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Spin-Orbit Interaction and Aharonov-Anandan Phase in Mesoscopic Rings

Tie-Zheng Qian[1,2] and Zhao-Bin Su[2]
[1] *CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, People's Republic of China*
[2] *Institute of Theoretical Physics, Academia Sinica, P.O. Box 2735, Beijing 100080, People's Republic of China*
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We show the existence of a nonadiabatic geometric phase, i.e., an Aharonov-Anandan (AA) phase, in the Aharonov-Casher (AC) topological interference effect in one-dimensional mesoscopic rings. We find the AC phase is the phase accumulated by the spin wave function during a cyclic evolution, and show it is the sum of a geometric AA phase and a dynamical phase. In the adiabatic limit, the AA phase becomes the spin-orbit Berry phase introduced by Aronov and Lyanda-Geller. By solving exactly the model of a quasi-one-dimensional ring formed by the 2DEG on a semiconductor heterostructure, we discuss the observability of the AA phase in the AC effect.

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A quantum holonomy phenomenon, known as the geometric phase, has received a lot of interest for years [1]. Berry [2] first discovered that there exists the geometric phase in adiabatic cyclic evolution. This adiabatic geometric phase, referred to as the Berry phase, can be interpreted as a holonomy associated with the parallel transport around a circuit in a parameter space [3]. In a fundamental generalization of Berry's idea, Aharonov and Anandan (AA) removed the adiabatic restriction and studied the geometric phase for any cyclic evolution [4]. Suppose a normalized state $|\psi(t)\rangle$ evolves according to the Schrödinger equation $i\hbar(d/dt)|\psi(t)\rangle = H(t)|\psi(t)\rangle$ such that $|\psi(T)\rangle = e^{i\lambda}|\psi(0)\rangle$. Then $|\psi(t)\rangle$ undergoes a cyclic evolution in a time interval [0, T]. By removing the dynamical part from the phase λ, AA defined their nonadiabatic geometric phase for the cyclic evolution as $\gamma = \lambda + \hbar^{-1}\int_0^T \langle\psi(t)|H(t)|\psi(t)\rangle dt$. They found γ takes the form of $\gamma = \int_0^T \langle\tilde{\psi}|i(d/dt)|\tilde{\psi}\rangle dt$ where $|\tilde{\psi}\rangle$ is given by $|\tilde{\psi}(t)\rangle = e^{-if(t)}|\psi(t)\rangle$ with $f(T) - f(0) = \lambda$ such that $|\tilde{\psi}(T)\rangle = |\tilde{\psi}(0)\rangle$. The AA phase γ can be interpreted as a holonomy associated with the parallel transport around the circuit $\mathcal{C}$ defined by $|\tilde{\psi}\rangle$ in the projective Hilbert space $\mathcal{P}$. In case $H(t) = H[\mathbf{R}(t)]$ is a function of a set of parameters R^i, which vary slowly around a circuit Γ in the parameter space, $|\tilde{\psi}(t)\rangle$ becomes an instantaneous eigenstate $|n[\mathbf{R}(t)]\rangle$ of $H[\mathbf{R}(t)]$, and the AA phase tends to the Berry phase $i\oint_\Gamma \langle n(\mathbf{R})|\nabla_\mathbf{R}|n(\mathbf{R})\rangle \cdot d\mathbf{R}$. The observation of the AA phase has already been done in optical and NMR experiments [5]. The well known Aharonov-Bohm (AB) effect [6] can be interpreted as a special realization of the AA phase [4]. This is a generalization of the result that the AB phase can be regarded as a special case of the Berry phase [2] since the former has its nonadiabatic origin [4].

There are many observable effects of the geometric phase including the AB phase. In mesoscopic systems, several manifestations of the AB effect have been predicted and verified [7–10]. Similar manifestations of the Berry phase and AA phase are currently under investigation. The persistent currents from the Berry phase in textured mesoscopic rings were studied by Loss, Goldbart, and Balatsky [11]. They also first predicted the effect that the Berry phase affects the conductivity [11]. In a further study, Stern demonstrated that the Berry phase affects the conductivity of the rings in a way similar to the AB effect, and a time-dependent Berry phase induces a motive force [12]. The AA phase in textured rings and its relation to the persistent currents were discussed in Ref. [13].

Recently, it has been recognized that spin-orbit (SO) interaction leads to a novel topological interference effect, which is an electromagnetic dual of the AB effect and

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called the Aharonov-Casher (AC) effect [14]. The observability of the AC effect in mesoscopic systems has been discussed by several authors [15–17]. Using the transfer matrix method, Meir, Gefen, and Entin-Wohlman [15] showed for the first time that SO interaction in one-dimensional disordered rings induces an effective spin-dependent magnetic flux, which leads to universal SO reduction factors of the harmonics. Mathur and Stone [16] then investigated the persistent-current paramagnetism and the conductance oscillations due to the effective magnetic flux, and pointed out that the effects of SO interaction are manifestations of the AC effect. By use of the same method as in Ref. [15], Balatsky and Altshuler [17] considered the AC effect in external electric field, and studied the persistent currents produced by SO interaction via the AC effect. In these works, the effective AC flux was obtained and several of its manifestations were predicted. However, as the spin motion of the electron has not been analyzed in connection with the quantum phase effect in cyclic evolution, the contribution of the geometric phase in the effect of SO interaction has not been specified yet. On the other hand, in a slightly different but closely related context, Aronov and Lyanda-Geller [18] considered the spin motion of the electron in conducting rings, and demonstrated that SO interaction results in a spin-orbit Berry phase in the adiabatic limit. As is well known, in the presence of SO interaction there is a momentum-dependent effective magnetic field coupled to the electron spin. If the momentum traverses a circuit in momentum space (the parameter space for the spin Hamiltonian), and if the spin orientation keeps in the direction of the effective magnetic field, the spin state acquires a Berry phase in its adiabatic cyclic evolution. This Berry phase is just the SO Berry phase.

The purpose of this paper is to show the existence of a nonadiabatic geometric phase, i.e., an AA phase, in the AC effect in mesoscopic rings. We first derive a spin cyclic evolution, which is governed by SO interaction and invokes no adiabatic approximation. Then, with the help of this cyclic evolution, we diagonalize the Hamiltonian in spin space. It is seen that the spin cyclic evolution determines both the AC phase and local spin orientation of the eigenstate of the Hamiltonian. The AC phase is found to be the phase acquired by the spin state in its cyclic evolution. In particular, we show the AC phase consists of a geometric AA phase and a dynamical phase. This implies that in the same sense as the AB effect is a quantum phase effect in an orbital cyclic evolution, the AC effect is a quantum phase effect in a spin cyclic evolution. In the adiabatic limit, the AA phase becomes the SO Berry phase introduced in Ref. [18]. The connection between the results in Refs. [15] and [18] is therefore established. As an illustrative example, we find the exact solution for the system proposed in Ref. [18] by making use of the invariant theory, which is suitable for the study of the geometric phase in cyclic evolution [19]. The AA phase and dynamical phase, which are the two parts of the AC phase for the system, are derived. The condition for the validity of the adiabatic approximation is then discussed. It turns out that the AA phase is observable in the experiments proposed for measuring the AC effect.

In the presence of SO interaction, the Anderson Hamiltonian for electrons confined to a one-dimensional ring of N sites reads

$$H = \sum_{l\mu} \epsilon_l C^\dagger_{l\mu} C_{l\mu} + \sum_{l\mu\rho} V_l C^\dagger_{l+1\mu} (S_l)_{\mu\rho} C_{l\rho} + \text{H.c.}, \quad (1)$$

where S_l is a 2×2 SU(2) matrix in spin space. For a ring of radius a,

$$S_l = \mathcal{P} \exp\left(i \frac{ea}{4m_e c^2} \boldsymbol{\sigma} \cdot \int_l^{l+1} d\phi \, \mathbf{e}_\phi \times \mathbf{E} \right), \quad (2)$$

where ϕ is the angular coordinate of the ring, $\mathbf{e}_\phi$ is the corresponding unit vector, and $\mathcal{P}$ is the path ordering operator. In fact, the Hamiltonian (1) with S_l given by (2) is the discrete version of the single electron Hamiltonian

$$\frac{\mathbf{p}^2}{2m_e} - eV(\mathbf{r}) + \frac{e\hbar}{4m_e^2 c^2} \boldsymbol{\sigma} \cdot \mathbf{E} \times \mathbf{p},$$

where $\mathbf{E} = -\nabla V$ is the electric field which leads to SO interaction.

Consider a Schrödinger-type equation

$$i \frac{\partial}{\partial \phi} \psi(\phi) = -\frac{ea}{4m_e c^2} \mathbf{e}_\phi \times \mathbf{E}(\phi) \cdot \boldsymbol{\sigma} \psi(\phi), \quad (3)$$

which describes the evolution of the spin state ψ of the electron during its hopping. In fact, S_l in Eq. (2) is the discretized version of the evolution operator for Eq. (3). A spin cyclic evolution is defined to satisfy the condition that when the electron traverses the whole ring once, the initial and final spin wave functions only differ in phase factors. Following Ref. [4] and introducing $\tilde{\psi}(\phi)$, we solve the cyclic evolution formally as

$$\psi^{(\mu)}(\phi) = e^{i[\gamma_\mu(\phi) + \delta_\mu(\phi)]} \tilde{\psi}^{(\mu)}(\phi), \quad (4)$$

with the cyclic condition $\tilde{\psi}^{(\mu)}(2\pi) = \tilde{\psi}^{(\mu)}(0)$, and the phases

$$\begin{aligned} \gamma_\mu(\phi) &= \int_0^\phi i \tilde{\psi}^{(\mu)\dagger}(\varphi) d\tilde{\psi}^{(\mu)}(\varphi), \\ \delta_\mu(\phi) &= \int_0^\phi \tilde{\psi}^{(\mu)\dagger} \frac{ea}{4m_e c^2} \mathbf{e}_\varphi \times \mathbf{E} \cdot \boldsymbol{\sigma} \tilde{\psi}^{(\mu)} d\varphi, \end{aligned} \quad (5)$$

where $\mu = \pm$;

$$\tilde{\psi}^{(+)} = \begin{bmatrix} \cos\frac{\alpha}{2} \\ e^{i\beta} \sin\frac{\alpha}{2} \end{bmatrix}$$

and

$$\tilde{\psi}^{(-)} = \begin{bmatrix} -e^{-i\beta} \sin\frac{\alpha}{2} \\ \cos\frac{\alpha}{2} \end{bmatrix}$$

are spin coherent states. α and β satisfy the auxiliary equation

$$\frac{\partial}{\partial\phi}\mathbf{n}(\phi) = \left[-\frac{ea}{2m_ec^2}\mathbf{e}_\phi \times \mathbf{E}\right] \times \mathbf{n}(\phi), \qquad (6)$$

with $\mathbf{n} = (\cos\beta\sin\alpha, \sin\beta\sin\alpha, \cos\alpha)$. The AA phase associated with the cyclic evolution is $\gamma_\mu(2\pi)$ while the dynamical phase is $\delta_\mu(2\pi)$ [4]. From the above results, S_l can be expressed as $S_l = \sum_\mu \tilde{\psi}^{(\mu)}(\phi_{l+1})e^{i\mu\lambda_l}\tilde{\psi}^{(\mu)\dagger}(\phi_l)$ with $\mu\lambda_l = [\delta_\mu(\phi_{l+1}) + \gamma_\mu(\phi_{l+1})] - [\delta_\mu(\phi_l) + \gamma_\mu(\phi_l)]$. ϕ_l is the angular coordinate of the lth site.

Now we demonstrate that the spin cyclic evolution is responsible for the AC effect by diagonalizing explicitly the Hamiltonian in spin space. We first introduce a unitary transformation

$$U = \prod_{l=1}^{N} \exp\left[i\frac{\alpha_l}{2}C^\dagger_{l\mu}(\sigma_1\sin\beta_l - \sigma_2\cos\beta_l)_{\mu\rho}C_{l\rho}\right], \qquad (7)$$

where $\alpha_l = \alpha(\phi_l)$ and $\beta_l = \beta(\phi_l)$ are given by Eq. (6). The corresponding transformation property of the electron creation operators is $UC^\dagger_{l\mu}U^\dagger = [C^\dagger_{l+}\ C^\dagger_{l-}]\tilde{\psi}^{(\mu)}(\phi_l)$, which leads to $UC^\dagger_{l\mu}(\sigma_3)_{\mu\rho}C_{l\rho}U^\dagger = \mathbf{n}(\phi_l)\cdot C^\dagger_{l\mu}\boldsymbol{\sigma}_{\mu\rho}C_{l\rho}$. So U operator transforms the local spin orientation at lth site from z axis to $\mathbf{n}(\phi_l)$. From the above results, we obtain that H is transformed into $H_0 = U^\dagger HU$, which is diagonal in spin space:

$$H_0 = \sum_{l\mu} \epsilon_l C^\dagger_{l\mu}C_{l\mu} + V_l C^\dagger_{l+1\mu}e^{i\mu\lambda_l}C_{l\mu} + \text{H.c.} \qquad (8)$$

In the derivation, it is readily seen that the cyclic condition is necessary for the fulfillment of the periodic boundary condition for the ring. The phase factor $e^{i\mu\lambda_l}$ in the hopping term of H_0 indicates that the ring is threaded by a spin-dependent magnetic flux which leads to the AC effect. $\mu\lambda = \mu\sum_l \lambda_l = \gamma_\mu(2\pi) + \delta_\mu(2\pi)$ is the AC phase. For an eigenstate of H, the local spin orientation at lth site is $\mu\mathbf{n}(\phi_l)$. It is straightforward to verify that the AA phase

$$\gamma_\mu(2\pi) = -\frac{\mu}{2}\int\int_{\partial S=C} \mathbf{n}\cdot d\mathbf{S} \qquad (9)$$

is $-\frac{1}{2}$ of the solid angle subtended by the circuit traced on a sphere by the spin orientation along the ring. In the adiabatic limit, the spin orientation approaches the direction of the local momentum-dependent effective magnetic field, i.e., $\mathbf{n}(\phi_l) \parallel \mathbf{e}_\phi(\phi_l) \times \mathbf{E}(\phi_l)$, and the AA phase becomes the SO Berry phase introduced in Ref. [18]. We would like to emphasize here that, by introducing explicitly the spin cyclic evolution (4) and expressing S_l in terms of the evolution, we demonstrate the AC phase is actually the phase acquired by the spin state during its cyclic evolution. Consequently, the geometric AA phase in the AC effect is clearly identified without adiabatic approximation.

In the following, we discuss the observability of the AA phase in the experiments proposed for measuring the AC effect. Consider a quasi-one-dimensional ring of radius a formed by the two-dimensional electron gas (2DEG) on a semiconductor heterostructure. If the normal to the 2DEG plane in a A_3B_5 crystal is directed along $z' \| (111)$, the Hamiltonian is

$$\mathcal{H} = \frac{\mathbf{p}_\perp^2}{2m} + \hbar(b_1 + b_2\epsilon)(\boldsymbol{\sigma}\times\mathbf{p})_{z'}, \qquad (10)$$

where m is the effective mass, $b_1 + b_2\epsilon$ is the SO coefficient, $\mathbf{p}_\perp \perp z'$, and the Zeeman term is absent. This Hamiltonian was first proposed in Ref. [18] with the eigenvalue problem unsolved. Here we solve this model exactly to exhibit explicitly the relation of the spin evolution to the AC effect. In cylindrical coordinates, the Hamiltonian (10) can be written as

$$\mathcal{H} = \frac{\hbar^2}{2ma^2}\left[-i\frac{\partial}{\partial\phi} + ma\kappa(\cos\phi\sigma_1 + \sin\phi\sigma_2)\right]^2 - \frac{m\hbar^2\kappa^2}{2}, \qquad (11)$$

with $\kappa = b_1 + b_2\epsilon$. The Schrödinger-type equation corresponding to Eq. (3) now takes the form of

$$i\frac{\partial}{\partial\phi}\psi = H_s\psi, \qquad (12)$$

with $H_s = ma\kappa(\cos\phi\sigma_1 + \sin\phi\sigma_2)$. It is easy to see that there is an invariant

$$I_s = \cos\phi\sin\chi\sigma_1 + \sin\phi\sin\chi\sigma_2 + \cos\chi\sigma_3, \qquad (13)$$

satisfying $i\frac{\partial}{\partial\phi}I_s + [I_s, H_s] = 0$ and $I_s(2\pi) = I_s(0)$, with $\tan\chi = -2ma\kappa$. The eigenvalue equation of I_s is $I_s\tilde{\psi}^{(\mu)} = \mu\tilde{\psi}^{(\mu)}$, with

$$\tilde{\psi}^{(+)} = \begin{bmatrix} \cos\frac{\chi}{2} \\ e^{i\phi}\sin\frac{\chi}{2} \end{bmatrix}$$

and

$$\tilde{\psi}^{(-)} = \begin{bmatrix} -e^{-i\phi}\sin\frac{\chi}{2} \\ \cos\frac{\chi}{2} \end{bmatrix}.$$

From the expectation value of $\boldsymbol{\sigma}$ as a function of ϕ, it is readily seen that χ is the angle by which the spin orientation deviates from the z axis, i.e., the azimuthal angle of $\mathbf{n}$ in Eq. (6). The exact solution of Eq. (12) is found to be

$$\psi^{(\mu)}(\phi) = \exp\left(i\int_0^\phi d\varphi\left[-\frac{\mu}{2}(1-\cos\chi) - \mu ma\kappa\sin\chi\right]\right)\tilde{\psi}^{(\mu)}(\phi), \qquad (14)$$

which satisfies the cyclic condition $\tilde{\psi}^{(\mu)}(2\pi) = \tilde{\psi}^{(\mu)}(0)$. The AA phase and dynamical phase associated with the cyclic evolution are

$$\int_0^{2\pi} i\tilde{\psi}^{(\mu)\dagger}(\varphi)d\tilde{\psi}^{(\mu)}(\varphi) = -\mu\pi(1-\cos\chi) \qquad (15)$$

and

$$-\int_0^{2\pi} \tilde{\psi}^{(\mu)\dagger} H_s \tilde{\psi}^{(\mu)} d\varphi = -2\mu\pi ma\kappa \sin\chi, \qquad (16)$$

respectively [19]. From these results, we obtain the AC phase $\Phi_\mu = -\mu\pi(1-\cos\chi) - 2\mu\pi ma\kappa\sin\chi$, and the solution of the eigenvalue equation $\mathcal{H}\Psi_{n\mu} = \mathcal{E}_{n\mu}\Psi_{n\mu}$:

$$\Psi_{n\mu} = e^{in\phi}\tilde{\psi}^{(\mu)}/\sqrt{2\pi}, \quad \mathcal{E}_{n\mu} = \frac{\hbar^2}{2ma^2}\left(n - \frac{\Phi_\mu}{2\pi}\right)^2 - \frac{m\hbar^2\kappa^2}{2}, \qquad (17)$$

where n is an integer.

It is interesting to notice that Hagen [20] has established elegantly an equivalence between the AB and AC effects for the relativistic spin-$\frac{1}{2}$ particles in two-dimensional space. For the AC effect, he showed that if the particles are confined to a plane, then the cyclic phase factor can be calculated exactly. We would like to emphasize that in those cases discussed in Ref. [20], not only the particles are relativistic, but also the electric field can have only "in plane" components. However, in our discussion, not only do we have the nonrelativistic situation, but also the electric field is allowed to have nonvanishing component perpendicular to the plane of the ring. Especially in our solved example, the electric field is perpendicular to the 2DEG plane. Here the AC effect is in fact induced by SO interaction different from that discussed in Ref. [20].

Now we turn to the adiabatic limit of the solution. From Eq. (12), we see that the condition for the validity of the adiabatic approximation is $ma\kappa \gg 1$ which requires that SO interaction be sufficiently strong. Under this condition, we get $\chi \to \pi/2$, from which the SO Berry phase $-\frac{\mu}{2}\int_0^{2\pi}(1-\cos\frac{\pi}{2})d\varphi = -\mu\pi$ as well as the adiabatic limit of other quantities can be obtained. As the Zeeman term is absent in Hamiltonian (10), the SO Berry phase here is identical to that calculated for $\omega_s = 0$ in Ref. [18]. From the difference between the adiabatic and nonadiabatic results, we find χ is a quantity which characterizes the nonadiabatic property of the exact solution. For a InAs ring of radius a ($m = 0.023m_e$, the SO coefficient $\hbar^2\kappa = 6\times 10^{-10}$ eV cm [21]), we have $ma\kappa = (1.8\ \mu\text{m}^{-1})a$. Since a is of the order 1 μm, we see that the relativistic nature of SO interaction makes it so weak that a nonadiabatic treatment of the problem is necessary.

We conclude this paper with a few discussions.

(1) Similar to the case studied in Ref. [13], here the cyclic condition is imposed on the spin evolution by requiring that the wave function of the system satisfies the spatial periodic boundary condition for the ring. This is to say that in this paper the spin cyclic evolution appears as a natural consequence of the ring geometry of the system. Note that in Ref. [18] an adiabatic spin cyclic evolution was obtained quasiclassically from a Hamiltonian traversing adiabatically along a circuit in its parameter space, i.e., the momentum space.

(2) In the derivation of the AC phase for a magnetic dipole diffracting around a charged line, it is assumed that the neutrons are polarized along an axis parallel to the line and only the dynamical phase contributes to the AC phase [14]. As noted by Goldhaber [22], there is an extra degree of freedom which deserves further attention. That is the spin orientation of the particle as the spin is a quantum operator with noncommuting components. However, this has not been taken into account seriously. In Ref. [17], the uncommutability of adjoining S_l matrices was neglected in the derivation of the AC phase and consequently the AA phase therein was not obtained. In the present paper, we made it clear that the rotation of spin orientation results in a nonadiabatic geometric AA phase in addition to the conventional dynamical phase. In some mesoscopic systems, the electric field inside solids can cause such rotation and the observation of the AA phase is possible.

(3) It is interesting to notice that S_l in Eq. (2) can be expressed as $g_{l+1}g_l^{-1}$ with $[g_l]_{\sigma\mu} = \psi_\sigma^{(\mu)}$, so that S_l is essentially a pure gauge in the sense of SU(2) non-Abelian gauge theory. This follows from the fact that the effective SU(2) gauge field associated with S_l has only one spatial component.

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