

# Harry Bateman: from Manchester to Manuscript Project\*

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## Abstract

**T**he life and work of Harry Bateman is outlined. He was a Mancunian mathematician who spent half of his life in California. He was an expert in what might be called "classical" applied mathematics, especially in exact methods for solving partial differential equations and in the application of mathematical techniques to the problems of fluid dynamics. Nowadays, he is perhaps best known because of the "Bateman Manuscript Project", an activity designed to organise the accumulated knowledge of "special functions". Such activities are unfashionable, but they are of immense value to the progress of science.

## 1 Introduction and biographical outline

Harry Bateman was an English mathematician, born in Manchester in 1882. He went from Manchester Grammar School to Trinity College, Cambridge, in 1900, graduating as Senior Wrangler in 1903. (Roughly speaking, Wranglers obtained first-class degrees, and the Senior Wrangler was the top first.) As was common at the time, Bateman then stayed in Cambridge where he studied under Edmund Whittaker, and he travelled: he attended lectures on integral equations by Émile Picard in Paris and by David Hilbert in Göttingen. He returned to England in 1906, spent one year as a lecturer at the University of Liverpool and then three years back in Manchester, as a Reader in Mathematical Physics. This was a new position, set up to support Ernest Rutherford's appointment as Langworthy Professor of Physics. Rutherford moved from Canada to Manchester in 1907, and he was awarded the Nobel Prize for Chemistry in 1908.

Then, in September 1910, Bateman emigrated to the USA. He spent two years at Bryn Mawr College, a private university for women, having been recruited by Charlotte Scott, an English mathematician who had earned a DSc from the University of London under Arthur Cayley's guidance. According to Rutherford, Bateman "was very amusing for he seemed to think at first that Bryn Mawr was a mixed institution of men and women. I pointed out the dangers of his situation". Nevertheless, he returned to England in the summer of 1912, marrying Ethel Dodd in July of that year. They travelled back to the USA, to Johns Hopkins University in Baltimore, where Bateman spent the next five years. While there, he obtained a PhD in 1914 for a thesis in geometry; his supervisor was Frank Morley, another English mathematician and friend of Charlotte Scott. Apparently, Bateman felt that he needed to have a PhD (even though he had already published over 50 papers): the PhD degree did not become established in England until after World War I, whereas it was commonly seen as a necessary qualification if one wanted an academic position in the USA.

Finally, Bateman moved in 1917 from the east coast to the west coast, to an institution that was soon to become the California

Institute of Technology (Caltech). Bateman remained at Caltech for the rest of his career, dying suddenly on a train in January 1946.

## 2 What was Bateman interested in?

Bateman wrote about 200 papers and several books. Three books are noteworthy. First, there is his little 1915 book on *The Mathematical Analysis of Electrical and Optical Wave-Motion on the Basis of Maxwell's Equations*. Second, there is his 1932 opus on *Partial Differential Equations of Mathematical Physics*, described by Clifford Truesdell as being "for virtuosi, profitless to those not already familiar with the field as it stood in the 1920s". Third, also in 1932, Bateman wrote most of *Hydrodynamics* (his co-authors were Hugh Dryden and Francis Murnaghan, Bateman's first PhD student). This should be compared with Horace Lamb's *Hydrodynamics* (6th edition, 1932): surprisingly, there is very little overlap. Lamb was Beyer Professor of Mathematics at the (Victoria) University of Manchester while Bateman was there. We know that they discussed mathematics; we give an example below.

As mentioned earlier, Bateman studied integral equations. This was a "hot topic" 100 years ago, and was Hilbert's main focus when Bateman visited him in 1906. Bateman wrote a substantial review article in 1910, covering the theory and applications of integral equations. He was mainly interested in finding exact solutions of particular integral equations rather than in developing general theories. In fact, this characterises much of Bateman's mathematics: he was not comfortable with approximations (especially those based on physical arguments) and he did not use asymptotic methods.

Bateman used Laplace transforms much as we would today. For example, Rutherford asked him to solve the system of ordinary differential equations

$$\frac{du_i}{dt} = \lambda_{i-1}u_{i-1}(t) - \lambda_i u_i(t),$$

for  $i = 1, 2, \dots, n$  with  $\lambda_0 = 0$  and initial conditions  $u_i(0) = v_i$ . This system arose in simple models of radioactive decay. In 1910, Bateman solved it by a now-familiar method: he multiplied by  $e^{-st}$ , integrated from  $t = 0$  to  $t = \infty$ , solved for what we would call the Laplace transform of  $u_i$ , and then "inverted". This was the first time this method had been used. Keep in mind that, at that time, engineers were using Oliver Heaviside's "operational calculus", despite the protestations of mathematicians! Bateman did not publish much else on Laplace transforms, but, later in life, he complained that his contribution had not been given due credit. For more on the fascinating history of Laplace transforms, see articles by Michael Deakin.

Returning to Lamb, one of his papers from 1910 contains an integral equation,

$$\int_0^\infty u(x - y^2)dy = f(x), \quad \text{all } x,$$

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where  $f$  is given and  $u$  is to be found. Lamb gives a solution in the form of a double integral, but then adds a footnote: "Mr. H. Bateman, to whom I submitted the question, has obtained a simpler solution", followed by a formula but with no indication of how Bateman obtained it. Here is Bateman's solution:

$$u(x) = \frac{2}{\pi} \int_{-x}^{\infty} \frac{f'(-y)dy}{\sqrt{x+y}}.$$

The reader may wish to derive this solution using Fourier transforms.

### 3 Bateman and the wave equation

Inevitably, Bateman was interested in partial differential equations (PDEs). In particular, he was interested in finding solutions involving arbitrary functions (not just arbitrary constants), and in finding transformations between solutions of the same PDE or between solutions of different PDEs. To give a taste of what Bateman liked, start with the one-dimensional wave equation,

$$\frac{\partial^2 w}{\partial x^2} = \frac{1}{c^2} \frac{\partial^2 w}{\partial t^2}.$$

It is an undergraduate exercise to verify d'Alembert's general solution,

$$w(x, t) = f(x - ct) + g(x + ct),$$

where  $f$  and  $g$  are arbitrary smooth functions of one variable. There is a similar result for the three-dimensional wave equation,

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} + \frac{\partial^2 w}{\partial z^2} = \frac{1}{c^2} \frac{\partial^2 w}{\partial t^2}.$$

For this PDE, the general spherically symmetric solution is given by

$$w(x, y, z, t) = R^{-1} \{ f(R - ct) + g(R + ct) \},$$

where  $R = \sqrt{(x^2 + y^2 + z^2)}$  is a spherical polar coordinate: the solution given does not depend on the other two spherical polar coordinates.

What happens in two dimensions? In this case, the PDE is

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \frac{1}{c^2} \frac{\partial^2 w}{\partial t^2}.$$

We can introduce plane polar coordinates,  $r$  and  $\theta$ , given by  $x = r \cos \theta$ ,  $y = r \sin \theta$ , and then look for cylindrically symmetric solutions that resemble the d'Alembert solution. It turns out that there are none. The closest one can get is

$$w((x, y, t) = r^{-1/2} \{ f(r - ct) + g(r + ct) \} \cos(\theta/2),$$

which involves the angle  $\theta$ . Bateman liked and used this solution (which he attributed to Poisson): it is simple but seldom seen in textbooks on PDEs.

To illustrate some of Bateman's transformation, suppose that  $w(x, y, z, t)$  solves the three-dimensional wave equation. Then so do  $s^{-2}w(x/s^2, y/s^2, z/s^2, t/s^2)$  and

$$\frac{1}{z - ct} w \left( \frac{x}{z - ct}, \frac{y}{z - ct}, \frac{s^2 - 1}{2(z - ct)}, \frac{s^2 + 1}{2c(z - ct)} \right),$$

where  $s^2 = r^2 - c^2 t^2$ . The first of these looks plausible (it resembles Kelvin inversion for Laplace's equation) but the second is surprising.

Bateman's fascination with transformations dates back to his studies of the Lorentz transformation, which leaves Maxwell's equations invariant. Bateman and Ebenezer Cunningham looked for generalisations of the Lorentz transformation; they found them, and they showed that they form a 15-parameter group.

Bateman was also interested in Maxwell's equations because, especially in the time before quantum mechanics, much of Cambridge mathematical physics was directed at "aether theories", the idea that electronic theories of matter could be built. Thus, he constructed elaborate exact solutions of Maxwell's equations: see his 1915 book for more details.

### 4 Bateman and fluid dynamics

In 1914, while he was at Johns Hopkins University, Bateman began working in fluid dynamics. His motivation was simple: money! To supplement his income, he worked for the U.S. Weather Bureau, initially reviewing papers. He wrote four papers for the Monthly Weather Review (MWR). These covered the behaviour of real fluids, such as sound propagation in the moving atmosphere, with detailed literature surveys: this is typical of Bateman's scholarly approach. The papers also include some results of his own researches. His 1915 MWR paper includes a study of the behaviour of solutions to the Navier-Stokes equations as the viscosity  $\nu \rightarrow 0$ . Thus, he considered the possibility that the limiting form may be discontinuous: this "possibility ... may perhaps be illustrated by a consideration of the equation"

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} = \nu \frac{\partial^2 u}{\partial x^2},$$

a nonlinear PDE for  $u$ . Bateman sought (non-constant) travelling-wave solutions,  $u(x, t) = f(x - Ut)$ , whence  $(f - U)^2 + A = 2\nu f'$ , where  $A$  and  $U$  are constants. If  $A > 0$ , there is no limit as  $\nu \rightarrow 0$ . If  $A = -a^2 < 0$ , we can integrate to obtain

$$\left| \frac{u - U - a}{u - U + a} \right| = \exp \left\{ \frac{a}{\nu} (x - Ut + C) \right\},$$

where  $C$  is another constant. Then,  $u \rightarrow U \pm a$  as  $\nu \rightarrow 0$ , where the sign is taken as the sign of  $x - Ut + C$ . Bateman remarks that it "is clear from this example that the question of the limiting form of the motion of a viscous fluid when the viscosity tends to zero requires very careful investigation".

Bateman's nonlinear PDE is usually known as the "Burgers equation", although J. M. Burgers did not study it until the 1940s. Again, Bateman was unlucky!

Bateman's work on fluid dynamics must have been a factor in his recruitment by Caltech. Southern California was home to a growing aviation industry, so it made sense for this to be reflected in a growing research university. Bateman wrote reports for the National Advisory Committee for Aeronautics (NACA, later NASA) and the book *Hydrodynamics* mentioned above. Unlike Lamb's *Hydrodynamics*, Bateman gives lengthy discussions of viscous flows and of turbulent flows.

Theodore von Kármán moved to Caltech in 1930. His vitality and influence are legendary, and he attracted many excellent

graduate students. One of his major research topics in the 1930s was high-speed compressible flow: it is not surprising that Bateman became involved with related theoretical studies. Two of von Kármán's students were Francis and Milton Clauser: they were identical twins. Unusually, they co-authored two PhD theses in 1937, one on wind-tunnel experiments and one entirely theoretical (on the hodograph method, supervised by Bateman). Today, one cannot imagine a University Registrar permitting such behaviour!

## 5 Special functions

In 1945, Bateman estimated that there were about 1400 documented special functions (including several that he had defined). He was not interested in special functions for their own sake, but only as they arose in the solution of various applied problems. For example, he once solved a differential equation for von Kármán, namely,

$$xu''(x) = (x - \nu)u(x),$$

where  $\nu$  is a parameter. Bateman gave one solution:

$$k_\nu(x) = \frac{2}{\pi} \int_0^{\pi/2} \cos(x \tan \theta - \nu\theta) d\theta.$$

This defines what is now known as "Bateman's function".

Nevertheless, Bateman accumulated facts and formulae involving special functions, much as a philatelist collects postage stamps. He recorded everything on index cards and scraps of paper, which he kept in dozens of cardboard boxes, famously described as "Bateman's shoe-boxes". He planned to compile all this information into some kind of encyclopedia of special functions: he was certainly familiar with all the relevant literature and so, in principle, he was the ideal man for the job. Two things thwarted his plan. First, he was probably incapable of organising everything into a coherent whole. Second, he died suddenly.

Bateman left behind his shoe-boxes and many unfinished attempts at writing books. The "Bateman Manuscript Project" (BMP) was created so as to disseminate some of this material. The Director of the BMP was Arthur Erdélyi. He was a Hungarian mathematician at the University of Edinburgh. He had been recommended for the position by Whittaker, who was the Professor of mathematics at Edinburgh. The other BMP Research Associates were Wilhelm Magnus, Fritz Oberhettinger and Francesco Tricomi. All four had published extensively on special functions and their applications.

The output of the BMP was published as five red books in the 1950s. Three of the five are on "Higher Transcendental Functions" (HTF). The other two give tables of integrals. (The first HTF volume describes the genesis of BMP in some detail.) The HTF volumes have proved to be very influential. They contain lots of formulae (as you might expect), which makes them useful to applied mathematicians and other scientists, but this does not account for their lasting influence, as other sources of formulae are readily available. Arguably, the strength of the HTF books is that they outline how the formulae can be obtained, and this can be suggestive of new formulae, thus promoting further research.

## 6 Why is Bateman interesting today?

By all accounts, Bateman was not an exciting individual: he lived for mathematics. The functional analyst Angus Taylor took

classes from Bateman when he was a graduate student at Caltech. He evokes the man thus: "Bateman was a mild, shy man with a kind and generous heart; to shake his hand was like grasping a leaf of wilted lettuce".

Bateman is interesting today because of his mathematics (of course!) and for his understated influence: he taught many students at Caltech, and many of those have noted his influence on their work. Examples are Linus Pauling, Charles Richter, Clifford Truesdell and John Miles. Bateman is also interesting because he was a pioneering American *applied* mathematician: there was very little applied mathematics in the USA until the middle of World War II. Indeed, Bateman was appointed at Caltech as "Professor of Aeronautical Research and Mathematical Physics", and this title does describe the areas where most of his "applied" contributions fall.

Today, if we remember Bateman at all, we usually associate his name with the Bateman Manuscript Project and its five well-known red books on the theory of special functions and integral transforms. These books have been used by several generations of mathematicians. Another five books, giving a new Encyclopedia of Special Functions (edited by Mourad Ismail and Walter Van Assche), are expected from the "Askey-Bateman Project". Therefore, we may anticipate a revival of interest in Bateman and his work.

We end with a few more extracts and assessments, beginning with one from Francis Murnaghan:

Bateman's best work centered around the development of the properties of special functions and the solution of the equations of Mathematical Physics. . . . The field in which [he] stood preeminent was that of electrodynamics. . . . Modest, reserved, cultured and self-effacing he was, in its true sense, a gentle man. He was very English and he must have long looked back over his shoulder, before becoming, in 1927, a citizen of the new exuberant country in which he spent more than half his life, to the quiet England of Victoria and Edward VII.

E. T. Bell was one of Bateman's colleagues at Caltech; today, he is perhaps best known for his lively (but flawed) history, *Men of Mathematics*, first published in 1937 and still in print. His assessment is as follows:

Bateman was an almost unique combination of erudition and creativeness. It is most unusual for a mathematician to have the extraordinary range of exact knowledge that Bateman had, and not be crushed into sterility by the mere burden of an oppressive scholarship. But, as his numerous publications testify, Bateman retained his creative originality till his death. In pure mathematics, his dominating interest was in the analysis that has developed from classical mathematical physics. His technical skill in this broad field was unrivalled. His numerous contributions to mathematical physics are marked by a vivid, at times almost romantic, imagination.

These extracts are taken from two of Bateman's obituaries. Writing 30 years later, Truesdell gave the following assessment:

[Bateman] was not an interdisciplinarian, because for him there were no disciplines. He saw the mathematical sciences as a continuum with no compartments; the center of his interest was the mathematics of physical problems and the physical interpretations of mathematics. . . . He saw no difference in style or standard between pure mathematics and applied, between mathematics and theoretical physics, between physics and its application. From the great range of Bateman's interest you might expect his papers to be superficial, but they are by no means so. Each one displays virtuosity, but the choice of problem is sometimes more for its difficulty or curiosity than for its importance. In a period of increasing specialization Bateman remained a natural philosopher.

Perhaps Bateman would have liked to have been described as a “natural philosopher” with a “vivid, romantic imagination” and “unrivalled technical skills”?

## 7 Further information

The first place to go for information about mathematicians is the MacTutor History of Mathematics archive at the University of St. Andrews: <http://www-history.mcs.st-andrews.ac.uk/>

Bateman’s page contains three pictures and it cites obituaries by Erdélyi and Bell. There are others by Murnaghan and by R. C. Archibald. Truesdell’s essay, “Genius and the establishment at a polite standstill in the modern university: Bateman”, was published

in 1984. Bateman’s papers are held at Caltech; for information and many pictures, go to <http://www.archives.caltech.edu/>

Bateman’s 1915 book on Maxwell’s equations is freely available on the web, as is his 1918 textbook, *Differential Equations*, and his four papers in the Monthly Weather Review. His major book on PDEs and the book on hydrodynamics were both reprinted by Dover; used copies are easy to find.

I have accumulated much information about Bateman’s life and work; perhaps a book will emerge one day. If you have comments, suggestions or anecdotes, please contact me. □