§2.1: 13, 15, 19  §2.2: 12, 16, 20  §2.8: 21, 22, 24
§2.3: 9, 18  §3.1: 10, 13  §3.3: 6, 15, 27
§2.9: 10, 15, 17, 30  §3.2: 14, 40
§3.3: 14, 40  §3.3: 10, 13

Additional problems

1. True or false questions:
   (a) If rows of a matrix are linearly independent, so are the columns.
   (b) If the columns of an $n$-by-$n$ matrix span $\mathbb{R}^n$, so do the rows.
   (c) If $A$, $B$, and $C$ are $n$-by-$n$ invertible matrices, so is the matrix $AB^TC$.
   (d) If $AB = AC$ and $A$ is not equal to zero matrix, then $B = C$.
   (e) If $AC = BC$ and $C$ is invertible, then $A = B$.
   (f) Let $A$ and $B$ be same size square matrices. If $\det A = 3$ and $\det B = 5$, then $\det(A + B) = 3 + 5$.

2. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. True or false questions:
   (a) If $n \leq m$, then $T$ is one-to-one.
   (b) If $n \geq m$, then $T$ is onto.
   (c) If $n = m$, then $T$ is one-to-one and onto.
   (d) If $T$ is one-to-one, then $n \leq m$.
   (e) If $T$ is onto, then $n \geq m$.
   (f) If $T$ is one-to-one and onto, then $n = m$.

3. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation defined below. Determine whether $T$ is one-to-one, or onto, or invertible?
   (a) $f(x_1, x_2, x_3) = (x_2 + 7x_3, x_1 + 3x_2 - 2x_3)$.
   (b) $f(x_1, x_2, x_3) = (x_1 + 2x_3, 2x_1 - x_2 + 3x_3, 4x_1 + x_2 + 8x_3)$.
   (c) $f(x_1, x_2, x_3) = (x_1 + x_2 + x_3, x_1 + 2x_2, x_1 + 2x_3)$.

4. Compute $\det A$, $\det A^T$, $\det A^4$, $\det A^{-5}$, $A^{-1}$, $(A^T)^{-1}$, $(A^{-1})^T$, where

   $$A = \begin{bmatrix}
   1 & 1 & 1 & 1 \\
   1 & 2 & 2 & 2 \\
   1 & 3 & 6 & 8 \\
   1 & 4 & 8 & 9
   \end{bmatrix}.$$ 

5. Let $T : \mathbb{R}^n \to \mathbb{R}^m$ be a linear transformation. Then $T$ is one-to-one if and only if $T$ preserves linear independence (i.e., $T(v_1), \ldots, T(v_k)$ are linearly independent in $\mathbb{R}^m$ whenever $v_1, \ldots, v_k$ are linearly independent in $\mathbb{R}^n$).