

## SCIE1110 Exercises

### Exercise 1.

Draw the addition and multiplication tables for base 5.

### Exercise 2.

Convert the number 2503 to bases 2, 3, 5, 9, 12, 25, 27.

### Exercise 3.

Convert numbers between different bases (no subscript means base 10).

1.  $2038040 = ?_{[60]}$ .
2.  $2038040 = ?_{[100]}$ .
3.  $2038040_{[9]} = ?$ .
4.  $2038040_{[9]} = ?_{[3]}$ .
5.  $2038040_{[9]} = ?_{[5]}$ .
6.  $2038040_{[9]} = ?_{[25]}$ .
7.  $2, 3, 80, 40_{[144]} = ?$ .
8.  $2, 3, 80, 40_{[144]} = ?_{[12]}$ .
9.  $2038040_{[16]} = ?_{[2]}$ .
10.  $2038040_{[16]} = ?_{[8]}$ .

### Exercise 4.

Do the arithmetic calculations in base 2 (subscript [2] omitted). For the division, find the quotient and remainder.

1.  $1010011 + 10101$ .
2.  $1010011 - 10101$ .
3.  $1010011 \times 10101$ .
4.  $1010011 \div 10101$ .
5.  $10101 + 101 - 1001$ .
6.  $10101 \times 101 \div 1001$ .

**Exercise 5.** Do the arithmetic calculations in base 16.

1.  $5AB + E07 - C5$ .
2.  $5AB \times E07 \div C5$ .

### Exercise 6.

Draw the addition and multiplication tables for  $\mathbb{Z}_5$ . Compared with the addition and multiplication tables for base 5, what do you find?

**Exercise 7.**

Let  $a$  and  $b$  be integers. Explain that  $10a + b$  is divisible by 7 if and only if  $a - 2b$  is divisible by 7.

**Exercise 8.**

Find the rule for divisibility by 21.

**Exercise 9.**

For natural numbers expressed in base 5, find the rule for divisibility by 3.

**Exercise 10.**

How many multiplications are there in a Chinese multiplication table? Given the sexagesimal system, how big should the Babylonian multiplication table be?

**Exercise 11.**

We express numbers in base 11, and denote 10 by  $T$ .

1. Calculate  $235 + T3T - 1T9$  and  $3T7 \times 203$ .
2. Show that a number  $\cdots N_3N_2N_1N_0$  in base 11 is divisible by  $T$  if and only if  $N_1 + N_1 + N_2 + N_3 + \cdots$  is divisible by  $T$ .
3. Determine the divisibility of  $2T2T \cdots 2T$  ( $2T$  repeated  $n$  times) by  $T$ .

**Exercise 12.**

In the sexagesimal system used by the Babylonians, there is no equivalent to the decimal point. This is analogous to expressing  $\sqrt{2}$  as  $1414 \cdots$ , omitting the dot.

The following numbers are expressed in our usual decimal notation. Which ones will the Babylonians express in the same way.

$$2.25, \quad 13.5, \quad 135, \quad 225.$$

**Exercise 13.**

In Babylonian notation, the population of Hong Kong is  $36,6$ . Given that the population is about 7.8 million, what is the exact value of  $36,6$  in decimal expression?

**Exercise 14.**

Express 1 as a sum of three distinct fractions:  $1 = \frac{1}{n_1} + \frac{1}{n_2} + \frac{1}{n_3}$ . Then use this to explain that the fractional expression used by Egyptians is not unique<sup>1</sup>.

**Exercise 15.**

Find the age of Diophantus from the following epigram by Metrodorus:

*This tomb holds Diophantus. Ah, how great a marvel! The tomb tells scientifically the measure of his life. God granted him to be a boy for the sixth part of his life, and adding a twelfth part to this, he clothed his cheeks with down; He lit him the light of wedlock after a seventh part, and five years after his marriage He granted him a son. Alas! late-born wretched child; after attaining the measure of half his father's life, chill Fate took him. After consoling his grief by this science of numbers for four years he ended his life.*

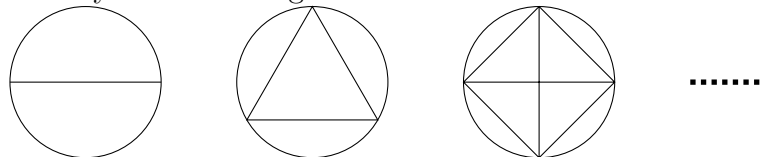
**Exercise 16.**

What is wrong with the following argument?

1. No dog has 5 legs.
2. A dog has 4 more legs than no dog.
3. A dog has 9 legs.

**Exercise 17.**

Take  $n$  evenly spaced out points on the circle and connect all the possible straight lines between them. The number of regions you get are 2, 4, 8, . . . . What do you think the general number is?



**Exercise 18.**

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<sup>1</sup>This leads to interesting question of “best Egyptian fractional expression”. Please check out <http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fractions/egyptian.html>.

In a party, two people are (mutual) friends if they met before, and they are (mutual) strangers if they meet for the first time. Show that in a party of at least 6 people, there are at least 3 people who are either mutual friends to each other or mutual strangers to each other.

[There are obvious extensions you can explore.]

**Exercise 19.**

Show that the odd harmonic series  $\frac{1}{1} + \frac{1}{3} + \frac{1}{5} + \cdots + \frac{1}{2n-1} + \cdots$  diverges.

**Exercise 20.**

What is the condition that an  $m \times n$  grid can be tiled by  $2 \times 2$  squares (dominos)?

**Exercise 21.**

Prove that an  $m \times n$  grid can be tiled by  $2 \times 3$  dominos if and only if  $mn$  is divisible by 6. How many tilings can you have?

**Exercise 22.**

Suppose  $F(n) = 2F(n-1) + F(n-2) - 2F(n-3)$ ,  $F(0) = 0$ ,  $F(1) = 1$ ,  $F(2) = 2$ . Find the general formula for  $F(n)$ .