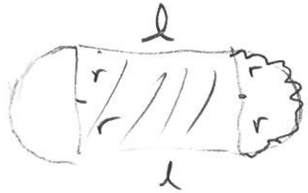
27 Track shaped as



(a) Perimeter is 5km

$5\text{km} = P = 2l + 2\pi r$ so $2l = 5 - 2\pi r$ $\quad l = \frac{5}{2} - \pi r$

$2\pi r = 5$

Area shade region $= 2rl$ Maximize

$A(r) = 2r\left(\frac{5}{2} - \pi r\right) = 5r - 2\pi r^2$ $\qquad 0 \le r \le \frac{5}{2\pi}$

$A(0) = 0 \qquad A\left(\frac{5}{2\pi}\right) = 0$

$A'(r) = 5 - 2\pi 2r = 5 - 4\pi r$

$A'(r) = 0$ when $r = \frac{5}{4\pi}$ Sign A' changes $+$ to $-$ as r increases pass $r = \frac{5}{4\pi}$

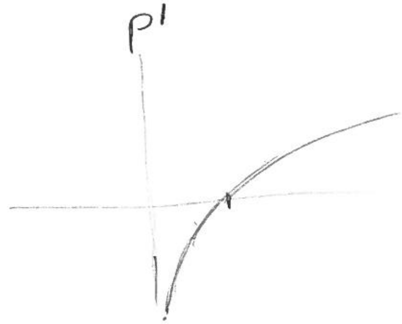
Max at $r = \frac{5}{4\pi}$, $l = \frac{5}{2} - \pi r = \frac{5}{2} - \frac{5}{4} = \frac{5}{4}$.

Area $= 2rl = \frac{5}{2\pi} \cdot \frac{5}{4} = \frac{25}{8\pi}$.

1b) Assume $A = \frac{25}{8\pi} = 2r \cdot l$, so $l = \frac{25}{16\pi}\frac{1}{r}$.

Find l, r so that perimeter $P = 2l + 2\pi r$ is minimal

$$P(r) = 2\left(\frac{25}{16\pi}\right)\left(\frac{1}{r}\right) + 2\pi r.$$ domain is $0 < r < \infty$.



To minimize.
$$p'(r) = \left(\frac{25}{8\pi}\right)(-1)r^{-2} + 2\pi$$
$$0 = -\left(\frac{25}{8\pi}\right)r^{-2} + 2\pi$$
$$\left(\frac{25}{8\pi}\right)r^{-2} = 2\pi$$
$$25r^{-2} = 16\pi^2 \quad \text{so} \quad r = \frac{5}{4\pi}$$

As r increase through critical pt p' changes $-$ to $+$

so min. $r = \frac{5}{4\pi}$ gives min perimeter, $l = \frac{25}{16\pi} \cdot \frac{4\pi}{5} = \frac{5}{4}$

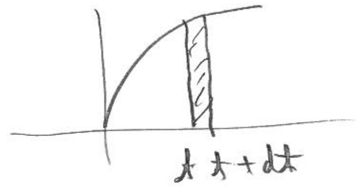
$$P = 2l + 2\pi r = 2\left(\frac{5}{4}\right) + 2\pi\left(\frac{5}{4\pi}\right) = \frac{5}{2} + \frac{5}{2} = 5.$$

#18 Definite integrals.

Tank

48 cubic meters.

rate $r(t) = 9\sqrt{t}$ meter^3/min.



How long to fill tank?

$r(t) \cdot dt$ = amount of water that follows in between t and $t+dt$

$$\int_0^{T_0} 9\sqrt{t}\, dt = 48 \text{ cubic meters.}$$

$$\left.\frac{9\, t^{3/2}}{(3/2)}\right|_0^{T_0} = 48$$

$$\frac{9}{3/2} = 6$$

$$6\, T_0^{3/2} = 48$$

$$T_0^{3/2} = 8$$

$$T_0 = (8)^{2/3} = (8^{1/3})^2 = 2^2$$

$$= 4 \text{ (minutes).}$$

#21 Find $\lim_{n \to \infty} \left(\frac{1}{2n+1} + \frac{1}{2n+2} + \cdots + \frac{1}{2n+n} \right) = \ln\left(\frac{3}{2}\right)$

Riemann Sums

$$\frac{1}{2n+1} + \frac{1}{2n+2} + \cdots + \frac{1}{2n+n} = \frac{1}{n}\frac{1}{(2+\frac{1}{n})} + \frac{1}{n}\frac{1}{(2+\frac{2}{n})} + \cdots + \frac{1}{n}\frac{1}{(2+\frac{n}{n})}$$

$\frac{1}{n} \longleftrightarrow \Delta x = \frac{b-a}{n}$

$a=0,\ b=1$

$f\left(\frac{1}{n}\right) = \frac{1}{2+\frac{1}{n}},\quad f\left(\frac{2}{n}\right) = \frac{1}{2+\frac{2}{n}},\ \ldots\ f\left(\frac{n}{n}\right) = \frac{1}{2+\frac{n}{n}}$

Alternative $a=2,\ b=3$
$f(x) = \frac{1}{x}$

$f(x) = \frac{1}{2+x}$

$$\int_0^1 \frac{1}{2+x}\,dx$$

Use FTC II.

$$\int_0^1 \frac{1}{2+x}\,dx = \ln(2+x)\Big|_0^1 = \ln(3) - \ln(2)$$

$$= \ln(3/2)$$

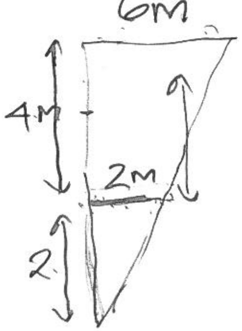
#22 FTC I

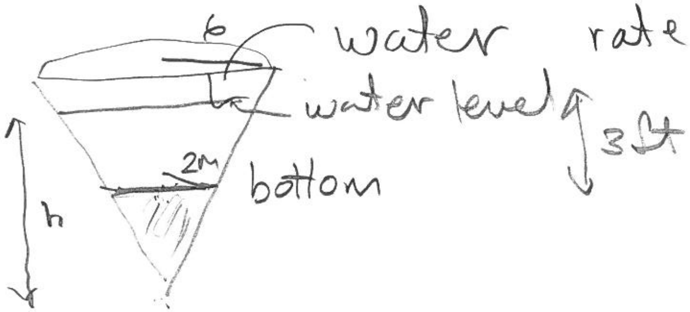
If f is continuous on $a \leq x \leq b$, then area function

$$A(x) = \int_a^x f(t)\,dt$$ is differentiable and $A'(x) = f(x)$

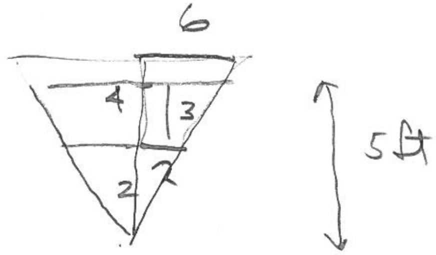
Since the graphs of all 4 functions are continuous, their area functions are differentiable.

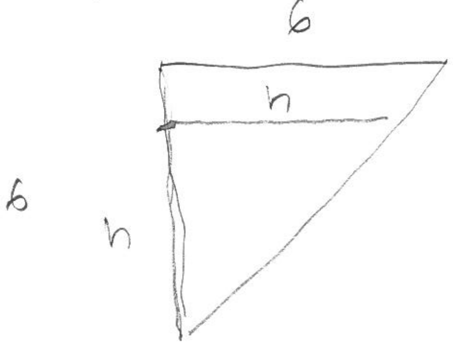
#13 related rates





water rate is $10\pi \frac{m^3}{min}$.





$\underline{\underline{r = h}}$ $\quad V = \frac{1}{3}\text{ base area} \cdot h = \frac{1}{3}\pi h^2 \cdot h = \frac{1}{3}\pi h^3.$

$$V = \frac{1}{3}\pi h^3$$

$$\frac{dV}{dt} = \frac{1}{3}\pi 3 h^2 \frac{dh}{dt}$$

$$10\pi \frac{m^3}{min} = \cancel{\frac{1}{3}}\pi \cancel{3} \cdot (3+2)^2 \cdot \frac{dh}{dt}$$

$$10\cancel{\pi} = \cancel{\pi} \cdot 5^2 \frac{dh}{dt} \quad \text{so} \quad \frac{dh}{dt} = \frac{10}{25} = \frac{2}{5} = 0.4 \frac{\text{meter}}{\text{min.}}$$

\# 14 linear approximation

$f(x) = \sqrt{1+x} + \sin x$ $\qquad$ $f'(x) = \frac{1}{2}(1+x)^{-1/2} + \cos x$

$f(0) = \sqrt{1+0} + \sin 0 = 1$ $\qquad$ $f'(0) = \frac{1}{2} + \cos 0 = \frac{3}{2}$

Estimate $f(0.02)$

Use tangent line. Two things $f(0) = 1$

$f'(0) = \frac{3}{2}$

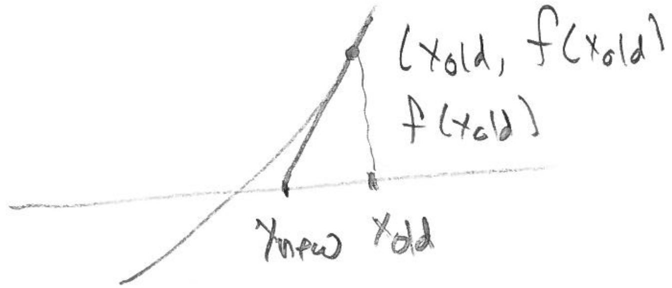
So tangent line a $x=0$ is $L_0(x) = f(0) + f'(0)(x-0)$

$= 1 + \frac{3}{2}x$

$L_0(.02) = 1 + \frac{3}{2}(.02) = 1 + .03 = 1.03$

#15 Newton's method.

Want to find $2^{1/3}$. It is root of $f(x) = x^3 - 2$.



$$\frac{f(x_{old})}{x_{old} - x_{new}} = f'(x_{old})$$

So $\quad x_{old} - x_{new} = \dfrac{f(x_{old})}{f'(x_{old})}$ $\quad$ so $\quad x_{new} = x_{old} - \dfrac{f(x_{old})}{f'(x_{old})}$

$$x_{new} = x_{old} - \frac{x_{old}^3 - 2}{3x_{old}^2} = \frac{3x_{old}^3 - (x_{old}^3 - 2)}{3x_{old}^2}$$

$$= \frac{2x_{old}^3 + 2}{3x_{old}^2} = \frac{2}{3} x_{old} + \frac{2}{3} \frac{1}{x_{old}^2}$$