Expected shortfall or median shortfall

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Abstract

In a recent consultative document, the Basel Committee on Banking Supervision suggests replacing Value-at-Risk (VaR) by expected shortfall (ES) for setting capital requirements for banks’ trading books because ES better captures tail risk than VaR. However, besides ES, another risk measure called median shortfall (MS) also captures tail risk by taking into account both the size and likelihood of losses. We argue that MS is a better alternative than ES as a risk measure for setting capital requirements because: (i) MS is elicitable but ES is not; (ii) MS has distributional robustness with respect to model misspecification but ES does not; (iii) MS is easy to implement but ES is not.

Keywords: Model uncertainty; robustness; elicitation; backtest; Value-at-Risk; expected shortfall; median shortfall; Basel accord; capital requirements.

JEL Classifications: C10; C44; C53; D81; G17; G18; G28; K23

1. Introduction

Elementary statistics teaches us that both mean and median measure the average size of a random quantity, but they have different properties. In particular, if we want to obtain a robust measurement, then median is a better choice than mean. This basic fact about mean and median is closely related to a newly proposed Basel Accord capital rule for trading books.

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There are some concerns that the proposed Basel III Accord may be too complicated and may not fully address the real issues; see a recent article by Kupiec (2013). We agree with his viewpoints. In particular, it is proposed in a recent consultative document by the Basel Committee on Banking Supervision (2013) that one of the major changes to the trading book capital rule is to move from value-at-risk (VaR) to expected shortfall (ES) mainly because of “the inability of the measure [VaR] to capture the tail risk of the loss distribution.” We fully agree that it is necessary to capture the tail risk beyond the loss level specified by VaR. However, how to achieve this is debatable. More precisely, should we use ES, defined as the mean of the size of loss beyond VaR (as suggested in the document), or median shortfall (MS), defined as the median of the size of loss beyond VaR (Kou et al., 2013)? This is related to the question about choosing between mean and median.

For example, if we want to capture the tail risk, e.g., the size of loss beyond VaR at 99% level, we can either use ES at 99% level, which is the mean of the size of loss beyond VaR at 99% level, or, alternatively, MS at 99% level, which is the median of the size of loss beyond VaR at 99%. Hence, just like ES, MS also measures the riskiness of random losses by taking into account both the size and likelihood of losses. However, MS has several advantages over ES, in face of statistical model uncertainty.

2. Model Uncertainty

In the internal models based approach for determining trading book capital requirements, regulators impose the risk measure and allow institutions to use their own internal risk models and private data in the calculation. There can be several statistically indistinguishable models for the same instrument or portfolio due to limited availability of data. In particular, the heaviness of tail distributions cannot be identified in many cases. For example, Heyde and Kou (2004) show that it is very difficult to distinguish between exponential-type and power-type tails with 5,000 observations (about 20 years of daily observations) because the quantiles of the two types of distributions may overlap. Therefore, the tail behavior may be a subjective issue depending on people’s modeling preferences.

3. The First Advantage of MS: Elicitability

MS satisfies a basic statistical property called elicitability (i.e., there exists an objective function such that minimizing the expected objective function yields the risk measure; see Gneiting, 2011), but ES does not. If a risk measure is not
elicitable, then it is hard to justify the use of a forecasting procedure for the risk measure.

More precisely, in face of model uncertainty, several forecasting procedures based on different models for the underlying risk can be used to forecast the risk measure. It is hence desirable to be able to evaluate which procedure gives a better forecast. The elicitability of a risk measure means that the risk measure can be obtained by minimizing the expectation of a forecasting objective function; hence, the forecasting objective function can be used for evaluating different forecasting procedures. On the other hand, if one cannot find such a forecasting objective function, then one cannot tell which one of competing point forecasts for the risk measurement performs the best by comparing their forecasting error, no matter what objective function is used.

Elicitability is closely related to backtesting, whose objective is to evaluate the performance of a risk forecasting model. If a risk measure is elicitable, then the sample average forecasting error based on the objective function can be used for backtesting the risk measure. MS is elicitable (Gneiting, 2011; Kou and Peng, 2014), but ES is not. The non-elicitability of ES “may challenge the use of ES as a predictive measure of risk, and may provide a partial explanation for the lack of literature on the evaluation of ES forecasts” (Gneiting, 2011).

4. The Second Advantage of MS: Robustness

MS has a desirable property of distributional robustness with respect to model misspecification in the sense of Hampel (1971), which means that a small deviation of the model only results in a small change in the risk measurement; but ES does not (Kou et al., 2013; Kou and Peng, 2014). This means that MS leads to “more stable model output and often less sensitivity to extreme outlier observations”, a desirable property mentioned on p. 18 of the consultative document of Basel Committee on Banking Supervision (2013).

To further compare the robustness of MS with ES, Kou and Peng (2014) carry out a simple empirical study on the measurement of the tail risk of S&P 500 daily return. They consider two IGARCH(1, 1) models similar to the model of RiskMetrics, one with the noise having a normal distribution and the other a t-distribution with the degree of freedom unknown. After fitting the two models to the historical data of daily returns of S&P 500 Index during 1/2/1980–11/26/2012 and then forecasting the one-day MS and ES of a portfolio of S&P 500 stocks, it is found that the difference of ES under the two models is much larger than that of MS, indicating that ES is more sensitive to model misspecification than MS.

Regulatory risk measures should demonstrate robustness with respect to model misspecification (Kou et al., 2013). From a regulator’s viewpoint, a regulatory risk
measure must be unambiguous, stable, and capable of being implemented consistently across all the relevant institutions, no matter what internal beliefs or internal models each may rely on. When the correct model cannot be identified, two institutions that have exactly the same portfolio can use different internal models, both of which can obtain the approval of the regulator; however, the two institutions should be required to hold the same or at least almost the same amount of regulatory capital because they have the same portfolio. Therefore, the regulatory risk measure should be robust; otherwise, different institutions can be required to hold very different regulatory capital for the same risk exposure, which makes the risk measure unacceptable to both the institutions and the regulators. In addition, if the regulatory risk measure is not robust, institutions can take regulatory arbitrage by choosing a model that significantly reduces the capital requirements.

The requirement of robustness for regulatory risk measures is not anything new; in general, robustness is essential for law enforcement, as is implied by legal realism, one of the basic concepts of law; see Hart (1994). Legal realism is the viewpoint that a law is only a guideline for judges and enforcement officers (Hart, 1994, pp. 204–205) and is only intended to be the average of what judges and officers will decide. Hence, a law should be established in a robust way so that different judges will reach similar conclusions when they implement the law. In particular, the risk measures imposed in banking regulation should also be robust with respect to underlying models and data.

It is also worth noting that it is not desirable for a risk measure to be too sensitive to the tail risk. For example, consider the random loss that could occur to a person who walks on the street. There is a very small but positive probability that the person could be hit by a car and lose his life; in that unfortunate case, the loss may be infinite. Hence, the ES of the random loss may be equal to infinity, suggesting that the person should never walk on the street, which is apparently not reasonable. In contrast, the MS of the random loss is a finite number.

5. The Third Advantage of MS: Easy Implementation

Kou and Peng (2014) show that, for any loss distribution, MS at a given confidence level is simply equal to VaR at a higher confidence level. For example, MS at 99% level is simply equal to VaR at 99.5% level.

Furthermore, the backtesting for MS can be easily done using the existing methods for backtesting VaR (see, e.g., Jorion, 2007; Gaglianone et al., 2011), while it is difficult to do backtesting for ES. In fact, the consultative document of
Basel Committee on Banking Supervision (2013) suggests to do backtesting by “comparing 1-day static value-at-risk measure at both the 97.5th percentile and the 99th percentile to actual P&L outcomes,” although it suggests to replace VaR by ES in measuring risk.

6. Conclusion

It is better to replace VaR by MS instead of ES. In fact, Kou and Peng (2014) prove that MS is the only tail risk measure that satisfies a set of axioms based on the Choquet expected utility theory (Schmeidler, 1989) and has the statistical property of elicitability. Furthermore, MS is robust with respect to model misspecification. ES is neither elicitatable nor robust; and it is difficult to implement ES and to do backtesting for ES.

In the last decade, financial institutions around the globe have spent considerable effort to develop capacities to compute VaR. The implementation of MS is as easy as that of VaR, as MS can be computed as VaR at a higher level. Shifting from VaR to ES, as proposed by the Basel Committee on Banking Supervision (2013), not only lacks sound justification, but also may lead to huge implementation problems in financial institutions.

In short, MS is a better alternative than ES as a risk measure for setting capital requirements in the Basel Accord.

References


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