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**Sample Midterm Exam, MAFS5140**

Totally 5 problems.

Name:

ID Number:

1. A qqnorm plot reveals that the plot is close to a straight line with intercept 2 and slope 5.
  - (a) How would you guess the mean and variance of the data?
  - (b) Explain why.

2. For linear regression model  $y_i = \beta_0 + \beta_1 x_{i1} + \dots + \beta_p x_{ip} + \epsilon_i$ ,  $i = 1, \dots, n$ . Show that the least squares estimator  $\hat{\beta}$  is uncorrelated with the residuals  $\hat{\epsilon}_i$ .

3. (About principal component) A four dimensional random vector  $X = (X_1, X_2, X_3, X_4)$  has variance matrix:

$$\mathbf{S} = \begin{pmatrix} 4 & 0 & -1 & 0 \\ 0 & .5 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & .1 \end{pmatrix}$$

- (a) Find all four principal components as linear combination of  $X$ .
- (b) What are the percentage of variation explained by each principal component.
- (c) If you are to retain the principal components that are enough to explain 80 percent of the total variation, which principal components will you retain.

4. (Portfolio theory) One risk free asset has return 0.01. Two risky assets have mean and variance of returns as the following

$$\mu = \begin{pmatrix} -0.01 \\ 0.02 \end{pmatrix} \quad \Sigma = \begin{pmatrix} 0.0003 & -0.0002 \\ -0.0002 & 0.0004 \end{pmatrix}$$

- (a) Find the tangency portfolio.  
(b) Find all efficient portfolio and their Sharpe's ratio.

5. Suppose the risk free return is 0. We observed, for the past 500 days that the daily returns of a security  $R$  and the daily returns of the market portfolio  $R_M$  have respectively sample mean 0.001 and 0.002 and sample standard standard deviation 0.004 and 0.002. Their sample correlation is 0.4.
- (a) Find the beta for this security.
  - (b) At significance level 0.01, is there sufficient evidence against the CAPM for this security.