18 + 1 \cdot 24 + 1 \cdot 32 + 1 \cdot 41$

The 1's are the 1 month time period. Other number was lowest pollution rate for that month.

Overestimate for pollution $1 \cdot 12 + 1 \cdot 18 + 1 \cdot 24 + 1 \cdot 32 + 1 \cdot 41 + 1 \cdot 51$

These two are examples of what are called Riemann Sums.

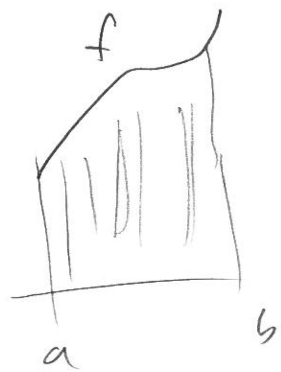
Riemann Sum Suppose f is a function on the interval $[a,b]$

Divide interval into n equal subintervals

$$\Delta x = \frac{b-a}{n}$$

Pick a point x_i these subintervals

$$\Delta x \cdot f(x_1) + \Delta x \cdot f(x_2) + \dots + \Delta x \cdot f(x_n)$$



is an approximation for the area below graph of function f, above x-axis, and between $x=a$ and $y=b$.

If we take x_i so that $f(x_i)$ has min value on subinterval we get underestimate. Similarly if $f(x_i)$ has max value on subinterval we get overestimate.