

SCIE1110 Exercises

Exercise 1.

Convert the number 2503 to bases 2, 3, 5, 9, 12, 25, 27.

Exercise 2.

Convert numbers between different bases (no subscript means base 10).

1. $2038040 = ?_{[60]}$.
2. $2038040 = ?_{[100]}$.
3. $2038040_{[9]} = ?$.
4. $2038040_{[9]} = ?_{[3]}$.
5. $2038040_{[9]} = ?_{[5]}$.
6. $2038040_{[9]} = ?_{[25]}$.
7. $2, 3, 80, 40_{[144]} = ?$.
8. $2, 3, 80, 40_{[144]} = ?_{[12]}$.
9. $2038040_{[16]} = ?_{[2]}$.
10. $2038040_{[16]} = ?_{[8]}$.

Exercise 3.

Draw the addition and multiplication tables for \mathbb{Z}_5 . Compared with the addition and multiplication tables for base 5, what do you find?

Exercise 4.

Let a and b be integers. Explain that $10a + b$ is divisible by 7 if and only if $a - 2b$ is divisible by 7.

Exercise 5.

Find the rule for divisibility by 21.

Exercise 6.

For natural numbers expressed in base 5, find the rule for divisibility by 3.

Exercise 7.

We express numbers in base 11, and denote 10 by T .

1. Show that a number $\cdots N_3 N_2 N_1 N_0$ in base 11 is divisible by T if and only if $N_1 + N_1 + N_2 + N_3 + \cdots$ is divisible by T .
2. Determine the divisibility of $2T2T \cdots 2T$ ($2T$ repeated n times) by T .

Exercise 8.

In the sexagesimal system used by the Babylonians, there is no equivalent to the decimal point. This is analogous to expressing $\sqrt{2}$ as $1414\cdots$, omitting the dot.

The following numbers are expressed in our usual decimal notation. Which ones will the Babylonians express in the same way.

$$2.25, \quad 13.5, \quad 135, \quad 225.$$

Exercise 9.

In Babylonian notation, the population of Hong Kong is $36,6$. Given that the population is about 7.8 million, what is the exact value of $36,6$ in decimal expression?

Exercise 10.

Express 1 as a sum of three distinct fractions: $1 = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}$. Then use this to explain that the fractional expression used by Egyptians is not unique¹.

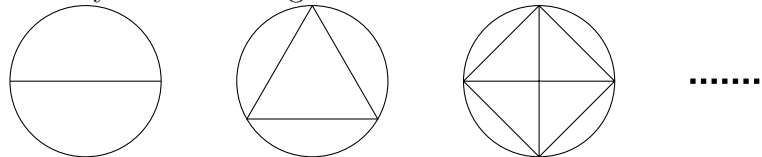
Exercise 11.

What is wrong with the following argument?

1. No dog has 5 legs.
2. A dog has 4 more legs than no dog.
3. A dog has 9 legs.

Exercise 12.

Take n evenly spaced out points on the circle and connect all the possible straight lines between them. The number of regions you get are $2, 4, 8, \dots$. What do you think the general number is?



¹This leads to interesting question of “best Egyptian fractional expression”. Please check out <http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fractions/egyptian.html>.

Exercise 13.

What is the condition that an $m \times n$ grid can be tiled by 2×2 squares (dominos)?

Exercise 14.

Prove that an $m \times n$ grid can be tiled by 2×3 dominos if and only if mn is divisible by 6. How many tilings can you have?

Exercise 15.

Suppose $F(n) = 2F(n-1) + F(n-2) - 2F(n-3)$, $F(0) = 0$, $F(1) = 1$, $F(2) = 2$. Find the general formula for $F(n)$.

Exercise 16.

Show that the three axioms are independent.

1. Each committee has three members.
2. Each member belongs to exactly two committees.
3. No two committees have the same two members.

Exercise 17.

If you take $n' = n + 1$ in all integers \mathbb{Z} . Which Peano's Axioms are still true?

Exercise 18.

Show that the axioms of the Peano's Axiom are independent.

Exercise 19.

Construct different I and n' on \mathbb{N} , such that the first two Peano's Axioms hold but the third fails. Can you still keep $I = 1$ but define new n' ? Can you still keep $n' = n + 1$ but take new I ?

Exercise 20.

1. Express 14205 in base 8.
2. Use the first part to determine whether the number is divisible by 7.
3. Use the first part to determine whether the number is divisible by 3.

Exercise 21.

Which grids can be tiled by 3×1 dominos? For the ones that can be tiled, show the tiling. For the ones that cannot be tiled, explain.

