Answer to Math Module of SCIE1110
(*not absolutely guaranteed to be correct*)

(1.1)

$$\begin{aligned}10205 &= 1275 \times 8 + 5 = (159 \times 8 + 3) \times 8 + 5 \\ &= (19 \times 8 + 7) \times 8^2 + 3 \times 8 + 5 \\ &= (2 \times 8 + 3) \times 8^3 + 7 \times 8^2 + 3 \times 8 + 5 \\ &= 2 \times 8^4 + 3 \times 8^3 + 7 \times 8^2 + 3 \times 8 + 5 = 23735_{[8]}.\end{aligned}$$

(1.2) In $\mathbb{Z}_7$, we have $\bar{8} = \bar{1}$. Therefore

$$\begin{aligned}\overline{28735_{[8]}} &= \bar{2} \times \bar{8}^4 + \bar{3} \times \bar{8}^3 + \bar{7} \times \bar{8}^2 + \bar{3} \times \bar{8} + \bar{5} \\ &= \bar{2} \times \bar{1}^4 + \bar{3} \times \bar{1}^3 + \bar{7} \times \bar{1}^2 + \bar{3} \times \bar{1} + \bar{5} \\ &= \bar{2} + \bar{3} + \bar{7} + \bar{3} + \bar{5} = \overline{2 + 3 + 7 + 3 + 5} = \bar{6} \neq \bar{0}.\end{aligned}$$

The number is not divisible by 7.

(1.3) In $\mathbb{Z}_3$, we have $\bar{8} = -\bar{1}$. Therefore

$$\begin{aligned}\overline{23735_{[8]}} &= \bar{2} \times \bar{8}^4 + \bar{3} \times \bar{8}^3 + \bar{7} \times \bar{8}^2 + \bar{3} \times \bar{8} + \bar{5} \\ &= \bar{2} \times (-\bar{1})^4 + \bar{3} \times (-\bar{1})^3 + \bar{7} \times (-\bar{1})^2 + \bar{3} \times (-\bar{1}) + \bar{5} \\ &= \bar{2} - \bar{3} + \bar{7} - \bar{3} + \bar{5} = \overline{2 - 3 + 7 - 3 + 5} = \bar{1} \neq \bar{0}.\end{aligned}$$

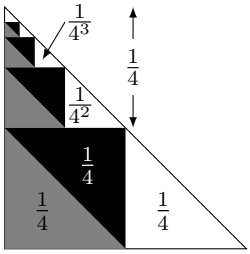
Since the result is $\bar{0}$, the number is divisible by 3.

(2) Let the triangle have area 1. Then we have 3 triangles at the first level, with area $\frac{1}{4}$ each. We have another 3 triangles at the second level, with area $\frac{1}{4^2}$ each, and so on. All levels fill the whole triangle. Therefore

$$1 = \frac{3}{4} + \frac{3}{4^2} + \frac{3}{4^3} + \cdots .$$

This means

$$\frac{1}{3} = \frac{1}{4} + \frac{1}{4^2} + \frac{1}{4^3} + \cdots .$$



(3) The first and third grids can be tiled.

For the second one, we color the tiles alternatively by white (w), gray (g), and black (b). The squares in any 3×1 domino should have one color each. Therefore a tilable grid should have equal numbers of w, g, b colors. Since the numbers are actually $12w, 10g, 11b$, which are not equal, the grid cannot be tiled.