Math398Q 2011 Summer
Introduction to Scientific Computation
Project 1

Paper I done individually should present a study of a limited aspect of some dynamical system. The tools which may be applied to this work include phase space diagrams, Poincare sections, power-spectra, bifurcation diagrams and fractal dimension.

Together with this file, you can find a compilation of many different dynamical systems. You can choose **one** of these systems and study it in detail.

The requirement for this paper is no more than **eight** pages in the standardized LaTeX format. To receive a high grade, your paper should be correct, interesting, original, and well-written. Any evidence of word-for-word copying from books or fellow classmates will result in a lowered grade. You should research ideas for your paper in the library, on the world-wide-web, and by looking at previous work done in earlier years. Your paper should reference papers, books, etc. that you use in your work.

A Compilation of Various Nonlinear Systems

1 Differential Equations

1. Free Brusselator Equations:

$$\begin{aligned}\dot{x} &= A - (B+1)x + x^2y \\ \dot{y} &= Bx - x^2y\end{aligned}$$

2. Rossler-chaos Equations:

$$\begin{aligned}\dot{X} &= -(Y+Z) \\ \dot{y} &= X + aY \\ \dot{Z} &= b + Z(X-c)\end{aligned}$$

3. Rossler-hyperchaos Equations:

$$\begin{aligned}\dot{X} &= -(Y+Z)\\ \dot{y} &= X+aY+W\\ \dot{Z} &= b+ZX\\ \dot{W} &= cW-dZ\end{aligned}$$

4. Chua's Circuit Equations:

$$\begin{aligned}\dot{x} &= \sigma(y-h(x))\\ \dot{y} &= x-y+z\\ \dot{z} &= -\beta y\end{aligned}$$

where $h(x) = m_1 x + (m_0 - m_1), m_0 x, m_1 x - (m_0 - m_1)$ fort $x \geq 1, |x| \leq 1$ and $x \leq -1$ respectively.

5. Forced Duffing Equations:

$$\begin{aligned}\dot{x} &= y\\ \dot{y} &= x-x^3-\epsilon y+\gamma\cos(\omega t)\end{aligned}$$

6. Unforced Duffing Equations:

$$\begin{aligned}\dot{x} &= y\\ \dot{y} &= \beta x-x^3-\epsilon y\end{aligned}$$

7. Quadratic Duffing Equations:

$$\begin{aligned}\dot{x} &= y\\ \dot{y} &= \beta x-x^2-\epsilon y\end{aligned}$$

8. Van Der Pol's Equations:

$$\begin{aligned}\dot{x} &= y\\ \dot{y} &= \frac{\epsilon}{\omega}(1-x^2)y-x\end{aligned}$$

9. Van Der Pol Oscillators coupled by weak linear interaction β and weak de-tuning δ:

$$\begin{aligned}\ddot{x}+\epsilon(x^2-1)\dot{x}+x &= \beta(y-x)\\ \ddot{y}+\epsilon(y^2-1)\dot{y}+y &= \beta(y-x)-\delta y\end{aligned}$$

10. Forced Van Der Pol's Equations:

$$\begin{aligned}\dot{x} &= y\\ \dot{y} &= \frac{\epsilon}{\omega}(1-x^2)y-\epsilon F\cos(\omega t)\end{aligned}$$

11. Henon-Heiles Equations:

$$\begin{aligned}\ddot{x} &= -x-2Dxy\\ \ddot{y} &= -y-Dx^2+Cy^2\end{aligned}$$

12. Bountis-Seger Equations:

$$\begin{aligned}\ddot{x} &= -x^3 - \xi(x-y)^3 \\ \ddot{y} &= \xi(x-y)^3 - y^3\end{aligned}$$

13. CSTR (Continuous Stirred Tank Reactor) Equations:

$$\begin{aligned}\dot{y_1} &= -y_1 + D(1-y_1)e^{y_2} \\ \dot{y_2} &= -y_2 + \beta D(1-y_1)e^{y_2} - \beta y_2\end{aligned}$$

14. Nerve Impluse Equations:

$$\begin{aligned}\dot{y_1} &= 3(y_1 + y_2 - y_1^3/3 + \lambda) \\ \dot{y_2} &= -(y_1 - 0.7 + 0.8y_2)/3\end{aligned}$$

15. Lorenz Equations:

$$\begin{aligned}\dot{X} &= \sigma(Y - X) \\ \dot{Y} &= X(R - Z) - Y \\ \dot{Z} &= XY - bZ\end{aligned}$$

16. Lorenz's 4th Order Equations:

$$\begin{aligned}\dot{y_1} &= -y_1 + 2\lambda - y_2^2 + (y_3^2 + y_4^2)/2 \\ \dot{y_2} &= -y_2 + (y_1y_2 - y_3y_4) + (y_4^2 - y_3^2)/2 \\ \dot{y_3} &= -y_3 + (y_2 - y_1)(y_3 + y_4)/2 \\ \dot{y_4} &= -y_4 + (y_2 + y_1)(y_3 - y_4)/2\end{aligned}$$

17. Ginzburg-Landau Equations:

$$\begin{aligned}\ddot{\theta} &= \theta(\theta^2 - 1 + \lambda\phi^2)k^2 \\ \ddot{\phi} &= \theta^2\phi\end{aligned}$$

18. Mathieu's Equations:

$$\ddot{\phi} + (\alpha^2 + \beta\cos t)\sin\phi = 0$$

19. Belousov-Zhabotinskii (BZ) Equations:

$$\begin{aligned}\dot{A} &= -k_fAB + k_rC - r(A - A_0) \\ \dot{B} &= -k_fAB + k_rC - r(B - B_0) \\ \dot{C} &= k_fAB - k_rC - rC\end{aligned}$$

20. Maxwell-Bloch Equations:

$$\begin{aligned}\dot{E} &= -kE + kP \\ \dot{P} &= -\nu_1ED - \nu_1P \\ \dot{D} &= \nu_2(\lambda + 1) - \nu_2D - \nu\lambda EP\end{aligned}$$

21. Morse Oscillator Equations:

$$\begin{aligned}\dot{x} &= y \\ \dot{y} &= -\mu(e^{-x} - e^{-2x}) + \epsilon\gamma\cos(\omega t)\end{aligned}$$

22. Holmes' Feedback Control System Equations:

$$\begin{aligned}\dot{x} &= y\\ \dot{y} &= x - x^3 - \delta y - z\\ \dot{z} &= \alpha\gamma x - \alpha z\end{aligned}$$

23. Euler's Equation of Motion For Free Rigid Body:

$$\begin{aligned}\dot{m_1} &= \frac{I_2 - I_3}{I_2 I_3} m_2 m_3\\ \dot{m_2} &= \frac{I_3 - I_1}{I_1 I_3} m_1 m_3\\ \dot{m_3} &= \frac{I_1 - I_2}{I_1 I_2} m_1 m_2\end{aligned}$$

24. Coupled Limit Cycle Oscillators Equations:

$$\begin{aligned}\dot{\theta_1} &= 1 - \gamma \sin 2\pi(\theta_1 - \theta_2)\\ \dot{\theta_2} &= \omega - \gamma \sin 2\pi(\theta_1 - \theta_2)\end{aligned}$$

25. Wind Induced Oscillation Equations:

$$\begin{aligned}\dot{x} &= -\xi x - \lambda y + xy\\ \dot{y} &= \lambda x - \xi y + (x^2 - y^2)/2\end{aligned}$$

26. Volterra Equations:

$$\begin{aligned}\dot{x} &= (c_1 + c_3 x + c_5 y)x\\ \dot{y} &= (c_2 + c_4 x + c_6 y)y\end{aligned}$$

27. Samardzija's Equations:

$$\begin{aligned}\dot{x} &= c_1 x + c_2 y - z\\ \dot{y} &= c_3 x + c_4 y - c_5 x^3\\ \dot{z} &= \rho x\end{aligned}$$

28. Tunnel Diode Circuit Equations:

$$\begin{aligned}\dot{x_1} &= \frac{1}{C}[-h(x_1) + x_2]\\ \dot{x_2} &= \frac{1}{L}[-x_1 - Rx_2 + U]\end{aligned}$$

where $h(x) = 17.76x - 103.79x^2 + 229.62x^3 - 226.31x^4 + 83.72x^5$.

29. Single-link Manipulator with Flexible Joints:

$$\begin{aligned}I\ddot{q_1} + MgL \sin q_1 + k(q_1 - q_2) &= 0\\ J\ddot{q_2} - k(q_1 - q_2) &= U\end{aligned}$$

30. Synchronous Generator Commected to an Infinite Bus:

$$\begin{aligned}M\ddot{\delta} &= P - D\dot{\delta} - \xi_1 E_q \sin\delta\\ \tau\dot{E_q} &= -\xi_2 E_q - \xi_3 \cos\delta + E_{FD}\end{aligned}$$

31. Wien-Bridge Oscillator

$$\begin{aligned}
\dot{x_1} &= \frac{1}{C_1R_1}[-x_1 + x_2 - g(x_2)] \\
\dot{x_2} &= \frac{-1}{C_2R_1}[-x_1 + x_2 - g(x_2)] - \frac{1}{C_2R_2}x_2
\end{aligned}$$

where $g(x) = 3.234x - 2.195x^3 + 0.666x^5$.

2 Mapping

1. Bouncing Ball With Moving Wall Equations:

$$\begin{aligned}
\phi_{j+1} &= \phi_j + V_j \\
V_{j+1} &= \alpha V_j - \gamma \cos(\phi_j + V_j)
\end{aligned}$$

2. Complex Cubic Map:

$$z_{j+1} = \rho z_j^3 + (c_1 + ic_2)z_j^2 + (c_3 + ic_2)z_j + c_5 + ic_6$$

where $z = x + iy$.

3. Ikeda Map:

$$z_{j+1} = \rho + c_2 z_j e^{i(c_1 - c_3/(1+|z_j|)}$$

where $z = x + iy$.

4. Tinkerbell Map:

$$\begin{aligned}
x_{j+1} &= x_j^2 - y_j^2 + c_1x_j + c_2y_j \\
y_{j+1} &= 2x_jy_j + c_3x_j + c_4y_j
\end{aligned}$$

5. Chossat Map:

$$z_{j+1} = (c_1|z_j^2| + c_2Re(z_j^{c_5}) + \rho)z_j + c_3(z^{c_5-1})^*$$

where z complex and z^* its conjugate and c_5 is integer.

6. Gumowski Map:

$$\begin{aligned}
x_{j+1} &= c_1(1 + c_2y_j^2)y_j + F(x_j) \\
y_{j+1} &= -x_j + F(x_{j+1}) \\
F(u) &= \rho u + 2(1-\rho)u^2/(1+u^2)
\end{aligned}$$

7. Henon Map with 5th Order Polynomial:

$$\begin{aligned}
x_{j+1} &= c_2 + c_3x_j + c_4x_j^2 + c_5x_j^3 + c_6x_j^4 + c_7x_j^5 + c_1y_j \\
y_{j+1} &= x_j
\end{aligned}$$

8. Hitzl Map:

$$\begin{aligned}
x_{j+1} &= 1 + y_j - z_jx_j^2 \\
y_{j+1} &= c_1x_j \\
z_{j+1} &= z_j - 0.5 + +\rho x_j^2
\end{aligned}$$

9. Piecewise Linear Map:
   If $x \leq 0$:

$$\begin{aligned} x_{j+1} &= c_1 x_j + y_j + \rho \\ y_{j+1} &= c_2 x_j \end{aligned}$$

   If $x > 0$:

$$\begin{aligned} x_{j+1} &= c_3 x_j + y_j + \rho \\ y_{j+1} &= c_4 x_j \end{aligned}$$

10. Yakubo Map:

$$\begin{aligned} x_{j+1} &= \frac{c_1 x_j}{x_j + y_j + c_2} \\ y_{j+1} &= e^{\rho - c_3(x_j + y_j)} \end{aligned}$$

11. Nordmark Truncated Map:
    If $x \leq 0$:

$$\begin{aligned} x_{j+1} &= c_1 \sqrt{x_j} + y_j + \rho \\ y_{j+1} &= c_2 c_4^2 x_j \end{aligned}$$

    If $x > 0$:

$$\begin{aligned} x_{j+1} &= c_3 x_j + y_j + \rho \\ y_{j+1} &= c_2 x_j \end{aligned}$$

12. Henon Map:

$$\begin{aligned} X_{n+1} &= 1 - aX_n^2 + Y_n \\ Y_{n+1} &= bX_n \end{aligned}$$

13. Twist Map:

$$\begin{aligned} J_{n+1} &= J_n \\ \theta_{n+1} &= \theta_n + 2\pi\alpha J_{n+1} (\text{mod } 2\pi) \end{aligned}$$

14. Quadratic Map:

$$\begin{aligned} y_{n+1} &= x_n \\ x_{n+1} &= 1 - y_n - ax_n^2 \end{aligned}$$

15. Quadratic Twist Map:

$$\begin{aligned} x_{n+1} &= x_n \cos\phi - (y_n - x_n^2) \sin\phi \\ y_{n+1} &= x_n \cos\phi + (y_n - x_n^2) \cos\phi \end{aligned}$$

16. Standard Map:

$$\begin{aligned} I_{n+1} &= I_n - K \sin\phi_n \\ \phi_{n+1} &= \phi_n + I_{n+1} (\text{mod } 2\pi) \end{aligned}$$

17. Fermi Map:

$$\begin{aligned} u_{n+1} &= |u_n + \sin\phi_n| \\ \phi_{n+1} &= \phi_n + \frac{2\pi M}{u_n + 1} (\text{mod } 2\pi) \end{aligned}$$

18. De Vogelaere Map:

$$\begin{aligned} x_{n+1} &= -y_n + g(x_n) \\ y_{n+1} &= x_n - g(x_{n+1}) \\ g(x) &= cx + x^2 \end{aligned}$$

19. Kaplan and Yorke Map:

$$\begin{aligned} x_{n+1} &= 2x_n (\text{mod } 1) \\ y_{n+1} &= \alpha y_n + \cos 4\pi x_n \end{aligned}$$

20. Whisker Map:

$$\begin{aligned} w_{n+1} &= w_n - \epsilon\pi w_0^2 \text{csch} \left(\frac{\pi w_0}{2}\right) \sin\phi_n \\ \phi_{n+1} &= \phi_n + w_0 \ln\left(\frac{16}{w_{n+1}}\right) \end{aligned}$$

21. Cometary Map:

$$\begin{aligned} P_{n+1} &= P_n + K\sin(g_n) \\ g_{n+!} &= g_n + \frac{4\pi}{3P_{n+1}} \end{aligned}$$

22. Kepler Map:

$$\begin{aligned} N_{j+1} &= N_j + \kappa \sin\phi_j \\ \phi_{j+1} &= \phi_j + 2\pi w_0 (-2w_0 N_{j+1})^{-3/2} \end{aligned}$$

23. Sine Circle Map:

$$\theta_{j+1} = \theta_j + \omega + \frac{K}{2\pi}\sin(2\pi\theta_j)$$

24. Doubling Map:

$$x_{n+1} = 2x_n (\text{mod } 1)$$

3 Fractal Set

1. Degree m Polynomial Sets (Including Julia Set and Mendelbrot Set):

$$z_{n+1} = z_n^m + c$$

   where $z = x + iy$.

2. Tricorn Set:

$$z_{n+1} = \bar{z_n}^2 + c$$

   where $z = x + iy$.

3. Exponential Set:

$$z_{n+1} = \lambda e^z$$

   where $z = x + iy$.

4. Sierpinski Triangle, Box Fractal and/or Koch Snowflake