MATH2023 Multivariable Calculus 2013

From the textbook Calculus of Several Variables (5th) by R. Adams, Addison Wesley.

Homework 7

(Total: 12 questions)

Ex. 14.5

4 Evaluate the triple integral $\iiint_R x\,dV$, where R is the tetrahedron bounded by the coordinate planes and the plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$.

Be alert for simplifications and auspicious orders of iteration.

11 Evaluate the triple integral $\iiint_R \frac{1}{(x+y+z)^3}\,dV$, where R is the region bounded by the six planes $z=1$, $z=2$, $y=0$, $y=z$, $x=0$, and $x=y+z$.

Be alert for simplifications and auspicious orders of iteration.

16 Sketch the region R in the first octant of 3-space that has finite volume and is bounded by the surfaces $x=0$, $z=0$, $x+y=1$, and $z=y^2$. Write six different iterations of the triple integral of $f(x,y,z)$ over R.

19 Express the iterated integral as a triple integral and sketch the region over which it is taken. Reiterate the integral so that the outermost integral is with respect to x and the innermost is with respect to z.

$$\int_0^1\int_z^1\int_0^{x-z} f(x,y,z)\,dydxdz.$$

27 Evaluate the iterated integral by reiterating it in a different order. (You will need to make a good sketch of the region.)

$$\int_0^1\int_z^1\int_0^{x} e^{x^3}\,dydxdz.$$

Ex. 14.6

19 Find the volume of the region above the xy-plane, inside the cone $z=2a-\sqrt{x^2+y^2}$ and inside the cylinder $x^2+y^2=2ay$.

25 Find $\iiint_B (x^2+y^2)\,dV$, where B is the ball given by $x^2+y^2+z^2\leqslant a^2$.

30 Evaluate $\iiint_R (x^2+y^2)\,dV$ over the region R, where R is the region which lies above the cone $z=c\sqrt{x^2+y^2}$ and inside the sphere $x^2+y^2+z^2=a^2$.

Ex. 14.7

2 Use double integral to calculate the area of the part of the plane $5z=3x-4y$ inside the elliptic cylinder $x^2+4y^2=4$.

6 Use double integral to calculate the area of the paraboloid $z=1-x^2-y^2$ in the first octant.

10 Show that the parts of the surfaces $z=2xy$ and $z=x^2+y^2$ that lie in the same vertical cylinder have the same area.

Qu Find the volume bounded by the surface with equation $x^{2/3}+y^{2/3}+z^{2/3}=a^{2/3}$.

* **Only hand in the underlined ones, the others are recommended exercises.**