Additional problems

1. True or false questions:
   Let $A\mathbf{x} = \mathbf{b}$ be a system of $n$ variables and $m$ equations.
   
   (a) If $m < n$, then the system is consistent.
   (b) If $m > n$, then the system is inconsistent.
   (c) If the system is consistent and $m = n$, then the system has unique solution.
   (d) If the system is consistent and $m < n$, then the system has infinitely many solutions.
   (e) If the system has unique solution, then $m = n$.

2. True or false questions:
   Let $v_1, v_2, \ldots, v_k$ be vectors of $\mathbb{R}^n$.
   
   (a) If $v_1, v_2, \ldots, v_k$ are linearly dependent, so are the vectors $v_1, v_2, \ldots, v_{k-1}$.
   (b) If $v_1, v_2, \ldots, v_k$ are linearly independent, so are the vectors $v_1, v_2, \ldots, v_{k-1}$.
   (c) If $k \geq n$, then $v_1, v_2, \ldots, v_k$ span $\mathbb{R}^n$.
   (d) If $k < n$, then $v_1, v_2, \ldots, v_k$ cannot span $\mathbb{R}^n$.
   (e) The vectors $v_1, v_2, v_3$ are linearly dependent if and only if the vectors $v_1 + v_2 + v_3, v_1 + v_2$ are linearly dependent.

3. Let $T : \mathbb{R}^n \rightarrow \mathbb{R}^m$ be a linear transformation, and let $v_1, v_2, \ldots, v_k$ be vectors of $\mathbb{R}^n$.
   
   (a) If the vectors $v_1, v_2, \ldots, v_k$ are linearly dependent, so are the image vectors $T(v_1), T(v_2), \ldots, T(v_k)$.
   (b) If the vectors $v_1, v_2, \ldots, v_k$ are linearly independent, so are the image vectors $T(v_1), T(v_2), \ldots, T(v_k)$.
   (c) If the image vectors $T(v_1), T(v_2), \ldots, T(v_k)$ are linearly dependent, so are the vectors $v_1, v_2, \ldots, v_k$.
   (d) If the image vectors $T(v_1), T(v_2), \ldots, T(v_k)$ are linearly independent, so are the vectors $v_1, v_2, \ldots, v_k$.

4. Consider the following linear system

   \[
   \begin{align*}
   x_1 - x_2 + (h - 1)x_3 + x_5 &= 1 \\
   x_1 + (h + 2)x_3 - x_5 &= 1 \\
   +2x_2 + 6x_3 + x_4 &= 2 \\
   -x_1 + 2x_2 + (4 - h)x_3 + x_4 + x_5 &= h + 4
   \end{align*}
   \]

   Determine the values of $h$ such that the system is consistent and find the general solution for those consistent cases.