Defaultable convertible bonds:

Binomial calculations
Callable feature

Issuer has the right to call back the bond at a pre-specified call price.
* Upon call, the holder can either convert the bond or redeem at the call price
* Restrictions on calling may apply; for example, notice period requirement, closing price of stock has been in excess of 150% of the conversion price on any 20 trading days within 30 consecutive days.

Put feature

The put feature allows the holder to sell back the bond to the issuer in return for a fixed sum.

Question: What should be the optimal call policies of the issuer and the optimal conversion strategies of the holder?

Impact of the credit ratings of the issuer on the convertible bond price!
Pricing of risky convertible bonds

One-factor binomial model

* stock price process follows binomial random walk
* interest rates to be deterministic

Two discount rates

1. If the convertible is certain to remain a bond, it is appropriate to use a discount rate corresponding to the creditworthiness of the issuer – risky rate.

2. Suppose the bond is certain to be converted, it is then appropriate to use the riskfree rate.

The holder will choose the maximum between the par value and the value of stocks upon conversion.
At each node, we test
a. whether conversion is optimal;
b. whether the position of the issuer can be improved by calling the bonds; dynamic programming procedure:
max(min(Q_1, Q_2), Q_3)

\[ Q_1 = \text{value given by the rollback} \]
\[ (\text{neither converted nor called back}) \]
\[ Q_2 = \text{call price} \]
\[ Q_3 = \text{value of stocks if conversion takes place} \]

The discount rate to be used when we roll back is given by

\[ pw_u + (1 - p)w_d \]

when \( p \) is the probability to a node where the discount rate is \( w_u \) and \( (1 - p) \) is probability to a node with \( w_d \).
Example
A 9-month discount bond issued XYZ company with a face value of $100. Assume that it can be exchanged for 2 shares of company’s stock at any time during the 9 months.
* It is callable for $115 at any time.
* Initial stock price = $50, $\sigma$ = 30\% per annum and no dividend; risk-free yield curve to be flat at 10\% per annum.
* Yield curve corresponding to bonds issued by the company to be flat at 15\%.
* Tree parameters are: $u = 1.1618$, $d = 0.8607$, $p = 0.5467$, $R = e^{0.1\Delta t} = 1.0253$.
* At maturity, the convertible is worth max (100, 2$S_T$).
Binomial tree for pricing a risky convertible bond

upper figure: stock price
middle figure: discount rate
lower figure: value of convertible
At node $D$
Roll back gives the bond value
\[(0.5467 \times 156.84 + 0.4533 \times 116.18)e^{-0.1 \times 0.25} = 134.98.\]
The bondholder is indifferent to conversion or hold, also the issuer is also indifferent as to whether the bond is called; the correct discount rate at node $D$ is 10%.

At node $F$
The correct discount rate is 15% since the convertible is contain not to be converted if node $E$ is reached.

At node $E$
The correct discount rate is
\[0.5467 \times 10\% + 0.4533 \times 15\% = 12.27\%.\]
The value of convertible at $E$
\[(0.5467 \times 116.18 + 0.4533 \times 100)e^{-0.1227 \times 0.25} = 105.56.\]
The bond should be neither converted nor called.
At node $B$

The discount rate is
\[0.5467 \times 10\% + 0.4533 \times 12.27\% = 11.03\%.
\]
and value of convertible is
\[(0.5467 \times 134.99 + 0.4533 \times 105.56)e^{-0.1103 \times 0.25} = 118.34.
\]
It is optimal to call the bond at node $B$ so that it causes immediate conversion and leads to $116.18$. The discount rate at node $B$ should be taken to be $10\%$, since conversion takes place at this node.

At node $A$

The discount rate is
\[0.5467 \times 10\% + 0.4533 \times 13.51\% = 11.59\%.
\]
The convertible value at node $A$ is
\[(0.5467 \times 116.18 + 0.4533 \times 98.00)e^{-0.1159 \times 0.25} = 104.85.
\]
If the bond has no conversion option, its value is
\[e^{-0.75 \times 0.15} = 89.36.
\]
The value of conversion option $= 104.85 - 89.36 = 15.49$. 