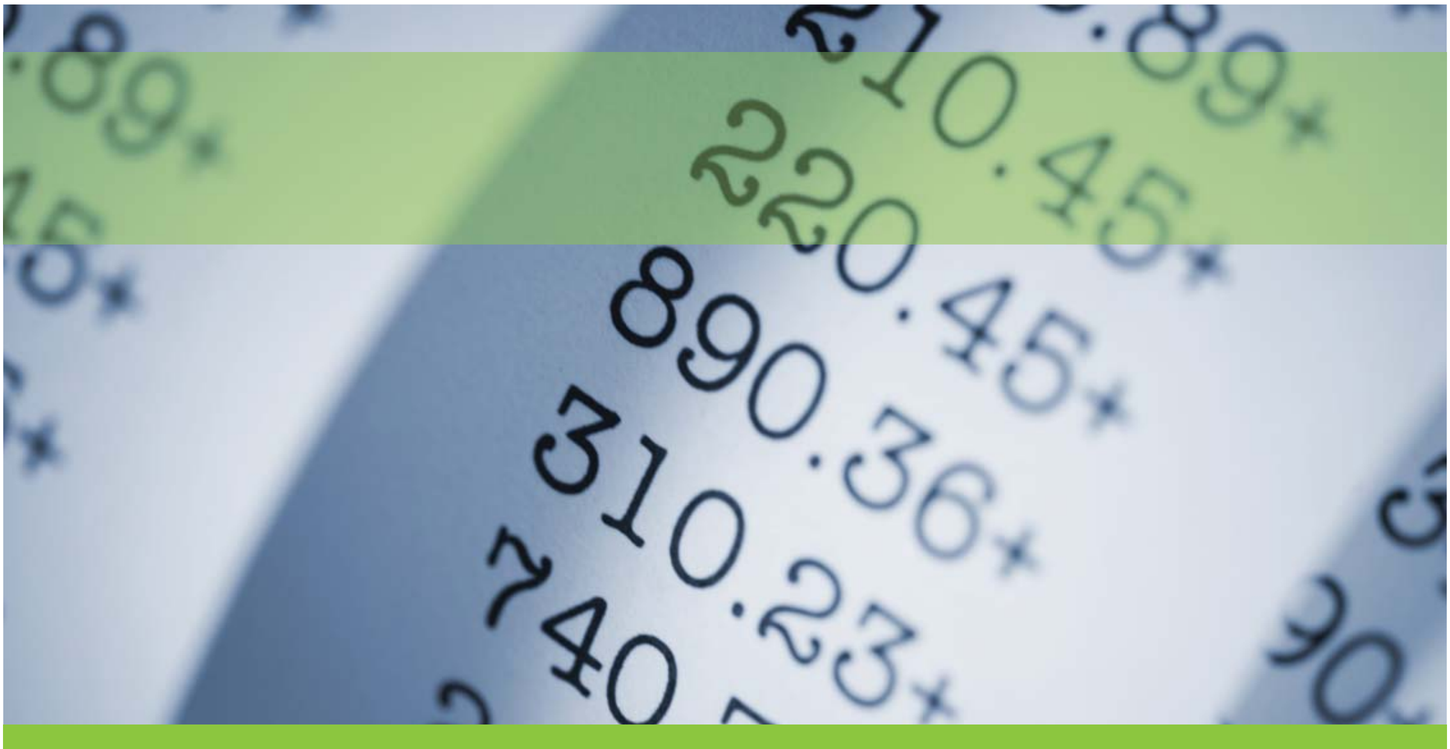

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Commentary

CPDOs Laid Bare: Structure, Risk and Rating Sensitivity

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DBRS

Insight beyond the rating.

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CPDOs Laid Bare: Structure, Risk and Rating Sensitivity

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Executive Summary

Constant proportion debt obligations (CPDOs) are one of the latest and most talked about innovations in the structured credit market. Since their debut in the summer of 2006, CPDOs have been frequently praised for providing an answer to investors searching for excess returns, but they have also been severely criticised for introducing more volatility into credit markets. The range of opinions of market participants has been wide indeed, and comments vary from "looks like a five- or ten-year mezzanine tranche," "similar to leveraged super-senior [LSS]," "a highly rated equity tranche," "a rated trading strategy" and "similar to CPPI [constant proportion portfolio insurance], just the opposite."

Although the level of understanding of the product varies, a common concern is that early CPDOs have received AAA ratings at 200 basis points (bp) over LIBOR from certain rating agencies. Indeed, one journalist states, "But theoreticians – pundits in general and columnists in particular – would seem to know a bigger truth. The rules of risk and reward, they say, state that a triple-A security can't pay 200 basis points (bp) over LIBOR. What's more, they add, is that basic common sense will tell you that a structure that includes up to 15 times leverage can't possibly be triple-A" (Salmon, F. (2007)). While there is little doubt that the attraction of CPDOs stems to a large extent from their ability to provide rated coupons, their growth highlights the importance of rating agencies in the development of the structured credit market, most of all their ability to assess both credit risk and market risk in the presence of a high degree of leverage. In a previous report, we outlined our initial views on the risks presented by CPDOs.[1] This report continues that discussion, and demonstrates DBRS's commitment to a rigorous and objective analysis of these innovative structures.

In this report, we provide an in-depth structural and risk analysis of CPDOs. In particular, we add to the existing (and extensive) body of CPDO publications by carrying out detailed empirical research in order to address the important issue of model risk; namely, the inherent sensitivity of CPDO performance to the choice of model and modelling parameters. After a brief introduction to the product, we focus on quantitative modelling aspects, including model risk and rating sensitivities, and discuss important insights gained from a back-testing experiment. In conclusion, we provide a brief product, performance and relative-value discussion.

The key findings of our CPDO research are outlined below.

CPDOs INVOLVE SIGNIFICANT MODEL RISK

Small changes in modelling assumptions, particularly regarding credit spread parameters, can have a significant impact on key risk measures, including probability of default (PD) and loss-given-default (LGD).

Higher spread volatility increases both the PD and LGD as a result of a greater chance of the CPDO being unwound when its net asset value (NAV) falls below a minimum threshold, known as a cash-out event. Similarly, the strength of spread mean reversion has a strong effect on PD, yet has little impact on LGD. We also find a very significant risk and ratings impact when varying the steepness of the credit curve, with model-implied ratings varying from AAA to BBB, depending on the choice of time period and data source used in the analysis.

The impact of changing bid-offer spreads is also surprising. Under an assumption of 0.25 bp, a typical model-implied rating might be as high as AA, but this falls to BBB (high) under a 1 bp assumption and BB under a conservative assumption of 2 bp. Such a high sensitivity implicitly requires taking a view on future liquidity in credit markets within any ratings or risk management framework.

Overall, we show that the rating is very sensitive to most modelling parameters and compare an "optimistic" calibration, assuming 25% spread volatility and 7% time decay,[2] with a calibration more in line with empirical findings across credit

[1] See "CPDOs: The DBRS Perspective": DBRS CDO Newsletter, December 2006.
[2] This corresponds to a 7% reduction in spreads over six months or 3.5% over a three-month period.

default swap (CDS) and bond markets. Using the empirical data considered here, the AAA model-implied rating under the optimistic calibration becomes BBB (high).

CREDIT SPREAD ANALYSIS REVEALS HIGH VOLATILITY

We have analysed spread volatility for a wide range of CDS and bond spread datasets – for different investment-grade (IG) rating classes and at different points in time – and find that spread volatility exceeds 35% in many cases. A lower assumption (say 25%) cannot generate sufficient fluctuation in long term spreads and would be unable to capture the spread widening observed during the last downturn in credit markets between 1998 and 2003. Unsurprisingly, this has a significant impact on CPDO risk and rating estimates, as very few cash-out events occur under a low volatility assumption.

TERM STRUCTURE DYNAMICS SHOW A LINK BETWEEN SPREADS AND SLOPE

Across all datasets, we find a strong link between the level of credit spreads and the steepness of the credit curve, with flatter term structures occurring in a high spread environment and steeper term structures in tight credit spread regimes (such as the one that exists at the time of writing). Incorporation of these dynamics into a CPDO model leads to a decrease in PD and an increase in LGD.

HIGH PATH DEPENDENCE AND MARK-TO-MARKET VOLATILITY CAN LEAD TO RATING VOLATILITY

Our back-testing exercise reveals very high path dependence in CPDOs, leading to a high implied level of volatility in NAV and implied ratings during the last credit cycle. For example, a CPDO issued in 2004 would show high NAV stability, as the structure would have experienced NAV gains and de-levered over the first three years. However, the high sensitivity of ratings to the initial level of spreads may still have resulted in significant ratings volatility. On the other hand, issuing a CPDO in 1997 shows a very different picture. A decline in NAV caused by immediate spread widening would have led to very high initial leverage, which would have caused significant MTM losses when spreads continued to gap out.

CPDOs ARE HERE TO STAY

Notwithstanding the above findings, we feel that CPDOs are an exciting addition to the structured credit market. Independent research shows that CPDOs may offer more value than investment alternatives in moderately bearish credit environments, whereas first-loss products may perform better in bullish credit scenarios. The high level of model risk, however, makes the rating and risk assessment of these products very challenging, and as a result, we take a more conservative view than other agencies on the risk assessment of the static CPDOs that have so far been issued in the market.

Nonetheless, we see considerable potential for mitigating some of these risks in the second generation of CPDO structures. For example, managed CPDOs can reference well-diversified, bespoke portfolios and introduce structural features that allow for risk mitigation and ratings stability. In addition, innovations in the CPDO leverage mechanism have been shown reduce MTM volatility while still generating significant carry. It is highly likely that some of these more innovative CPDO structures will be consistent with high investment grade ratings.

Overview of CPDOs

Figure 1 below shows the main features of a typical CPDO, and Appendix 2 provides a detailed description of the mechanics of a typical CPDO, including the leverage algorithm.

At trade inception, CPDO issuance proceeds are held in a deposit account that earns interest at the risk-free rate. The special-purpose vehicle (SPV) enters into a total return swap with the arranging bank, which simultaneously sells protection on a certain leveraged notional amount of a risky reference portfolio (typically a combination of the main credit indices, CDX and iTraxx, but as with CPPI, bespoke portfolios, hybrid assets or more complex credit products may be also referenced). Over time, CDS premium payments and MTM gains are paid into the deposit account, while MTM losses and default payments are taken out of the cash deposit. Principal and coupon payments are made to CPDO noteholders subject to sufficient funds being available in the deposit account. In contrast to credit CPPI (where only principal payments receive a rating), the arranging bank does not enter a zero coupon bond that guarantees principal investment, hence investors rely on – among other things – CPDO credit ratings to assess the likelihood of full principal and interest payments. In short, CPDOs put as much principal at risk as is needed to meet certain promised return, whereas the primary aim of CPPI is to maximise returns without putting principal at risk.

LEVERAGE

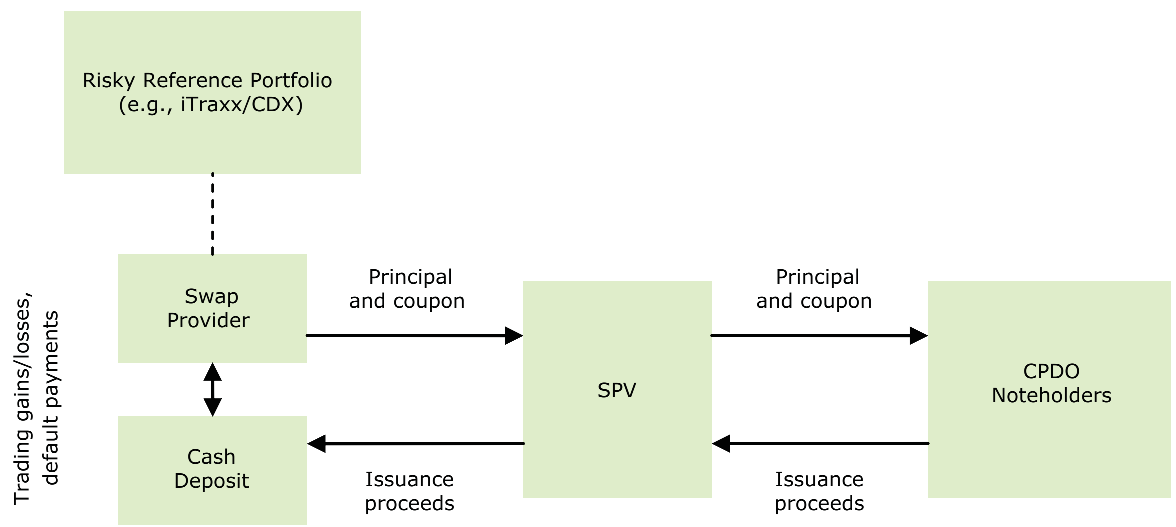
CPDOs provide returns to noteholders through leverage; namely, the selling of protection on a much larger notional amount than the note proceeds. The leverage factor is essentially a multiple of the difference – or shortfall – between the NAV of the CPDO strategy (the sum of the value of the cash deposit and the MTM value of the risky portfolio) and the present value of all future payments (Target Value) to be made by the SPV, including fees.[3] The portfolio is "rebalanced" when the calculated or required leverage differs from the current leverage by a certain preset amount.

In practice, losses due to defaults and spread widening lead to an increase in leverage compared with a decrease in leverage for a CPPI structure. As a result, CPDOs have been frequently called leveraged bets on credit quality and market spreads. In contrast, when the CPDO is performing well (increasing NAV), the structure is de-levered.

CASHING IN AND CASHING OUT

A so-called cash-in event takes place when the shortfall decreases to zero, in which case the strategy is unwound completely and the proceeds are held in the deposit account in order to make all future payments promised by the SPV. On the other hand, if the NAV falls below a certain threshold (typically 5% or 10% of the notional of the reference portfolio), the strategy is unwound

Figure 1: Structure at Closing of a Typical CPDO Transaction



[3] Leverage is therefore purely formulaic (as opposed to discretionary) but will clearly vary over time depending on the performance of the strategy. Leverage is typically capped at around 15 to prevent unacceptably high leverage in periods of poor strategy performance.

and the proceeds are distributed to CPDO noteholders. This is known as a cash-out event.

INITIAL INDEX-BASED STRUCTURES AND INDEX ROLL

The first CPDOs referenced "on-the-run" IG credit indices, which means that on or close to each roll date (20 March and 20 September), the arranging bank must buy protection on the "off-the-run" indices (up to the full leveraged notional amount) and sell protection on the new on-the-run indices. The difference in off-the-run index spread compared with the contractual spread entered at the previous roll date determines the MTM gain or loss experienced by the strategy. Contracting at a new index spread also has an impact on CPDO performance due to the new CDS premium the SPV earns over the next roll period. This impact may be positive if the new spread is high enough to offset unwind costs. Hence, the dynamics of index spreads around roll periods (e.g., off-the-run spread widening due to replacement of credits that fall below IG) is very important.

This contractual requirement to roll every six months into the new on-the-run indices has been a concern for many market participants. Indeed, many believe that both index and tranche markets were affected by the expectation of CPDO issuance (see, for example, Isla, Willemann and Soulier (2006)).[4] This concern, along with related concerns over MTM volatility, has led to the development of more actively managed structures. Lightly managed structures allow the manager to (temporarily) apply leverage different from the formulaic level or to add flexibility in the response of the CPDO to the rolling of the indices. The holy grail of CPDOs remains fully managed structures in which single name CDS or fully bespoke portfolios can be implemented while still achieving an attractively rated coupon.

Assessing the Risks in CPDOs

The need for CPDOs to provide a rating on principal and coupon has resulted in a demand for rating agencies to develop methodologies that go beyond their core competency; namely, credit risk. CPDOs are an extension of the trend toward non-credit assets and MTM risk faced by the agencies in recent years, starting with equity default swaps in 2004, followed by LSS in 2005 and commodity-linked CDOs and CPDOs in 2006. Similarly, agency methodologies used to analyse market-value CDOs and credit derivative product companies (CDPCs) utilise many of the aspects required for modelling the risks in CPDOs. While agencies have a strong track record in assessing credit risk, their ability to assess market risk, particularly combined with high leverage, is less clear. We believe that the research summarised here demonstrates our ability to meet the challenge of analyzing these complex products. While we may not have all of the answers, we provide an objective and empirically grounded analysis of all of the key risks faced by investors.

In this section, we provide an overview of an approach to modelling the main components of long-only, index-based CPDOs and assess the sensitivity of the resulting model-implied ratings[5] to changes in the underlying assumptions and methodology. The quantitative approach discussed below is by no means unique, and several extensions or alternatives can be developed. Nevertheless, we feel that the approach is capable of addressing the main risks in CPDOs, with conclusions (particularly with respect to rating stability) that remain valid under alternative frameworks.

In particular, we describe the following quantitative models and econometric techniques applied to address a CPDO's main economic risks:

- Default risk.
- Credit spread risk.
- Roll risk and liquidity risk.
- Interest rate risk.

A particular focus is given to the two primary risk factors: default risk and the evolution of the term structure of credit spreads.

We discuss the general models that can be used to assess these risks, provide insights into the parameterisation of these models and show the impact of changes to both models and parameters on a typical CPDO. By "typical," we refer to a

[4] The roll of the indices on March 20, 2007, was relatively benign, with CPDOs experiencing a 0.25 bp bid-offer spread. However, this was hardly surprising, given the relatively small number of CPDOs in the market at the time (less than USD 2 billion of issuance).
[5] Note that these are purely quantitative rating estimates, whereas DBRS ratings include both quantitative and qualitative factors.

static CPDO that is long semi-annual iTraxx and CDX five-year risk. The CPDO pays a coupon of LIBOR plus 200 bp, cashes out at 10% of par, can be levered 15 times (x) at most and matures in ten years. Standard fees are assumed, including a 1% upfront fee and various ongoing administration fees. Unless otherwise stated, bid-offer spreads of 1 bp are assumed.

2.1 DEFAULT RISK

We apply DBRS's standard default methodology, based on the Gaussian copula framework, to arrive at correlated default times for each of the underlying credits in both credit indices.[6]

The credit quality of an obligor *(i)* is described via a single normally distributed latent variable (V_i), often referred to as the obligor's asset value. Dependence between various obligors is introduced through their dependence on common factors. For example, in the simplest case of a single factor *F*, $V_i = \sqrt{\rho}\,F + \sqrt{1-\rho}\,\varepsilon_i$, where ρ denotes the correlation between two obligors and the standard normal variables *F* and ε_i are global and idiosyncratic risk factors, respectively. In practice, a single correlation factor is often too restrictive for risk management purposes, and generalisations to a multifactor framework are preferred.

DBRS employs the following multifactor model: $V_i = \sqrt{\rho}\,F + \sqrt{\rho^S - \rho}\,F_{S(i)} + \sqrt{\rho^R}\,F_{R(i)} + \sqrt{\rho^{SR} - \rho^R}\,F_{SR(i)} + \sqrt{1-\rho^{SR}-\rho^S}\,\varepsilon_i$, where *F* denotes the global, $F_{R(i)}$ a regional, $F_{S(i)}$ a sector-specific and $F_{SR(i)}$ a region-sector factor. For corporate obligors, we use ρ = 2%, ρ^S = 15%, $\rho^R = \rho^{SR}$ = 4%, which implies the following correlations between two obligors.

Table 1: Corporate Correlation Assumptions

	Within Sector	Between Sectors
Same region	0.15	0.06
Different regions	0.11	0.02

Given *N* different sectors in *M* different regions, (*N*+*M*+ *NM*+1) different factors would need to be simulated. Default occurs prior to maturity *(T)* if the asset value (V_i) falls below the default barrier – $d_i = N^{-1}(PD_T^i)$ – of obligor *i*, where PD_T^i denotes the obligor's cumulative *T*-year default probability.

Rather than computing default events prior to maturity, a default time (τ_i) can be computed for each obligor as follows:

(1) Calculate $u_i = \Phi(V_i)$.

(2) Calculate a default time – $\tau_i = S^{-1}(u_i)$ – for each asset.[7]

If τ_i is less than the maturity of the transaction, the loss L_i is determined as $L_i = N_i \times (1-\delta_i)$, where N_i and δ_i are the exposure-at-default and recovery,[8] respectively, for the *i*th asset. We can therefore write the portfolio loss up to time *t*, *L(t)*, as $L(t) = \sum_i N_i \times (1-\delta_i) \times 1_{\{\tau_i \le t\}}$, where $1_{\{\tau_i \le t\}}$ is the default indicator for the *i*th asset.[9]

As an example, a single default in a typical portfolio comprising 250 names leads to a 0.24% credit loss (or reduction in NAV), assuming 40% recovery. On a 15x levered portfolio, this loss amplifies to 3.6% of the initial notional.

To account for the fact that CDS indices roll every six months, we have generated a new set of default curves reflecting the fact that upon a transition below IG, the respective name is removed from the index at the next roll date and replaced with an IG name. A conservative treatment, which compensates for the fact that we do not explicitly model rating migrations, is to assume that the credit replacing the downgraded name is rated BBB (low). Table 2 on the following page shows the results of adjusting the six-month transition matrix accordingly and applying the usual Markov property leads to the following modified cumulative default probabilities.

For a standard index-based CPDO referencing 50% iTraxx and 50% CDX, these assumptions lead to approximately 0.50 to 0.65 names defaulting on average per annum.[10] We provide some insight into the sensitivity of CPDO risk and ratings to default risk in Section 2.6.

Due to the high number of sensitivity tests, defaults are simulated in a computationally efficient way using the six-month loss distribution generated from Table 2. Portfolio default rates are then simulated repeatedly for all six-month periods by sampling this distribution. The impact

[6] The same methodology is employed in the CDO Toolbox, our standard Monte Carlo CDO portfolio credit model (see *The CDO Toolbox*, a DBRS methodology report, available at www.dbrs.com).
[7] S^{-1} is used to denote the quasi-inverse of the survival function derived from the default curves PD_t^i.
[8] The recovery can either be assumed to be constant or drawn from a distribution.
[9] The default indicator equals 1 if the expression within parentheses is true and 0 if it is false.
[10] Note that this is roughly intermediate between the average annual default rate observed historically for a portfolio of 250 IG and BBB corporates (0.34 and 0.74 defaults, respectively) over the period 1994 to 2004 (Table 13).

Table 2: Cumulative Default Probabilities for Adjusted Transition Matrix

	Year 1	Year 2	Year 3	Year 4	Year 5	Year 6	Year 7	Year 8	Year 9	Year 10
AAA	0.02%	0.04%	0.07%	0.10%	0.14%	0.19%	0.24%	0.29%	0.35%	0.41%
AA (high)	0.04%	0.08%	0.12%	0.17%	0.23%	0.28%	0.34%	0.41%	0.48%	0.56%
AA	0.04%	0.09%	0.14%	0.19%	0.25%	0.32%	0.39%	0.47%	0.55%	0.64%
AA (low)	0.05%	0.11%	0.17%	0.24%	0.32%	0.40%	0.49%	0.58%	0.69%	0.79%
A (high)	0.06%	0.13%	0.20%	0.28%	0.37%	0.46%	0.57%	0.68%	0.80%	0.92%
A	0.07%	0.15%	0.24%	0.34%	0.45%	0.57%	0.70%	0.84%	0.99%	1.15%
A (low)	0.08%	0.18%	0.31%	0.46%	0.63%	0.80%	1.00%	1.19%	1.40%	1.61%
BBB (high)	0.19%	0.40%	0.62%	0.85%	1.10%	1.35%	1.60%	1.85%	2.11%	2.36%
BBB	0.28%	0.61%	0.95%	1.29%	1.62%	1.94%	2.25%	2.55%	2.83%	3.11%
BBB (low)	0.76%	1.34%	1.81%	2.21%	2.57%	2.89%	3.20%	3.50%	3.78%	4.05%

on average default rates over the simulation horizon is negligible, as the default rates shown in Table 2 increase roughly linearly over time.[11]

2.2 MODELLING THE TERM STRUCTURE OF CREDIT SPREADS

At least as important as the risk of default and subsequent credit losses is the future evolution of the term structure of credit spreads. CPDOs based on indices selling new protection every six months have a decreasing maturity profile between [T^I, T^I-0.5], where T^I denotes the index maturity in years. As a result, we need to specify the whole term structure of credit spreads, or at least the part of the term structure to which the transaction is exposed (e.g., six months of time-decay for index-based structures).

Spread Data and Estimation

Modelling the credit spread dynamics for CPDOs (or any other synthetic spread or MTM trade) is challenging for various reasons, among them the lack of a long-time series of CDS spread data. For this reason, we have investigated a number of different data sources, including CDX/iTraxx index spread data (Markit Partners), yield spreads (Bloomberg) and bond-implied CDS (BCDS) spreads (Lehman Brothers). The reason for the latter two datasets is the limited amount of data available on CDS indices, which start as recently as March 2004.

A more detailed description and comparison is given in Appendix 1, but essentially BCDS spreads (just like actual CDS spreads) are based on the concept of default arrival (and its probability) and therefore are directly comparable with CDS spreads. We believe that BCDS spreads provide a suitable proxy for spread analysis, given the paucity of CDS historical data. As a result, most of the subsequent results are based either on a limited time series of CDX/iTraxx data or on a much more extensive time series of BCDS data of different frequencies and duration (monthly between May 1994 and November 2006, bi-monthly from May 1997 and daily from January 2002 onward).

Choice of Spread Model and Maximum Likelihood Estimation (MLE)

Following Prigent et al. (2001), we have estimated a general mean reverting (MR) stochastic process in order to gain some insight into the family of models suitable for modelling credit spreads:

$$(1)\quad dx = (a + bx)dt + \sigma x^{\gamma} dW .$$

The MR level (long term spread, or LTS) is given by $\theta = -\frac{a}{b}$ and the MR speed by $\beta = -b$ · γ is a scalar measuring the degree of non-linearity between the level of spreads and volatility. Depending on the choice of γ, several well-known models can be derived. For example, $\gamma = 0$ leads to the Vasicek (1977) process, $\gamma = ½$ results in the Cox, Ingersoll and Ross (1985) (CIR) process and $\gamma = 1$ the Brennan & Schwartz (1980) process. γ therefore tells us a lot about the relationship between spread volatility and level.

By estimating this general process on all datasets, we obtain estimates for γ consistently above 80%, indicating that a Brennan & Schwartz process is reasonable. In order to enhance the comparative analysis, we restrict the model to $\gamma = 1$ and estimate the process on different datasets. Table 3 above shows a comparison of daily CDS index

[11] This approximation may have an impact on the tail of the loss distribution slightly. However, our tests confirm that this does not affect our conclusions on default rate sensitivity.

Table 3: MLE Results for Daily BDCS and CDS Index Data, March 2005 to January 2007

Data	a	b	σ	LTS (bp)	MRS (%)
CDX	0.51	-1.07	36%	47	107%
iTraxx	0.31	-0.86	33%	36	86%
BCDS BBB	8.04	-16.85	35%	48	1685%
BCDS A	6.45	-25.04	36%	26	2504%

LTS = long term spread. MLE = maximum likelihood estimation. MRS = mean reversion speed.

Table 4: MLE for BCDS Data of Different Frequency and Time Periods, Restricting γ = 1

Data	a	b	σ	LTS (bp)	MRS (%)
BCDS A (bi-monthly)	0.27	-0.61	60%	45	61%
BCDS A (monthly)	0.18	-0.44	58%	40	44%
BCDS BBB (daily)	0.23	-0.25	39%	91	25%
BCDS BBB (bi-monthly)	0.34	-0.38	45%	88	38%
BCDS BBB (monthly)	0.21	-0.27	46%	78	27%

LTS = long term spread. MLE = maximum likelihood estimation. MRS = mean reversion speed.

and BCDS data (five-year term) over the period March 2005 to January 2007, whereas Table 5 focuses on the longer time series available for the BCDS data (five-year term).

Table 3 reveals fairly consistent estimates for spread volatilities, around 35%. Mean reversion speeds are high (85% to 100%) for CDS data and extremely high for BCDS data.

Table 4 shows A and BBB estimates for daily, bi-monthly and monthly data, covering the periods 2002–2006, 1997–2006 and 1994–2006, respectively.

Overall, we can summarise the findings on the main model parameterisation as follows:

- **Spread volatility** ranges from 40% to 45% for BBB data, 60% to 70% for "A" spreads and 33% to 36% for a very short time series of CDS index observations.
- **Mean reversion speeds** are fairly consistent, between 25% and 40% for BBB data and above 40% for "A" data.
- **Long term average spreads** are very difficult to assess for various reasons, such as the change in liquidity in credit markets in recent years and the limited amount of CDS data. We believe that an assumption between 65 bp and 80 bp for a typical CPDO transaction referencing 50% CDX and 50% iTraxx is reasonable. Empirically, longer-term BCDS spreads average at around 80 bp to 90 bp for BBB spreads and 45 bp for "A" spreads (see Table 5).

Table 5: Average BCDS Spreads (in bp)

	Monthly (1994–2006)	Bi-Monthly (1997–2006)	Daily (2002–2006)
BBB	78	88	91
A	40	45	46

Spread Simulation

For CPDOs and other market-value structures, spreads often need to be simulated for long horizons (e.g., the ten-year term of a CPDO note). In order to gain further insight into the suitability of a model and its parameterisation, we conducted a series of goodness-of-fit tests that compare the statistical properties of the historical time series with the properties the simulation model produces.

As the above analysis indicates that a MR process where volatility is related to the level of spreads is adequate (indicated by strong levels of MR and high values of γ), we have chosen a common process discussed in detail in Schwartz (1997) and Geman (2005):

$$(2) \quad \frac{dS}{S} = \beta\left(\xi - \ln S\right)dt + \sigma dW.$$

Introducing the new variable $x = \ln(S)$, leads to $dx = \beta(\theta - x)dt + \sigma dW$, where $\theta = \xi - \frac{\sigma}{2\beta}$.

The solution to the stochastic process in equation (2) is given by

$$x(t) = x(s)e^{-\beta(t-s)} + \theta\left(1 - e^{-\beta(t-s)}\right) + \sigma\int_{u=s}^{t} e^{-\beta(t-u)}dW(u).$$

One drawback of this model is that the simulation does not converge to the required equilibrium spread level. For this reason, we simulate the adjusted process (3)

$$x_{i+1} = x_i e^{-\beta(t_{i+1}-t_i)} + \left(\ln \bar{S}^{LTS} - \frac{\sigma^2}{4\beta}\right)\left(1 - e^{-\beta(t_{i+1}-t_i)}\right) + \sigma\sqrt{\frac{1}{2\beta}\left(1 - e^{-2\beta(t_{i+1}-t_i)}\right)}Z_i,$$

where Z_i is dawn from a standard normal distribution. Computing $S_i = e^{x_i}$ ensures that the simulation model converges to a desired long term spread ($\bar{S}^{LTS}$).[12]

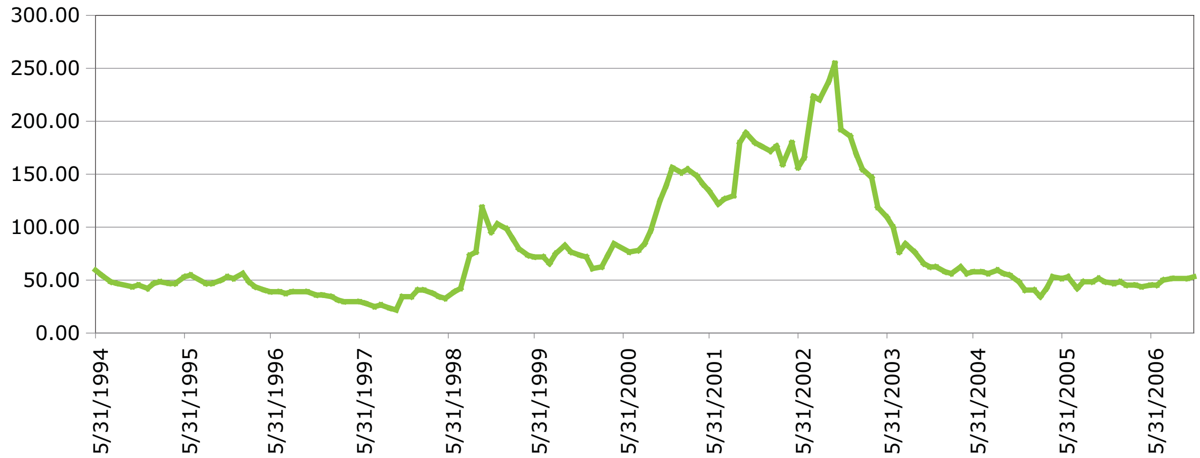
In the following example, we simulate this model and test its ability to replicate the behaviour observed in the time series data. We focus on monthly BCDS data, but the main results hold across all datasets.[13]

Figure 2 plots the time series for BBB BCDS data, and Table 6 reports some summary statistics on spread returns. Returns are calculated over one month, three months, six months and one year using overlapping intervals (moving forward in monthly time steps). For example, Table 6 shows that the average annualised three-month return had a volatility of 58%. In addition, the lowest monthly return was -24%, and the 99th percentile of six-month returns was 180%.

In the following example, we simulate equation (3) for different assumptions on spread volatility, as this is an important parameter for CPDO risk analysis.[14] We then compare some summary statistics with Figure 2 and Table 6 below. We start with a popular parameterisation, assuming LTS = 80 bp, MRS = 40%, Volatility = 25%, Starting Spread = 35 bp.

Figure 3 on the following page plots the average spread paths, as well as the minimum, maximum and various percentiles computed from the 10,000 simulations conducted over a horizon of ten years (120 periods).

Figure 2: BBB BCDS History (Five-Year Term)



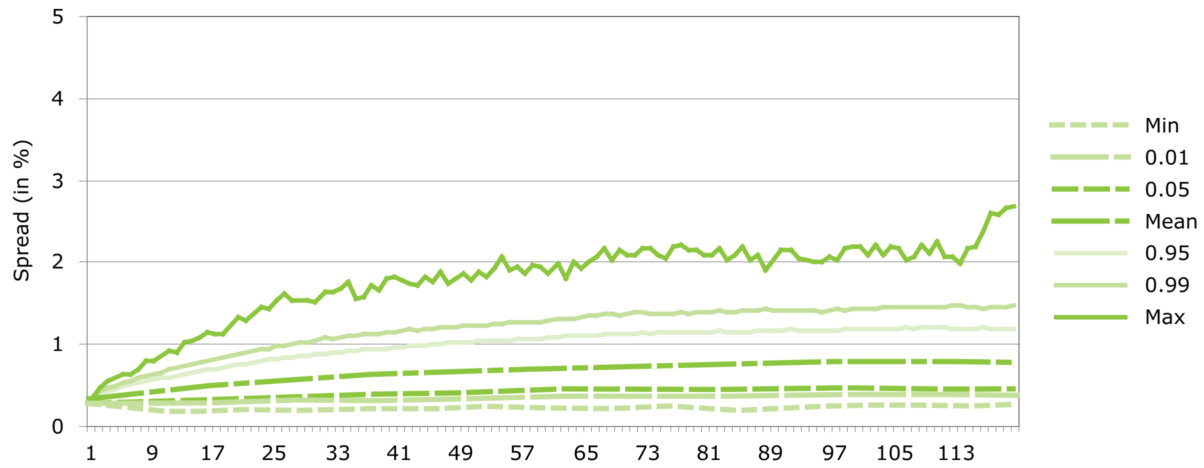
Source: Lehman Brothers.

Table 6: Credit Spread Returns for Monthly BBB BCDS Data

Returns		Time Horizon (Months)			
		1	3	6	12
Standard deviation (annualised)		46%	58%	62%	67%
Minimum		-24%	-36%	-54%	-74%
Percentile	0.01	-22%	-33%	-51%	-68%
	0.05	-15%	-26%	-43%	-62%
	0.95	21%	49%	83%	138%
	0.99	54%	110%	180%	220%
Maximum		75%	184%	243%	428%

[12] Generalisations toward a jump-diffusion setting can be easily incorporated following, for example, Prigent et al. (2001). Furthermore, multiple bond indices can be modeled or simulated separately following equation (3) by introducing correlation between the normal random variables (i.e., $Corr(Z_i, Z_j) = \rho_{ij}$).
[13] See Jobst et al. (2007) for details.
[14] Note that the impact of changes in other model parameters is available from the authors on request.

Figure 3: Spread Simulation



Note: Simulated spread quantiles for a mean reverting process with 25% volatility, LTS of 80 bp, MRS of 40% and initial spread of 35 bp (monthly discretisation).

The figure reveals that this model does not reproduce spread levels observed over the last ten years, with the maximum simulated paths barely reaching levels observed in 2002 (i.e., 250 bp). Furthermore, the 99th percentile produces spreads below 150 bp, while the 1st percentile does not fall below 23 bp over the entire simulation horizon.

In addition to the spread-level comparison, we also gain an understanding of the simulation model's ability to capture changes in spreads (returns). In order to do so, we have computed one-, three-, six- and 12-month returns and the same return statistics as those shown in Table 6 (using overlapping observations) for all simulation paths. Table 7 shows the average (left-hand side (LHS)) of these statistics across each simulation path, as well as the 99th percentile (right-hand side (RHS)) of all paths (which gives an indication of the most extreme 1% outcomes within the simulations).

The table reveals that the simulated percentiles do not match the empirical estimates in Table 6 very well for any horizon, consistently producing less extreme outcomes than those observed historically.

We now increase volatility to a more reasonable 35%. In our opinion, a spread volatility of less than 35% is unjustified, and a critical CPDO risk analysis should consider volatilities in the range of 35% to 40%. With all other parameters unchanged, this model reveals a better agreement to empirical observations (see Figure 4 and Table 8 on the following page).

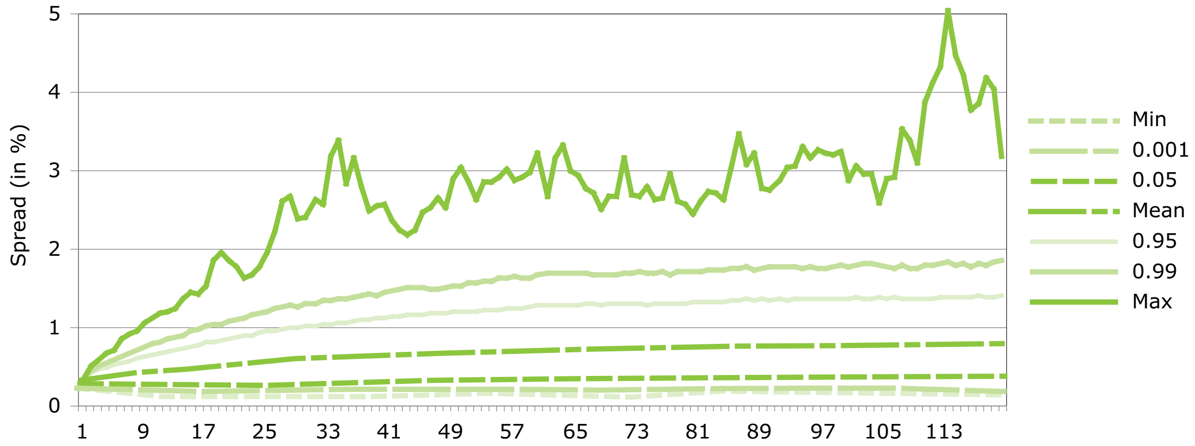
The spread simulation seems to produce more extreme outcomes in line with experience, where the 99th percentile converges on approximately 200 bp. Assessing the return estimates also shows a significant improvement. Note, however, that even using a volatility of 35%, the average of all simulations (LHS) appears low compared with

Table 7: Simulated Changes in Credit Spreads and Returns for a Mean Reverting Process

Returns		Averages Across Simulations Time Horizon (Months)				1/99 Percentile of Simulations Time Horizon (Months)			
		1	3	6	12	1	3	6	12
Standard deviation (annualised)		25%	25%	25%	26%				
Minimum		-16%	-25%	-31%	-35%	-23%	-35%	-45%	-54%
Percentiles	0.01	-15%	-23%	-29%	-33%	-19%	-23%	-42%	-52%
	0.05	-11%	-17%	-22%	-26%	-13%	-22%	-31%	-44%
	0.95	13%	25%	38%	58%	17%	33%	56%	105%
	0.99	19%	35%	52%	77%	26%	53%	85%	141%
Maximum		21%	39%	56%	81%	32%	62%	93%	149%

Note: Assuming 25% volatility, LTS of 80 bp, MRS of 40% and initial spread of 35 bp (monthly discretisation).

Figure 4: Spread Simulation



Note: Simulated spread quantiles for a mean reverting process with 35% volatility, LTS of 80 bp, MRS of 40% and initial spread of 35 bp (monthly discretisation).
Source: Lehman Brothers.

Table 8: Simulated Changes in Credit Spreads and Returns for a Mean Reverting Process

Returns		Averages Across Simulations Time Horizon (Months) 1	3	6	12	1/99 Percentile of Simulations Time Horizon (Months) 1	3	6	12
Standard deviation (annualised)		35%	36%	36%	36%				
Minimum		-22%	-33%	-41%	-46%	-31%	-46%	-57%	-66%
Percentiles	0.01	-20%	-31%	-38%	-45%	-27%	-31%	-54%	-65%
	0.05	-15%	-23%	-30%	-36%	-18%	-30%	-42%	-56%
	0.95	19%	35%	53%	82%	24%	49%	83%	154%
	0.99	27%	52%	77%	112%	38%	79%	134%	219%
Maximum		21%	39%	56%	81%	47%	92%	149%	236%

Note: Assuming 35% volatility, LTS of 8 0bp, MRS of 40% and initial spread of 35 bp (monthly discretisation).

empirical data, whereas the 99th percentile (RHS) covers the data reasonably well.

Summary of Simulation Insights

This simulation analysis confirms the MLE results; namely, that volatilities below 35% are unjustified and a lower assumption (e.g., 25%) may significantly underestimate credit spread risk, rendering simulation models incapable of producing outcomes seen during the period of spread widening from 1998 to 2002. This conclusion also holds for all other datasets, including daily CDX and iTraxx analysis, and can be found in Jobst et al. (2007).

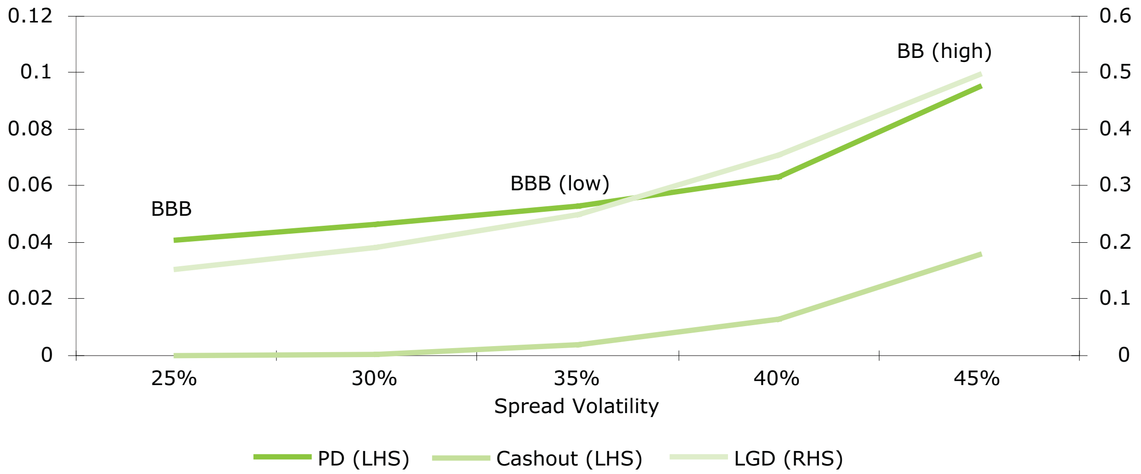
2.3 IMPACT OF CREDIT SPREAD PARAMETERS ON CPDO RISK AND RATINGS

After gaining insights into adequate credit spread parameterisations, we now investigate the impact of changes in these parameters on a hypothetical CPDO transaction. We consider a standard static CPDO that is long semi-annual iTraxx and CDX five-year risk. The CPDO pays a coupon of LIBOR plus 200 bp, cashes out at 10% of par, can be levered 15x at most and matures in ten years. Standard fees are assumed, including a 1% upfront fee and various ongoing administration fees. Unless otherwise stated, we assume that the CPDO is issued at an average spread of 35 bp for the combined index (50% CDX, 50% iTraxx).

Sensitivity to Spread Volatility

We consider firstly the sensitivity of our standard CPDO to changes in spread volatility. We plot the probability that the CPDO will fail to redeem par on or before maturity (PD), the probability of cash-out (CO) and the subsequent loss given default (LGD), using spread volatilities in the range 25% to 45%. Note that we also attach an indicative model-implied rating, derived under a first-dollar-of-loss (or PD) ratings approach. These model-implied ratings are simply derived by comparing the PD (failure to redeem par) to DBRS's benchmark default probabili-

Figure 5: PD, CO and LGD by Volatility (Average Time Decay)



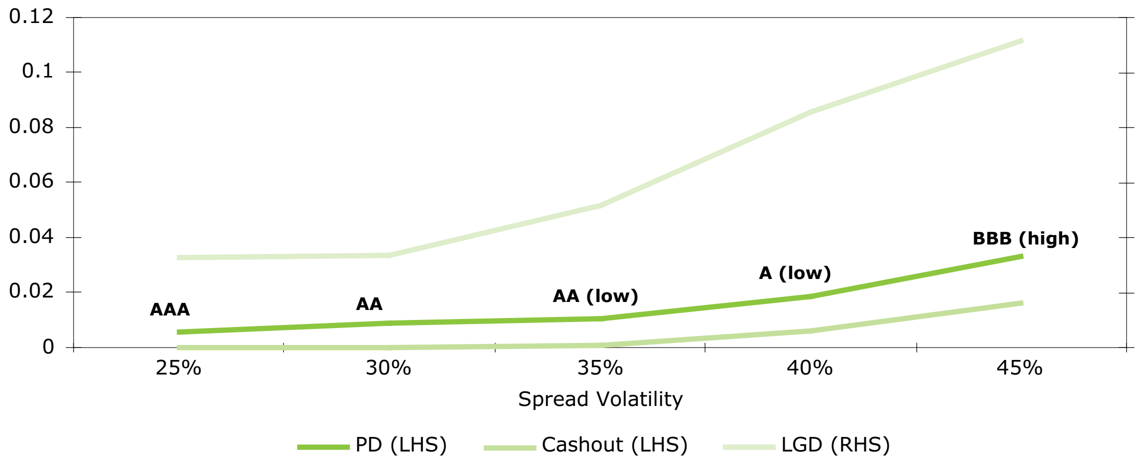
Note: Sensitivity of a CPDO (200 bp, 15x) to changes in spread volatility. Standard assumptions are applied to all other parameters (MRS = 40%, LTS = 70 bp, α = 0.45 (average time decay) and S_0 = 35 bp).

ties (see Appendix 3) and should not be mistaken as the rating DBRS would assign to such a transaction. The main objective of this investigation is to examine the potential mark-to-model sensitivity of CPDO ratings.[15]

Figure 5 above shows the three risk measures under an assumption of moderate, constant time decay $\alpha(S) = \alpha = 0.45$, corresponding to approximately 2.25% or 4.5% roll-down over three or six months, respectively. Standard assumptions of MRS = 40% and LTS = 70 bp are applied for the remaining parameters.

Figure 5 reveals that increasing volatility increases all risk measures significantly. Cash-out events typically result from credit losses and significant spread widening, which cannot be attained under a very low volatility assumption. Hence, all risk measures would be underestimated compared with realistic volatility assumptions in the range of 35% to 40%. For these levels of volatility, we observe a significant increase in cash-out events, resulting in very high LGDs that have an impact on the overall CPDO LGD estimate. The figure shows that the probability of not redeeming at par more than doubles, from 4% to approximately 9.5% (LHS), when volatility increases from 25% to 45%, whereas LGD more that triples, from 15% to 50% (RHS). This tripling in LGD stems from an increase in the probability of cash-out from 0% to 3.6%, respectively. Under a PD

Figure 6: PD, CO and LGD by Volatility (High Time Decay)



Note: Sensitivity of a CPDO (200 bp, 15x) to changes in spread volatility. A higher time decay of $\alpha = 0.7$ is assumed and standard assumptions are applied to all other parameters (MRS = 40%, LTS = 70 bp and S_0 = 35 bp).

[15] Owing to the computational complexity, these results are based on a limited number of simulations (10,000). While convergence is not guaranteed for such a low number of simulations, selected tests with a much higher number show that our general conclusions still hold. We also assume that default and credit spread processes are independent, and further work is needed to adequately incorporate these risk factors and assess their impact on CPDO risk and ratings analysis.

ratings framework, the indicative rating would vary between BBB and BB (high), respectively.

Figure 6 on the previous page repeats the analysis under a different (and popular) assumption of $\alpha(S) = \alpha = 0.7$ (or approximately 7% roll-down benefit over six months).

This exercise confirms the previous results, but shows an even more dramatic picture. A relatively low volatility assumption of 25%, combined with a higher roll-down assumption of 7% over the index holding period of 6 months, reduces the PD to 0.5%, which is consistent with a AAA rating. Across various volatility assumptions, an increase of roll-down from 0.45 to 0.7 essentially decreases the PD by 3 to 7 times. These results give a first indication of the importance of not only specifying the parameters of the stochastic spread process correctly, but also of modelling the dynamics of the (local) term structure adequately. A more detailed discussion follows in Section 2.4.

Sensitivity to Long Term Spread (LTS)

Next, we consider the sensitivity of varying the LTS between 60 bp and 80 bp under a moderate time-decay assumption, leaving other model parameters unchanged ($\sigma = 35\%$, $\alpha(S) = \alpha = 0.45$, $MRS = 40\%$).

Figure 7 reveals once again a significant impact on CPDO risk measures, with opposite effect on PD and LGD however. Overall, with increasing LTS, the PD decreases, whereas the probability of a cash-out event increases slightly, resulting in higher LGDs. These results appear intuitive, in that a higher LTS leads to more extreme spread simulations, which in turn result in an increase in extreme NAV declines and cash-out rates. On the contrary, a higher mean spread leads on average to higher contractual spreads on each roll date, when protection on the old index is bought back and the CPDO sells protection on the new, on-the-run index contracts. Overall, however, the impact of changes in LTS appears more moderate (and linear) compared with changes in volatility.

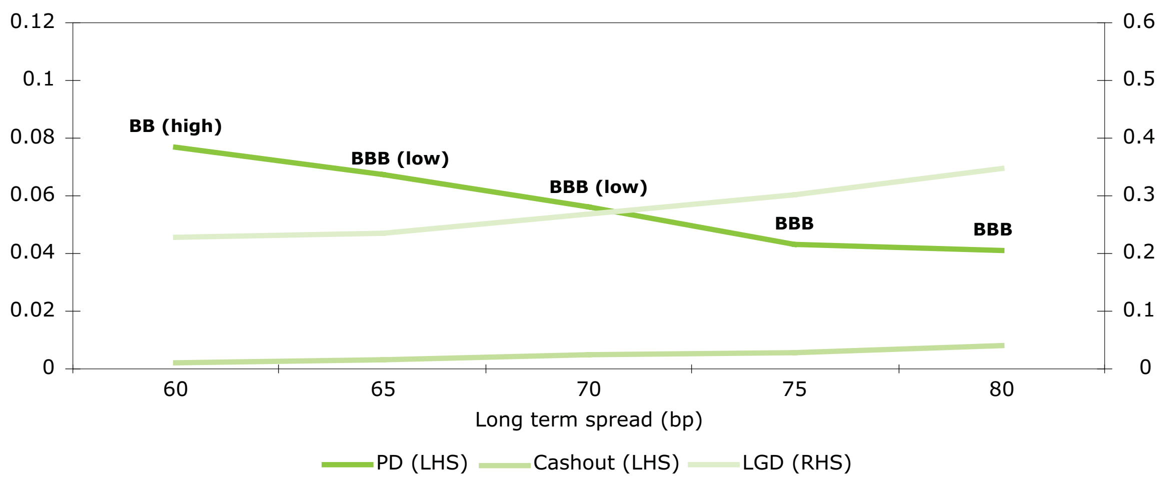
Sensitivity to Mean Reversion Speed (MRS)

We now assess the impact of varying MRS from 25% to 45% by leaving all other parameters unchanged: $\alpha = 35\%$, $\alpha(S) = \alpha = 0.45$, $LTS = 70$ bp. Figure 8 reveals a very high sensitivity of PD to MRS, whereas the LGD is fairly insensitive. This stems from the fact that the ratio of cash-out events to PD (failure to redeem) is quite stable, varying between 7% and 8% under all different MRS assumptions considered. In contrast, when varying volatility between 25% and 40%, the ratio of cash-out to PD changes from 0% to approximately 40%, which has a considerable impact on the LGD.

2.4 MODELLING THE LOCAL TERM STRUCTURE OF CREDIT SPREADS

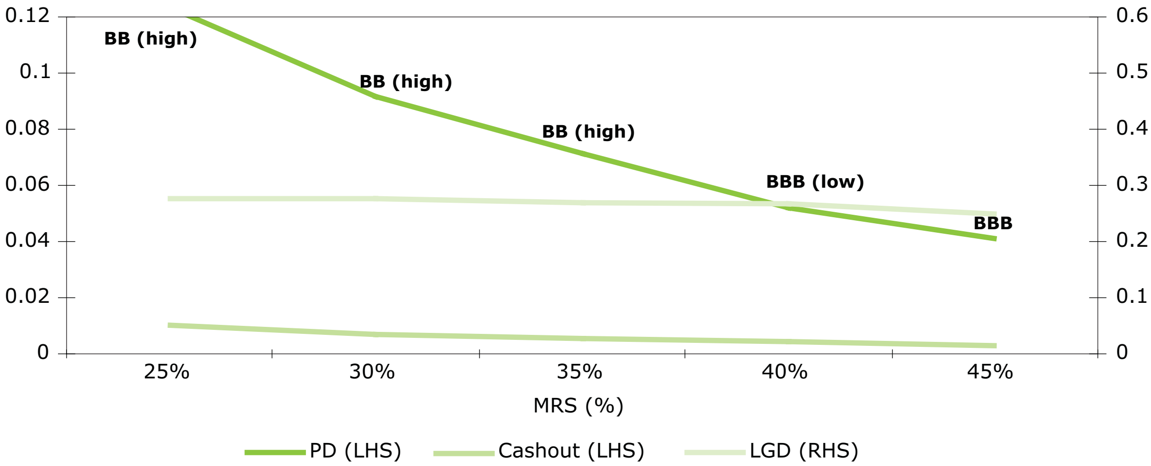
As indicated above, index-based static CPDOs are required to buy back protection on the roll date and sell protection on the new on-the-run index. As CPDOs are market-value products, the steepness of the credit curve plays an important part in determining the MTM gain the strategy cumulates as a result of rolling down the curve over each six-month exposure period in between roll dates. Furthermore, preliminary results in Section 2.3 have shown that CPDO risk measures and ratings may be very sensitive to such assumptions.

Figure 7: PD, CO and LGD by Volatility (Average Time Decay)



Note: Sensitivity of a CPDO (200 bp, 15x) to changes in long-term spread.
Standard assumptions are applied to all other parameters (σ = 35%, MRS = 40%, α = 0.45 and S_0 = 35 bp).

Figure 8: PD, CO and LGD by MRS (Average Time Decay)



Note: Sensitivity of a CPDO (200 bp, 15x) to changes in mean reversion speed. Standard assumptions are applied to all other parameters (σ = 35%, LTS = 70%, α = 0.45, S_0 = 35 bp).

Modelling the whole term structure dynamics of credit indices is a non-trivial exercise for various reasons, among them the availability of spread data for maturities below three and above seven years. While this is an area of ongoing research for DBRS, we focus here on term-structure dynamics based on three-, five- and seven-year observations, reflecting the fact that a CPDO referencing five-year CDX and iTraxx is typically exposed to the [5.25y, 4.75y] part of the credit curve.

Empirical Observations

In this section, we estimate and then simulate a constant maturity T^I-year spread $S_t(T^I)$ and reflect term-structure effects by adjusting the simulated spread according to the following relationship:

$$S_t(T) = S_t(T^I)\left(\frac{T}{T^I}\right)^{\alpha(S)}.^{16}$$

Here, $\alpha(S)$ denotes an adjustment or time-decay factor that depends on the level of spreads (i.e., the steepness of the credit curve is a function of spread levels). We start this analysis by considering CDX, iTraxx and BCDS spreads for maturities T = three, five and seven years and compute a time series of time-decay factors α_t.

iTraxx

Figure 9 shows the five-year iTraxx composite spread and the time series of three-to-five-year and five-to-seven-year slope factors over the period March 2005 to January 2007. It is apparent that there is an inverse relationship between slope and spreads, reflecting the fact that when spreads are wider, perceived default risk is higher in the near term, leading to a flatter term structure. Over the period reported, five-year spreads are tight, averaging at around 35 bp. This results in relatively high time-decay parameters α_t, averaging at around α_t^{35} = 1.09 for the three-to-five-year part of the curve and α_t^{57} = 0.78 for the five-to-seven-year data. Note that in May 2005, when spreads widened to around 60 bp, these time-decay parameters fell to a range of 0.5% to 0.6%. The correlation between five-year spread and slope is minus 70% and minus 90%, respectively, with a 75% correlation between α_t^{35} and α_t^{57}.

Lehman Brothers BCDS Data

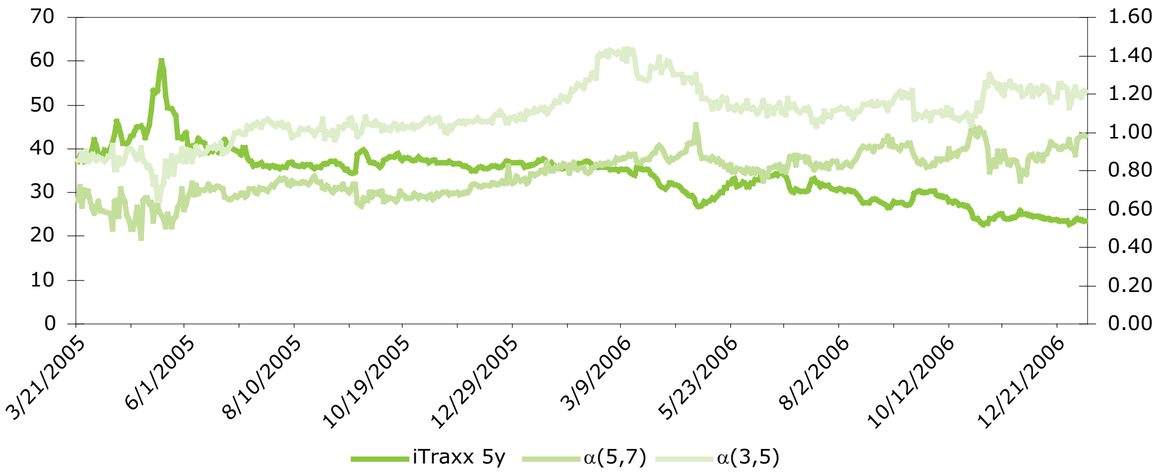
Once again, the BCDS dataset provided by Lehman Brothers provides a good proxy to CDS data and spans a much longer time period, particularly for monthly observations. Figure 10 shows the time series of spreads and time-decay factors for monthly BBB observations.

It is apparent that the inverse relationship holds across the whole sample period,[17] and one can see that BCDS term structures appear on average flatter than CDX or iTraxx term structures (e.g., they never exceed 1). Furthermore, the difference between slopes from three-to-five and five-to-seven years also appears to be lower, which is confirmed by a 99% correlation between α_t^{35} and α_t^{57} and is shown Table 9, which looks at the average time-decay factors for different datasets.

[16] As an example, a constant α = 0.4 corresponds to approximately 4% reduction in spreads over six months or 2% over a three-month period. In general, α_t/20 is a very good approximation to a linear estimate (in %) over a three-month period and α_t/10 over a six-month period

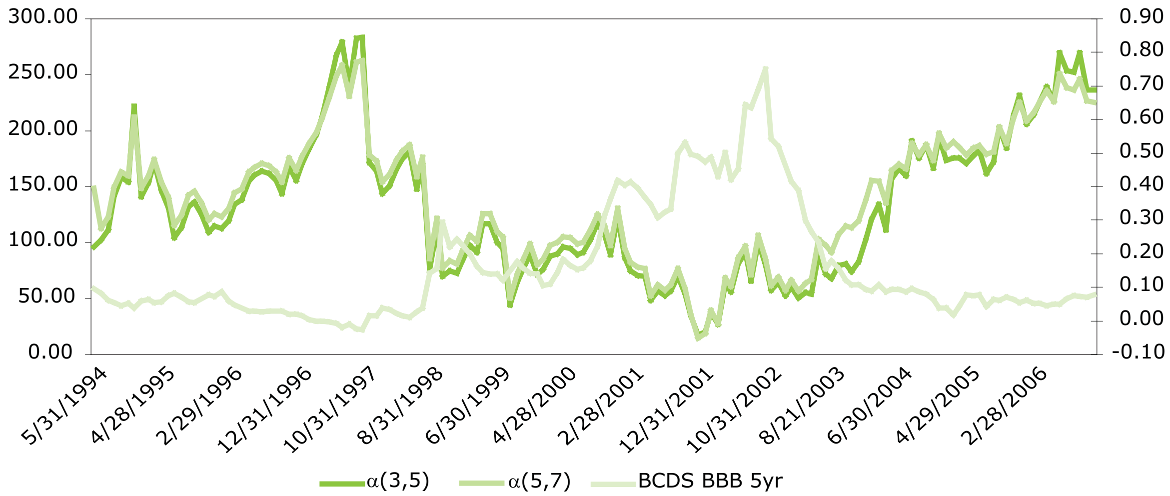
[17] The results also hold for all other datasets considered, including daily BCDS and CDX data (see Jobst et al. (2007) for further details).

Figure 9: iTraxx Europe: Spread (LHS) Versus Slope (RHS), March 2005 to January 2007



Note: Historic spread and time decay factors (a measure of slope/steepness of the three-to-five- and five-to-seven-year part of the credit curve) for iTraxx Europe.

Figure 10: BCDS BBB Monthly: 1994–2006



Note: Historic spread and time-decay factors (a measure of slope/steepness of the three-to-five-year and five-to-seven-year part of the credit curve) for monthly BBB BCDS data.
Source: Lehman Brothers.

Modelling Time Decay (Roll-Down)

Given the strong relationship between spreads and the steepness of the credit curve across all datasets, we conducted a more in-depth regression analysis that incorporates the link between spreads and slope. We focus on modelling the relationship between five-year spread and five-to-seven-year slope factor, which results in a conservative modelling approach, as α_t^{57} tends to be on average lower than α_t^{35} for CDS indices. Whether or not this is a result of CDS market technicals or factors specific to our data sources needs to be investigated further.

Across various datasets, we have considered a number of different regression models, among them two very simple specifications that appear to yield good results.

Model 1 is a very simple regression model that is extended to include an autoregressive term in Model 2:

$$\text{Model 1: } \alpha_t = c + \beta_1\left(\frac{1}{\ln(S_t)}\right) + \varepsilon_t$$

$$\text{Model 2: } \alpha_t = c + \beta_1\left(\frac{1}{\ln(S_t)}\right) + \beta_2\alpha_{t-1} + \varepsilon_t$$

Tables 10 and 11 show the results of the regression analysis for monthly BCDS data and daily iTraxx data, respectively.

Table 9: Average Time-Decay Factor for BCDS and CDS Index Histories

Data	Average α (3,5)	Average α (5,7)	Average of α (5,7) and α (3,5)
BBB BCDS (m)	0.34	0.36	0.35
BBB BCDS (d)	0.38	0.40	0.39
A BCDS (m)	0.50	0.50	0.50
CDX (from March 2004)	0.95	0.50	0.73
iTraxx (from March 2005)	1.09	0.78	0.94
BBB BCDS (d from March 2005)	0.64	0.62	0.63

The results indicate that both models fit the observed data very well, with R^2 = 70% for Model 1 and 90% to 92% for Model 2. For both models, all coefficients are statistically significant at a 99% level as indicated by the relevant *t*-statistic. The pickup in R^2 for the autoregressive model is less relevant for our application, where the relationship between spreads and the term structure is incorporated within a simulation framework. For that reason, root mean square errors are computed, which can be seen as a simple out-of-sample performance measure. The results indicate that the additional complexity of an autoregressive component may not add significant value within our simulation framework, and we proceed in the remainder of this document with Model 1. Note also that for Model 1, the coefficients are very similar despite the difference in time periods and datasets considered.

Figure 11 shows the level of time decay α generated by different regression models as credit spreads vary. We focus here for completeness on CDX and daily BBB BCDS data. In line with our empirical results, the higher the credit spread, the flatter the term structure, as indicated by a lower α. Figure 11 also shows an Aggregate Model that aims to produce a single model combining the features of various models estimated across all different data sources. In general, the time series for CDS indices is limited and almost entirely corresponding to a tight spread environment, resulting in reasonably good estimates for low to moderate spreads. BCDS data, in particular the monthly time series, provide a good model across the spread spectrum, from very tight spreads before 1998 and after 2003 to high spreads from 1998 to 2002.

The Aggregate Model corresponds to $\alpha = -1.79 + 9(1/\ln(S_t))$. This model reflects the CDS-based dynamics for very tight spreads and is more in line with BCDS data for moderate to wide spreads.[18]

Table 10: Regression Results for Time-Decay Models to Monthly BCDS Data (BBB)

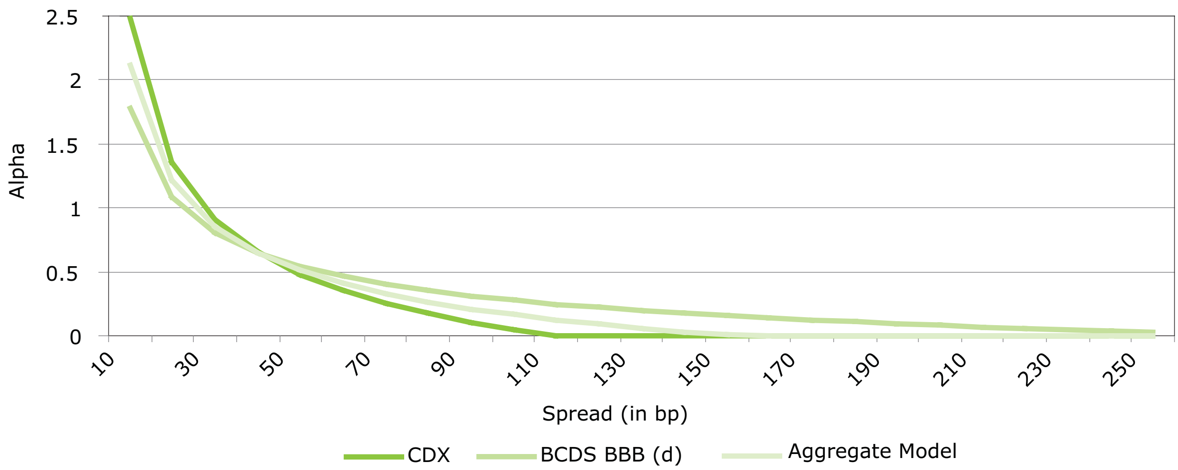
	Model 1			Model 2 (Autoregressive)		
Variable	Coefficient	Standard Error	*t*-Statistic	Coefficient	Standard Error	*t*-Statistic
c	-0.89	0.07	-13.34	-0.25	0.06	-4.36
β_1	5.16	0.27	18.89	1.38	0.29	4.83
β_2				0.76	0.05	16.12
R^2		70%			89%	
Root mean squared error (RMSE)		1.1%			1.2%	

Table 11: Regression Results for Time-Decay Models to Daily iTraxx

	Model 1			Model 2 (Autoregressive)		
Variable	Coefficient	Standard Error	*t*-Statistic	Coefficient	Standard Error	*t*-Statistic
c	-1.00	0.05	-18.36	-0.13	0.04	-3.75
β_1	6.23	0.19	32.80	0.83	0.17	4.81
β_2				0.87	0.02	37.08
R^2		70%			92%	
Root mean squared error (RMSE)		0.39%			0.44%	

[18] Another way of interpreting the model is that the BCDS data provided by Lehman Brothers corresponds to a constant credit-quality index and as such doesn't reflect possible credit quality deterioration and subsequent spread widening, which would reduce the benefit of time decay. By lowering the slope in a high spread environment, we implicitly capture credit deterioration.

Figure 11: Comparison of Time-Decay Functions by Credit Spread



Note: Comparison of various time-decay (regression) models for different data sources.

Table 12 tabulates the slope factor across a range of spreads, supplementing Figure 11. For example, in a very tight trading environment, we would assume a slope in the range of 0.8 to 0.9, broadly in line with data on CDX and iTraxx. At a spread level of 60 bp, the assumption of 0.41 may be viewed as slightly conservative, given the iTraxx slope was at around 0.6 when spreads were trading at this level in May 2005. However, concerns relating to the impact of high volumes of CPDO issuance on the term structure warrant a conservative treatment of this parameter.

Figure 12 shows the fit of the aggregate model to monthly BCDS.[19]

2.5 IMPACT OF ROLL-DOWN ON CPDO RISK AND RATINGS

While the previous section provides ample empirical support for jointly modelling the relationship between credit spreads and the steepness of the credit curve and Section 2.3 indicated that CPDO performance may be very sensitive to the assumption of curve steepness, we present a more detailed analysis of this sensitivity throughout this section.

Assuming the benchmark CPDO underlying all previous sensitivity results and the standard assumptions (σ = 35%, *LTS* = 70 bp, *MRS* = 40%), we vary the time-decay parameter α between 0.4 and 0.8 (i.e., assuming 4% and 8% roll-down over a six-month period) and plot PD, CO and LGD in Figure 13.

Figure 13 confirms the indications of Section 2.3 in that CPDO risk as measured by the inability to redeem par (PD) is extremely sensitive to the term-structure assumption. This of course has also a very strong impact on model-implied ratings, varying from AAA with α = 0.8 to non-investment grade (NIG) with α = 0.4. Notice that the former estimate is in line with CDS index estimates based on a very short time series of recent data, whereas the latter is more consistent with a much longer history of BCDS data.

Figure 14 shows the impact of modelling the inverse relationship between spreads and slope as outlined in the Aggregate Model. We also compare this more dynamic model with a risk analysis based on a constant α, where α is chosen to be in line with the average produced by the "coupled" spread-slope simulation. In order to gain some insight, we compare these two models for different levels of spread volatility.[20]

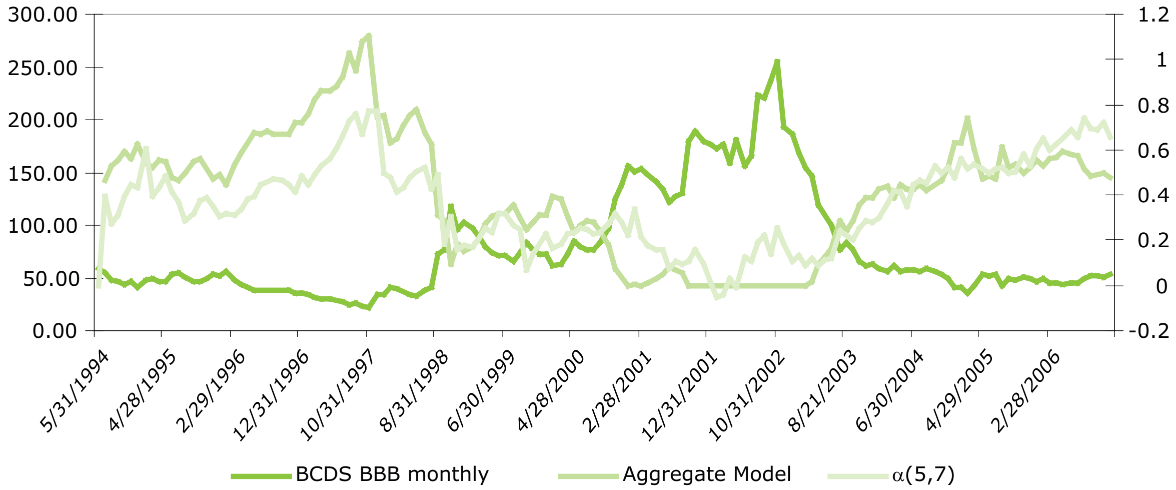
The impact of dynamically modelling spreads and slope is interesting. It appears that in a tight starting spread environment, the Aggregate Model

Table 12: Time-Decay Factor for Different Spread Levels for the Aggregate Model

Spread (bp)	20	30	40	50	60	70
Time-decay factor	1.21	0.86	0.65	0.51	0.41	0.33

[19] In these figures, it is assumed that the slope cannot become negative.
[20] Note that in the present simulation study, α is a deterministic function of credit spreads. However, extensions that incorporate uncertainty are straightforward.

Figure 12: BCDS BBB Monthly



Note: Historic spread, time-decay factors and Aggregate Model fit for monthly BBB BCDS data.
Source: Lehman Brothers.

reduces the overall probability of not cashing in, particularly under a low-volatility assumption, whereas LGD and CO both increase, particularly for higher volatilities. This matches our intuition, as the CPDO benefits more in a low-spread environment (through a steeper curve) and is penalised more in a wide-spread environment. Hence, the number of simulation paths where the CPDO just doesn't manage to cash in may decline when reflecting the dynamics adequately, whereas more extreme spread paths may lead to cash-out events as a result of less roll-down benefit for high spreads. Of course, in more volatile environments, the ratio of not redeeming at par to cash-out is smaller, leading to a convergence of both models at a PD level and a divergence for LGD. Overall, we can see that the ratings impact of a detailed modelling of the spread dynamics compared with a constant assumption is between one and two notches.

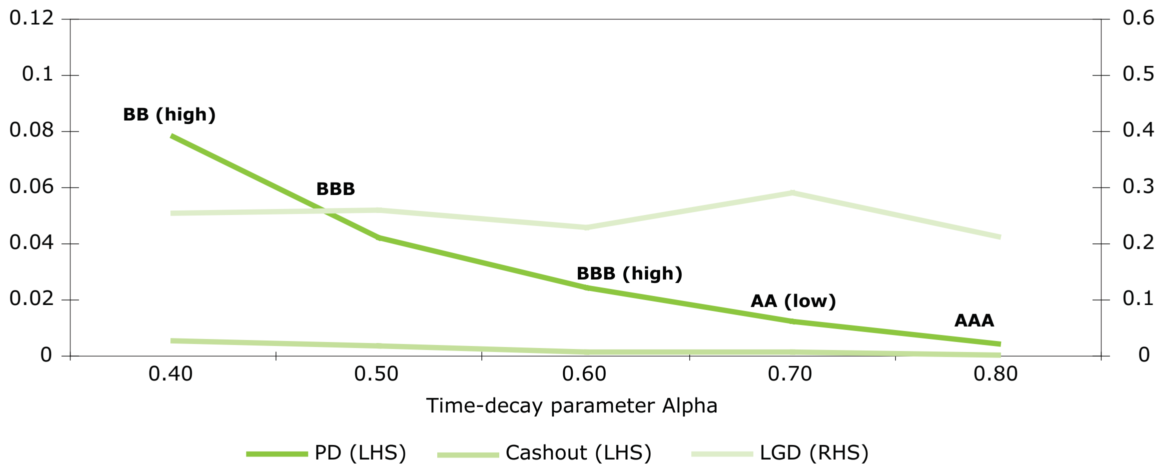
2.6 ADDITIONAL SENSITIVITIES: BID-OFFER SPREAD, DEFAULT RATE, NOTE COUPON AND LEVERAGE

Bid-Offer Spread Sensitivity

Index-based CPDOs that roll into the new index every six months incur significant amounts of transaction costs (frequently expressed and implemented as the difference between the bid and the offer spread). Figure 15 shows the bid-offer spread on iTraxx Europe Series 1 to Series 5 (June 2004 to September 2006).

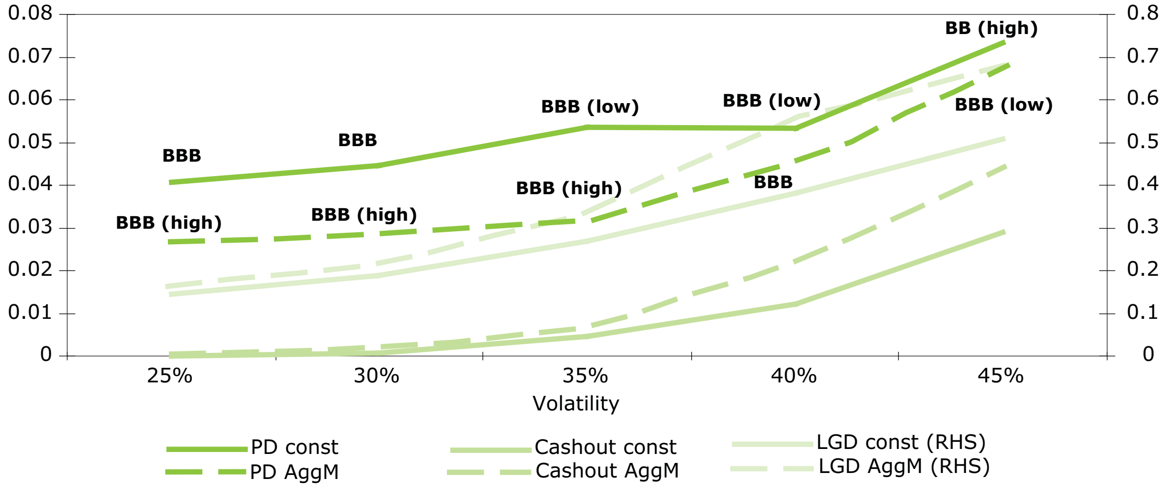
Figure 15 shows clearly how liquidity in this market improved with bid-offer spreads

Figure 13: PD, CO and LGD by Time-Decay Assumption



Note: Sensitivity of a CPDO (200 bp, 15x) to changes in time-decay parameter.
Standard assumptions are applied to all other parameters (σ = 35%, LTS = 70%, MRS = 40%, S_0 = 35bp).

Figure 14: PD, CO and LGD by Volatility – Constant Versus Dynamic Time Decay



Note: Comparison of a constant time-decay assumption versus a linked spread/time-decay approach (Aggregate Model = AggM) for a CPDO. Standard assumptions are applied to all other parameters. The constant is chosen to match the average of AggM (σ = 35%, LTS = 70%, MRS = 40%, S_0 = 35 bp).

continuously tightening from series to series. The average bid-offer spreads for Series 1 to Series 5 are 1.23 bp, 1.25 bp, 0.49 bp, 0.35 bp and 0.24 bp, respectively. Essentially bid-offer spreads declined from more than 1 bp to below 0.25 bp. It is also apparent that once the index series goes off the run, bid-offer spreads widen again, although the impact seems to get smaller over time as liquidity improves. Within our CPDO model, a constant bid-offer spread is assumed,[21] and Figure 16 shows the sensitivity of CPDO risk and ratings to bid-offer spreads.

We can observe a significant sensitivity to bid-offer spreads for index-based CPDOs that roll their entire levered position every six months. According to Figure 15, it appears that a 1 bp assumption is conservative. However, one should remember the tight credit spread environment over the reported period of 2004 to 2006. The sensitivity of CPDOs to bid-offer spreads clearly demands an assumption on future liquidity when assessing long term performance.

Default Rate Sensitivity

In order to test the sensitivity of CPDO risk and ratings to default rates, we reduced our six-month probabilities of default by 20% before generating the six-month loss distribution for the portfolio. This results, on average, in 0.50 defaults per annum, compared to the previous assumption of 0.65 defaults. Note that this is significantly lower than the average annual BBB default experience over the period 1994–2004 (Table 13), and higher than the annual IG default experience over the same period. We consider a standard CPDO for varying levels of spread volatility leaving all other parameters unchanged (*LTS* = 70%, *MRS* = 40%, α = *AggM*, *S_0* = 35 bp, bo = 1 bp). The experiment reveals some interesting insights, in that CPDO risk measures and ratings are much more sensitive to changes in default risk under low spread volatility, stemming from the fact that under very low volatilities, a CPDO essentially can only default when very high default rates are observed. As a result, higher default rates are more significant in low volatility settings. For example, a model-implied BBB (high) rating using 25% volatility would improve to AAA under the reduced default rate, compared to a change from BBB (high) to A (high) under a 35% volatility assumption. As the rating only improves to BBB under a 45% volatility, the (rating) impact of varying default rates is more moderate for higher volatilities.

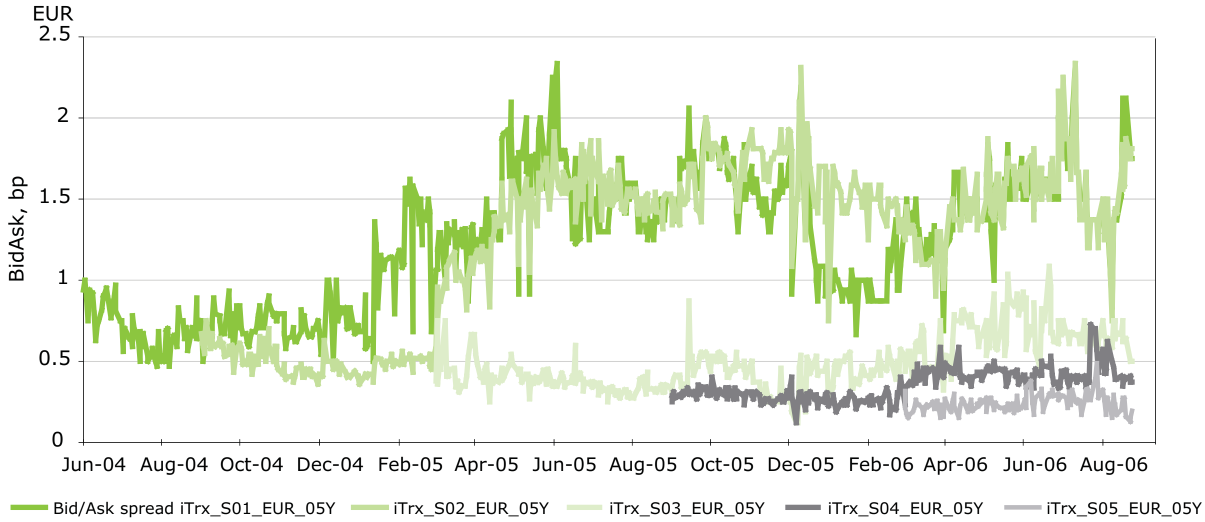
Sensitivity to Structural Features – Note Coupon and Leverage

Finally, we assess the impact of changing two structural features – note coupon and maximum leverage – on CPDO risk and ratings. Figure 17 shows a comparison of the usual risk measures for coupons between 40 bp and 200 bp and two levels of leverage – 15x (standard assumption) and 10x.

The figure provides some interesting insights. Firstly, using a CPDO model with parameters

[21] We have also experimented with a spread-dependent bid-offer spread reflecting the fact that in a wider trading market, bid-offer spreads are likely to be higher. The results, however, were fairly insensitive when compared with a constant bid-offer spread identical to the average of the spread-dependent approach.

Figure 15: iTraxx Bid-Offer Spread in bp for Different Series



Source: The Royal Bank of Scotland (RBS) and Bloomberg.

that are in line with empirical research (σ = 35%, *LTS* = 70 bp, *MRS* = 40%, spread-dependent α, 1 bp bid-offer spread) and assuming a 35 bp initial spread level, a AAA rating seems only to be consistent with an 80 bp (or lower) coupon, while a transaction offering a 200 bp coupon would be assigned a BBB (high) rating for the hypothetical 15x leveraged trade.

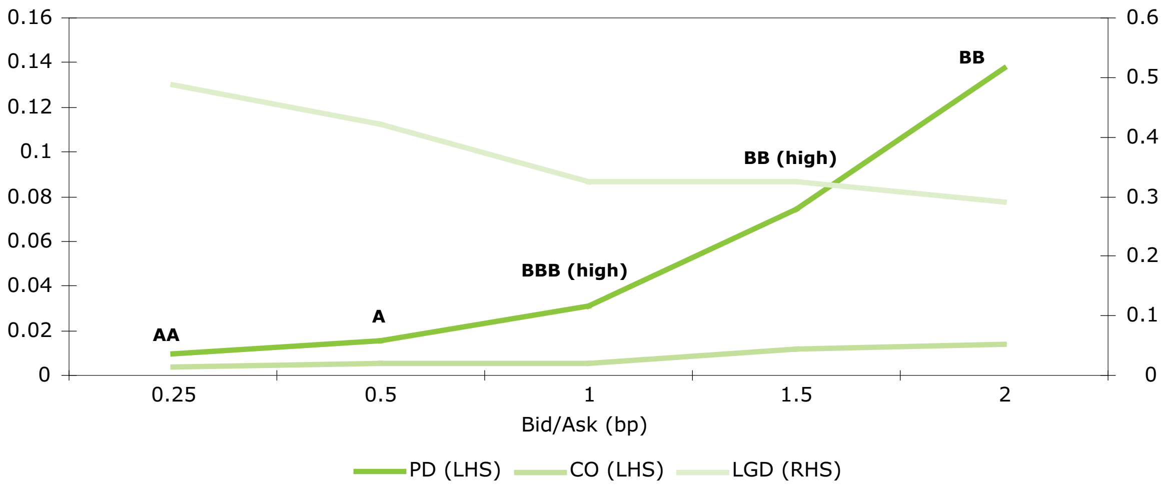
Reducing leverage essentially increases the PD, as there are more scenarios under which the CPDO just does not reach its target. On the contrary, lower leverage has a positive effect on LGD, as it essentially reduces the number of cash-out events (i.e., a CPDO with lower leverage is more resilient in a bad credit environment).

2.7 RATINGS SENSITIVITY: "EMPIRICAL" VERSUS "OPTIMISTIC" MODEL CALIBRATION

We now compare the risk profile of a number of CPDOs – differing only in the coupon they offer – and consider two models; one that is calibrated in line with the empirical findings presented above and one that takes a more optimistic view on some of the parameters; namely, σ = 25% and α = 0.7.

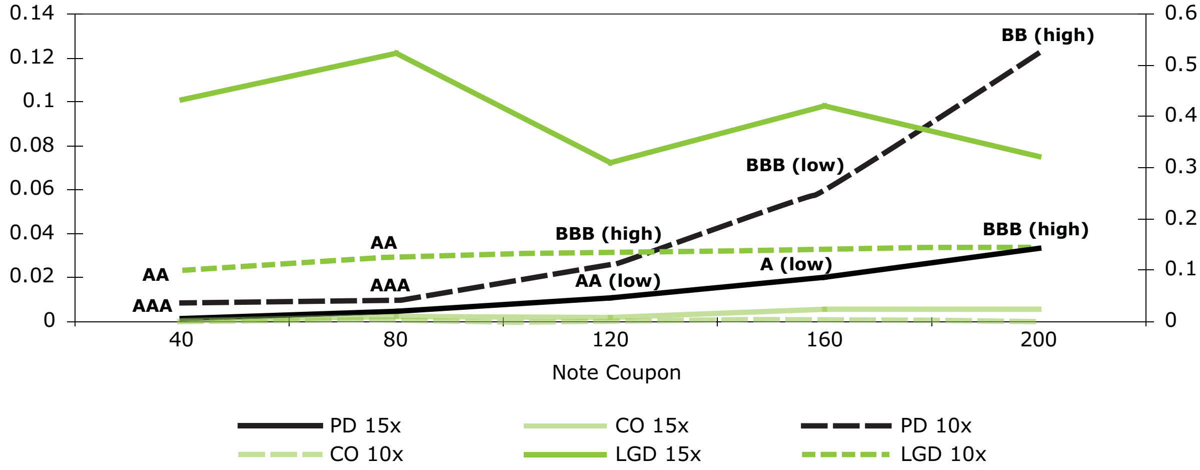
Figure 18 highlights once again the sensitivity of CPDO risk measures (PD and LGD) to modelling assumptions. The increase in LGD stems predominantly from the higher volatility of 35% (at the lower end of our empirical estimates) compared with the 25% assumed under the optimistic model calibration. The increase in PD (failure to redeem

Figure 16: PD, CO, and LGD by Bid-Ask Spread



Note: Sensitivity of a standard CPDO to changes in bid-offer spread. Standard assumptions are applied to all other parameters (σ = 35%, LTS = 70%, MRS = 40%, α = AggM, S_0 = 35 bp).

Figure 17: PD, CO and LGD – Impact of Leverage and Note Coupon (S_0 = 35 bp)



Note: Sensitivity of a CPDO to changes in leverage and note coupon.
Standard assumptions are applied (σ = 35%, LTS = 70%, MRS = 40%, α = AggM, S_0 = 35 bp).

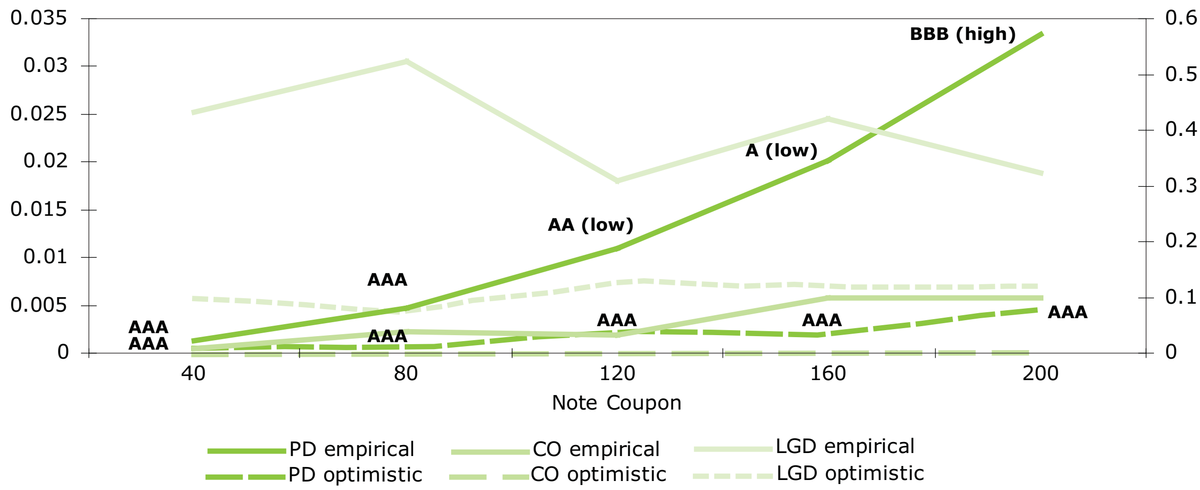
par) stems from this increase in volatility, but also from the assumption of lower average time decay. Figure 18 also reveals the difference in model-implied ratings under these two different model calibrations. Under the optimistic setting, CPDOs up to a coupon of 200 bp would be rated AAA, whereas the empirical setting only allows a rating of AAA for an 80 bp coupon, whereas a 200 bp coupon would imply a BBB (high) rating.

Figure 19 shows the sensitivity of a standard 200 bp CPDO (15x) to initial spread under the empirical parameterisation.

It is apparent that the lower the starting spread, the higher the PD and the lower the LGD, caused by two effects. Firstly, if the starting spread is lower, the mean reversion of credit spreads pushes average spreads toward the LTS, resulting in higher MTM losses for lower initial spreads. Secondly, under a lower starting spread, the excess spread is lower during the first few roll dates as the CPDO sells protection at a lower spread, which also has a negative impact on NAV.

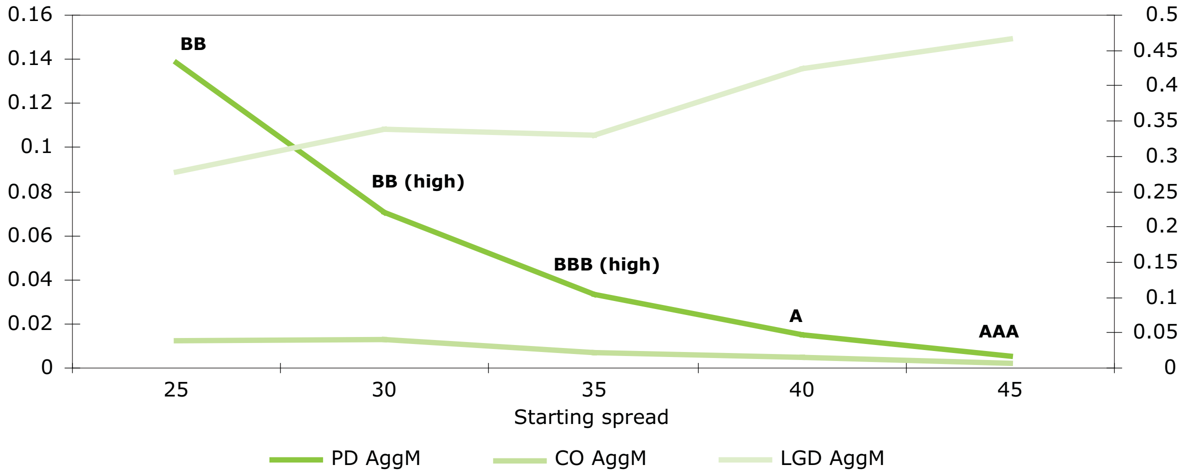
Interestingly, our empirical model parameterisation leads to results broadly in line with Varloot et al. (2006), where according to their model, a CPDO paying a 100 bp coupon would be rated BBB (high) given an initial spread of 30 bp. Although an exact comparison is not possible because the leverage mechanism and other structural features may vary slightly, the empirical model implies a BBB rating for a CPDO paying a coupon of 100 bp with maximum leverage of 10x and an A (low) rating for a maximum leverage of 15x.

Figure 18: PD, CO and LGD by Note Coupon (S_0 = 35 bp): Emprical Versus Optimistic Calibration



Note: Sensitivity of a CPDO with different note coupons to an "optimistic" and "empirical" spread model.

Figure 19: PD, CO and LGD by Starting Spread



Note: Sensitivity of a standard CPDO to changes in initial spread. Standard assumptions are applied (σ = 35%, LTS = 70%, MRS = 40%, α = AggM).

2.8 SUMMARY AND DISCUSSION

The previous sections provided a brief overview of the main economic risk factors affecting CPDOs, the empirical data available to estimate models and the sensitivity of CPDOs to model risk and assumptions on the relevant model parameters. From these experiments, we gained a deeper understanding of CPDO performance, allowing us to draw the following conclusions:

- CPDOs are very sensitive to all spread model parameters: PD and LGD increase with spread volatility, PD decreases while LGD increases with LTS, and PD decreases with MRS while LGD is relatively insensitive.
- CPDOs are extremely sensitive to assumptions on the steepness of the credit curve. Modelling the dynamics of flatter term structures when spreads are wider reduces PD but increases LGD considerably.
- CPDOs are very sensitive to bid-offer spreads, implicitly requiring any risk manager, analyst or investor to make an assumption on future liquidity.
- Spread model risk in general is very high, up to a point where a misspecification of one or more parameters increases risk multifold. In ratings terms, an overly optimistic parameterisation may lead to AAA ratings that, in the long run, might behave more like BBB ratings.
- Sensitivity to credit losses is also high, highlighting that the package of assumptions across default, spread and liquidity needs to be constructed adequately.
- Reducing leverage reduces LGD considerably and makes the CPDO more resilient to a severe credit downturn at the cost of increasing the PD.
- The sensitivity to changes in initial spread also deserves further comment, as it may mitigate the high rating volatility that would otherwise accompany the sudden tightening or widening of spreads. For example, if spreads widen over a period of time, NAV losses will be partially offset by the higher carry that the CPDO can generate, which in turn may lead to less significant rating changes, whereas spread tightening will lead to NAV increases, dampening somewhat the downgrade actions.

In light of the above, we strongly encourage market participants to engage in further discussions on model risk and mitigation strategies, aiming toward the highest possible level of transparency for complex structured credit products.

Back-Testing Insights

In addition to the previous analysis, we have conducted a series of back-testing experiments. Although index composition has changed over time and CDS indices have only been trading since 2004, we can still set up a number of experiments using observed aggregate default rates and historical BCDS spreads. The main objective of this section is to provide some insight into the MTM and ratings volatility of a CPDO. The latter, of course, is of great interest to investors and rating agencies alike.

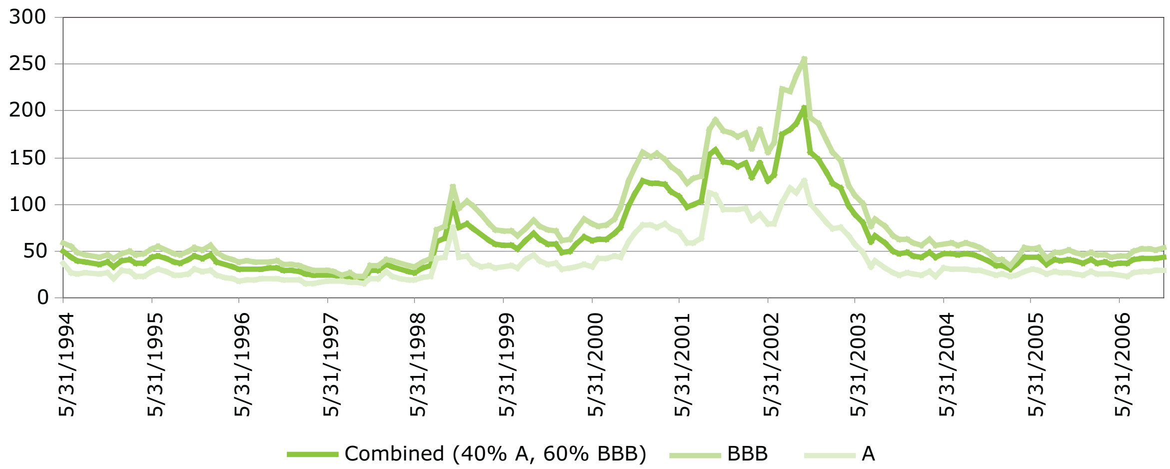
We assume that a CPDO with a certain coupon (e.g., 200 bp) is launched at various different points in the past (e.g., March 1997, June 1997, etc.). Within the analysis, we assume a constant 1 bp bid-offer spread, IG average default rates as published in Vazza et al. (2005) and Lehman Brothers' monthly BCDS data.[22] The aggregate slope model developed above is employed as a proxy for index term structure.[23] Adding data on the actual term structure of interest rates allows us to compute the CPDO NAV going forward. Figure 20 and Tables 13 and 14 show the respective spread and default data.

The spread evolution is approximated by a linear combination of observed BCDS spreads to reflect the average ratings distribution in the indices. Defaults are assumed to take place at the beginning of the year and are in line with the historic rating-specific default rates observed over this period (Table 13). We consider an IG default rate that translates into 3.4 defaults for the 250-name reference portfolio, as well as a more conservative BBB experience that translates into 7.4 defaults (see Table 14). Although the average credit quality in the index is only marginally better than BBB, the fact that the indices roll should reduce default risk considerably. We therefore consider frequently the IG experience in our back-testing exercise. Figure 21 shows the NAV performance of a number of CPDOs issued at different times. From left to right, each line represents a single CPDO issued six months after the previous transaction. Each CPDO offers a coupon of 200 bp and can be leveraged up to 15x.

Figure 21 reveals that NAV volatility can be very high, depending on the date the CPDO was issued. Although CPDOs appear very path-dependent, even under the BBB default rate, all CPDOs redeem par at maturity (i.e., cash in early). Reducing the underlying defaults to IG level would still lead to a very high NAV volatility, indicating that most volatility stems from spread widening (and, most importantly, its timing).[24]

Repeating the above analysis assuming that index spreads evolved according to BBB BCDS spreads

Figure 20: BCDS history (1994–2006)



Note: BCDS spread history for "A" and BBB. Combined denotes a hypothetical spread history comprising 40% "A" and 60% BBB credits.

[22] Using monthly data could lead to small inaccuracies caused by re-leverage and de-leverage mechanisms based on the NAV from the previous period. If we rebalance the positions daily, the NAV from the previous period will not be very different from the current NAV. However, in this case, as we only have monthly spread data, the MTM may have been changed dramatically within a month. Obviously, daily rebalancing allows the strategy to react faster to the NAV changes, thus better implementing the strategy. However, this means that transaction costs are also likely to be higher.
[23] Although the true slope could also have been taken into consideration, the good model fit provides a good and efficient proxy throughout the back-testing analysis.
[24] For details, see Jobst et al. (2007).

Table 13: Annual Default Rates for IG and BBB-Rated Corporates (1994-2004)

%	1994	1995	1996	1997	1998	1999	2000
IG	0.05	0.04	0	0.08	0.14	0.14	0.17
BBB	0	0.17	0	0.24	0.41	0.19	0.37
%	2001	2002	2003	2004			
IG	0.2	0.45	0.1	0			
BBB	0.33	1.01	0.22	0			

Source: Vazza et al., 2005.

Table 14: Defaults in 250-Name CDO Portfolio

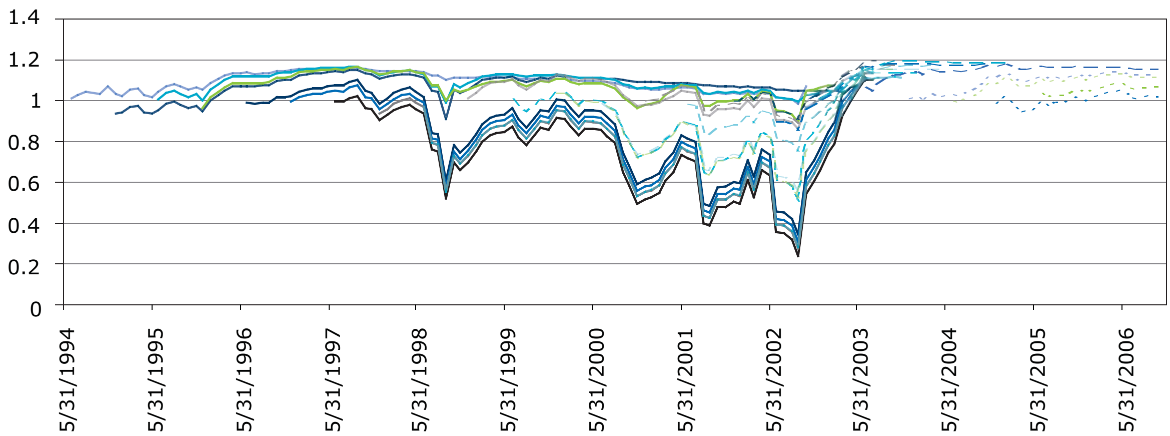
%	1994	1995	1996	1997	1998	1999	2000
IG	0.13	0.1	0	0.2	0.35	0.35	0.43
BBB	0	0.43	0	0.6	1.03	0.48	0.93
%	**2001**	**2002**	**2003**	**2004**		**Total Defaults**	
IG	0.5	1.13	0.25	0		3.43	
BBB	0.83	2.53	0.55	0		7.35	

Note: Derived from IG and BBB default rates by multiplying the annual default rate by 250.

has a more significant impact on CPDO performance. Although the use of BBB spreads may be very conservative, Figure 1 in Appendix 1 indicates that on average, BBB BCDS spreads are in line with CDX over the two-year period from November 2004 to November 2006. Although all CPDOs would cash in if no defaults occurred in the underlying indices, under moderate IG default rates, seven CPDOs fail to redeem par at maturity. Furthermore, under the more conservative BBB default assumption, 22 defaults occur, out of which 21 correspond to cash-out events. These cash-out events occurred during the third quarter of 2002, with a starting spread from November 29, 1996, to July 31, 1998, in the range of 20 to 30 bp, subsequently widening to around 220-255 bp.

Although the scenario leading to cash-out events is somewhat extreme, this example further highlights the NAV sensitivity and path dependence of CPDOs, and shows that the timing of issuance seems to be crucial. With that in mind, one can also reach an opinion about the severity of the 1998–2002 credit cycle and whether or not a product issued in 1997 should survive the subsequent events.[25]

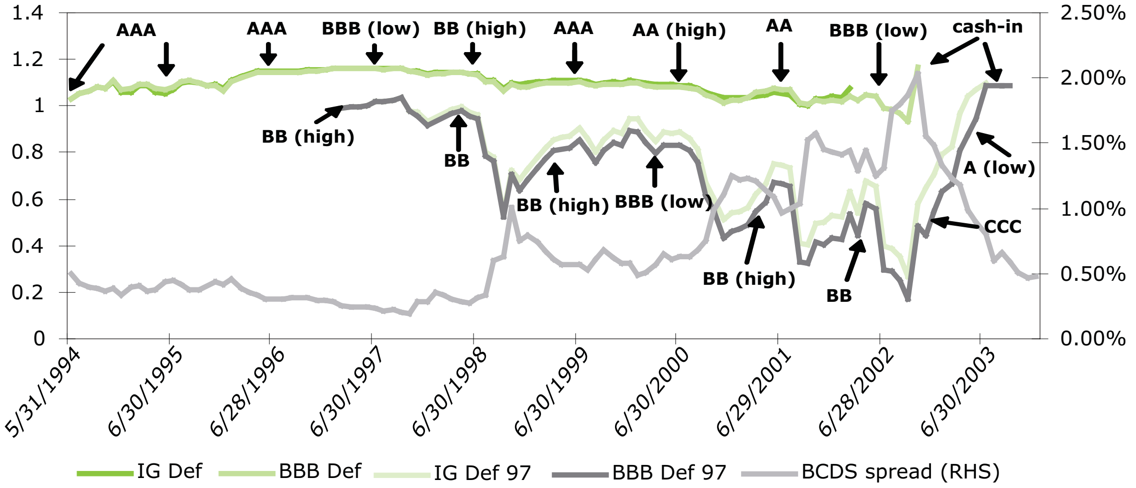
Figure 21: CPDO NAV Performance – BBB Defaults and Combined Spreads



Note: NAV evolution of standard CPDOs issued between June 1994 and November 2005 in six-month intervals. From left to right, each line represents a single CPDO, issued six months after the previous transaction. The combined BCDS spread history (60% BBB, 40% "A") and BBB default rates have been used in the NAV computation.

[25] One may argue that our back-testing data is highly conservative, as IG and especially BBB default rates are unlikely to have occurred within the index portfolios as a result of asset selection and index rolls. Similarly, one may argue that credit markets nowadays are so much more efficient that such extreme spread widening is not expected to occur again in modern markets. On the contrary, one may argue that the 1998–2003 period hardly corresponds to a AAA or AA environment, and as a result, a CPDO rated AAA or AA would have to withstand such stress tests.

Figure 22: CPDO NAV Performance for Two Different Issuance Dates



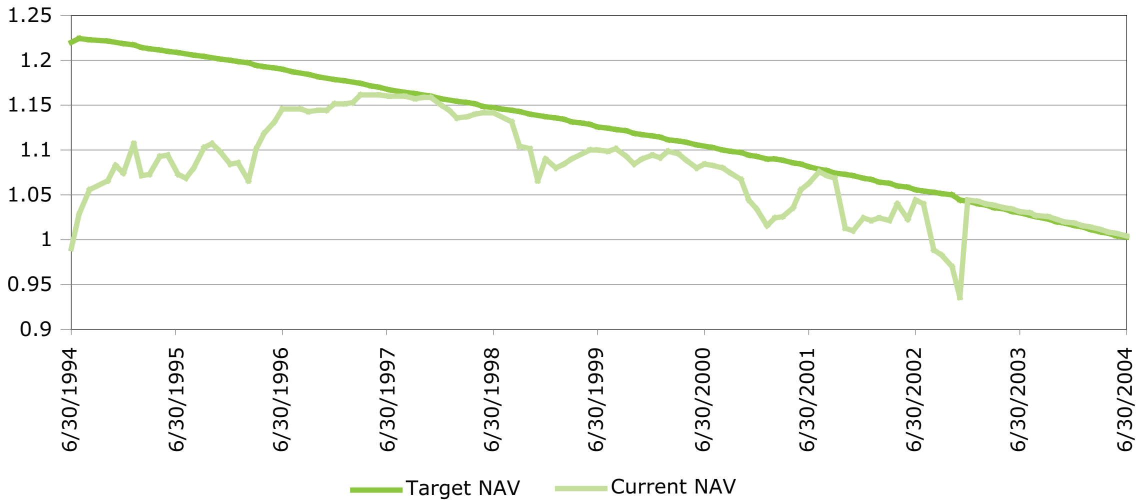
Note: NAV evolution of a standard CPDO issued at two points in time, 30 June 1994 and 28 February 1997. The combined spread history (60% BBB, 40% A) and both BBB and IG default rates are used in the NAV computation. The BCDS spread history is also shown.

We now look closer at the path dependence of CPDOs, focusing mostly on two different points in time of CPDO issuance: June 30, 1994, and February 28, 1997. Figure 22 shows the corresponding NAV evolution.

Although the CPDO redeems at par in each case, the NAV volatility and extreme path dependence is clearly visible. Starting in 1994, the structure would have generated enough carry and NAV gain to de-lever during the first three years, and subsequent spread widening would affect the transaction to a lesser extent. Figure 23 and Figure 24 illustrate this in more detail. The NAV plot (Figure 23) also shows the cash-in barrier (approximated assuming a constant 5% interest rate).

In contrast, issuing the CPDO in 1997 would lead to NAV deterioration below 20%, close to a cash-out event. This is caused by an NAV that declines from the outset, and as a result the CPDO runs at the maximum leverage for a prolonged period. As spreads continue to widen, high leverage leads to severe NAV deterioration. Figure 25 and Figure 26 illustrate this more clearly.

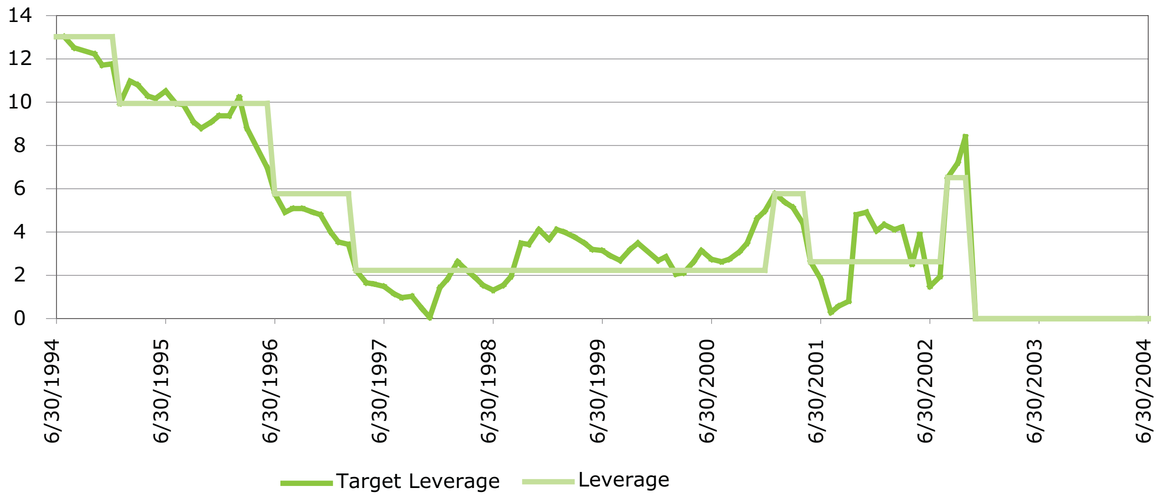
Figure 23: Evolution of NAV for a Standard CPDO Issued on 30 June 1994



Note: 200 bp coupon, 15x maximum leverage.

[25] One may argue that our back-testing data is highly conservative, as IG and especially BBB default rates are unlikely to have occurred within the index portfolios as a result of asset selection and index rolls. Similarly, one may argue that credit markets nowadays are so much more efficient that such extreme spread widening is not expected to occur again in modern markets. On the contrary, one may argue that the 1998–2003 period hardly corresponds to a AAA or AA environment, and as a result, a CPDO rated AAA or AA would have to withstand such stress tests.

Figure 24: Evolution of Leverage for a Standard CPDO issued on 30 June 1994



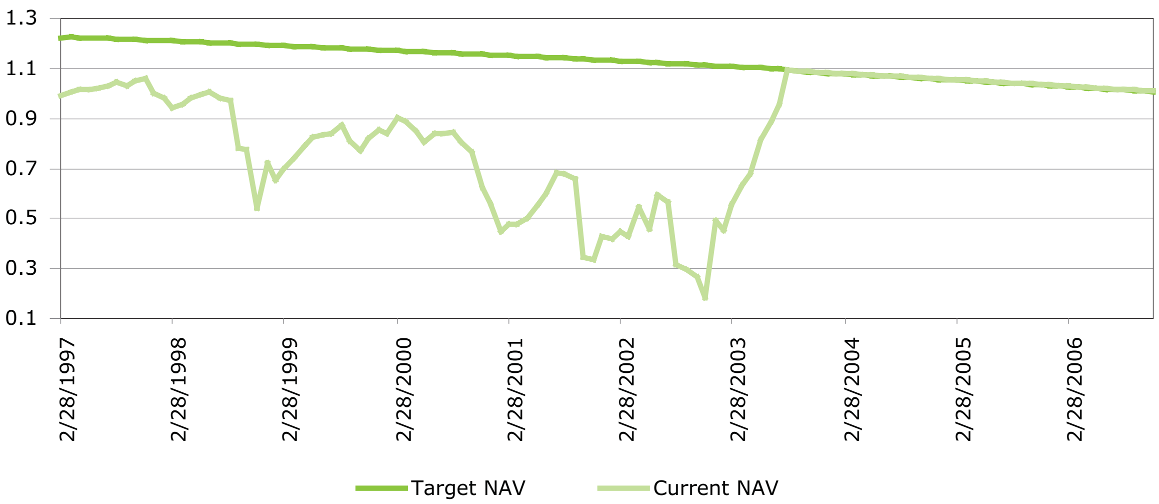
Note: 200 bp coupon, 15x maximum leverage.

RATING STABILITY

In addition to concerns about MTM and NAV volatility, many market participants are also concerned about the rating stability of the CPDO product. Figure 22 also provides some initial insight into a possible rating evolution. Using our CPDO model parameterised in line with the empirical findings, we have determined indicative ratings at issuance and then re-rated the transaction every year until cash-in takes place. For both the 1997 and 1994 issuance, we have assumed the CPDO has a coupon of 200 bp and maximum leverage of 15x, resulting in an initial rating of AAA in June 1994, when spreads were at around 50 bp, and BB (high) in February 1998, when spreads were trading between 20 bp and 30 bp.

Despite the stable NAV evolution for the deal issued in 1994, Figure 22 reveals that ratings may fluctuate considerably under this scenario. The transaction would still be rated AAA in May 1996, but would be downgraded to BBB (low) and BB (high) within two years because of the extremely tight spread environment of 24 bp and 28 bp in May 1997 and May 1998, respectively.[26] This volatility would be problematic for investors, particularly those that are forced to liquidate their positions at certain rating triggers. Note

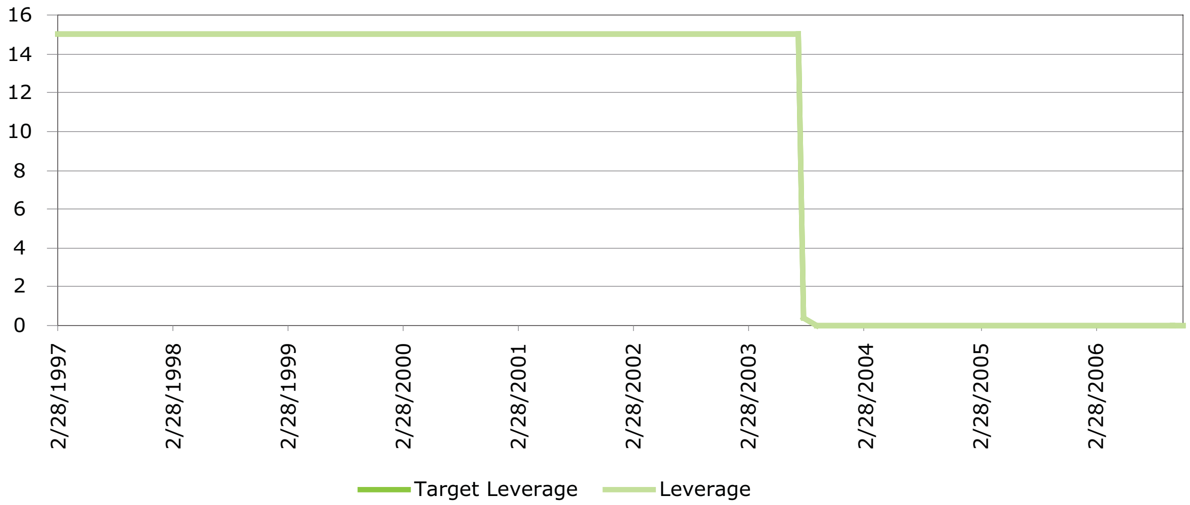
Figure 25: Evolution of NAV for a standard CPDO issued on 30 October 1997



Note: 200 bp coupon, 15x maximum leverage.

[26] This may seem counter-intuitive, given the high NAV, but results from the fact that very tight spreads automatically lead to lower ratings in a pure "mark-to-model" framework.

Figure 26: Evolution of Leverage for a Standard CPDO issued on 30 Ocotber 1997



Note: 200 bp coupon, 15x maximum leverage.

especially that one year later, with spreads at 55 bp to 60 bp, the model-implied rating moves back to AAA (the higher level of spread compensates for the NAV decline over that period). This example highlights the possible ratings volatility in the CPDO product, as well as the importance of a transparent and consistent surveillance approach by rating agencies.

The transaction issued in February 1997 would have had ratings varying between NIG and IG throughout its life. Given the initial low rating of BB (high), such high rating volatility is perhaps not unreasonable.

Additional back-testing experiments reveal that lower leverage generally stabilises the structure, in that MTM and NAV volatility are reduced and the likelihood of cash-out events decreases, which also results in lower LGD. On the other hand (and possibly against our intuition at first sight), the probability of investors not receiving full principal may increase, resulting in more situations where the structure does not cash in before maturity, but with lower LGD. Similarly, decreasing the coupon stabilises the structure and significantly increases the probability of cashing in before maturity. Unfortunately, in an environment where a cash-out event is more likely, a lower coupon has little effect on NAV stability and CPDO performance.

Concluding Remarks

We have shown that there are significant challenges in the estimation of suitable models and risk parameters for accurate analysis of CPDO risk and ratings, due in large part to the relative paucity of CDS index spread data. These challenges are exacerbated by a high level of model risk, where CPDO risk measures such as PD and LGD can be very sensitive to both the choice of model and modelling assumptions. Market participants, therefore, need to derive proxies for missing data and/or enrich the modelling paradigm by qualitative judgement (e.g., on future liquidity and bid-offer spreads of CDS indices). This combination of quantitative and qualitative analysis is crucial, as we have shown that – depending on the level of coupon and leverage – an indicative rating of a typical CDPO can differ by multiple rating categories between optimistic modelling assumptions and those in line with the empirical estimates derived in this study.

DBRS encourages all market participants, and particularly investors, to review all modelling assumptions very carefully and to insist on the maximum level of transparency and disclosure possible. Although CPDOs clearly involve multiple risks, we believe that there is potential to mitigate some of these risks in the next wave of CPDO structures, driven by a high demand from arrangers and asset managers toward bespoke CPDO strategies. Managed CPDOs may reference well-diversified, bespoke portfolios; introduce flexibility around the roll date and variable coupon and/or leverage mechanics; and execute early hedging of reference obligations with declining credit quality, all of which show potential for risk mitigation and additional ratings stability. For example, lower volatility collateral such as asset-backed securities (ABS) or commercial mortgage-backed securities (CMBS) may mitigate some of the cash-out risk, and efficient credit selection (highly liquid collateral) may help reduce transaction and roll costs. However, it goes without saying that the risk management and quantitative framework required to address many of these enhancements needs to be at least as transparent and empirically grounded as the one for static, index-based products. One of the key remaining challenges is the development of an integrated spread and default risk engine, applicable at both an index and single-name level.

Model risk and ratings aside, the path dependence and automatic-trading rules lead to interesting relative-value considerations. For example, Varloot et al. (2006) show that CPDOs offer a fair amount of carry, similar to a traditional mezzanine tranche or rated equity. Furthermore, the authors also provide the following observations in a series of interesting scenario analyses:

- CPDOs behave very well in good but also mildly bearish credit environments. An adequate risk assessment is still required, however, as no principal protection (as in CPPI) is guaranteed and in extreme scenarios, CPDOs do not redeem at par.
- CPDOs perform best in mildly bearish credit environments and if an investor is bullish, he or she would prefer first-lost products such as rated equity or zero equity. For bearish investors, CPDO is a good investment unless they don't want to be long credit at all.
- CPDOs can be potentially volatile, but on average their risk profile is not too far from a CDO mezzanine 6%-9% tranche for a buy-and-hold investor.

The fact that CPDOs are similar to a range of different products (e.g., rated equity or mezzanine tranches) for different credit environments highlights the complexity of modern structured credit transactions. If the risk and return profile is properly assessed, CPDO notes can provide a high level of capital efficiency within an investment portfolio.

Overall, DBRS is both able and willing to provide rating opinions on these products, but feels that more transparency, disclosure and dialogue among market participants are necessary to adequately assess the risks in complex structured products. Given the high path dependence, NAV and potential ratings volatility, DBRS feels that clear and transparent ratings and surveillance processes, combined with measures that quantify ratings volatility, are necessary in order to provide market participants with a good indication of the "true" risk in complex structured credit products such as CPDOs.

While this report focuses on index-based CPDOs, the market is moving toward managed structures that allow structural enhancements (e.g., short buckets) and the consideration of bespoke portfolios. DBRS is currently expanding its quantitative platform to address these proposals.

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Appendix 1: Spread Data

The analysis presented in this document has been based on a number of different spread data sources.

CDX/ITRAXX INDEX SPREADS – MARKIT

Index spread data is available on iTraxx and CDX from March 2004. From March 2005 onward, quotes for both indices for maturities of three, five, seven and ten years are available.

BOND YIELD SPREADS – BLOOMBERG

We have compiled data on U.S. industrial bond yields aggregated by rating and deducted the yield-to-maturity of a risk-free treasury benchmark with similar maturity. Index level data and U.S. and euro-zone industrial yields are available from August 1991 and 2001, respectively. As a risk-free benchmark, we have employed interest rate swap quotes (LIBOR or Euribor) rather than yields on Treasury bills, which is consistent with empirical research (see Hull and White (2005)) on CDS markets.

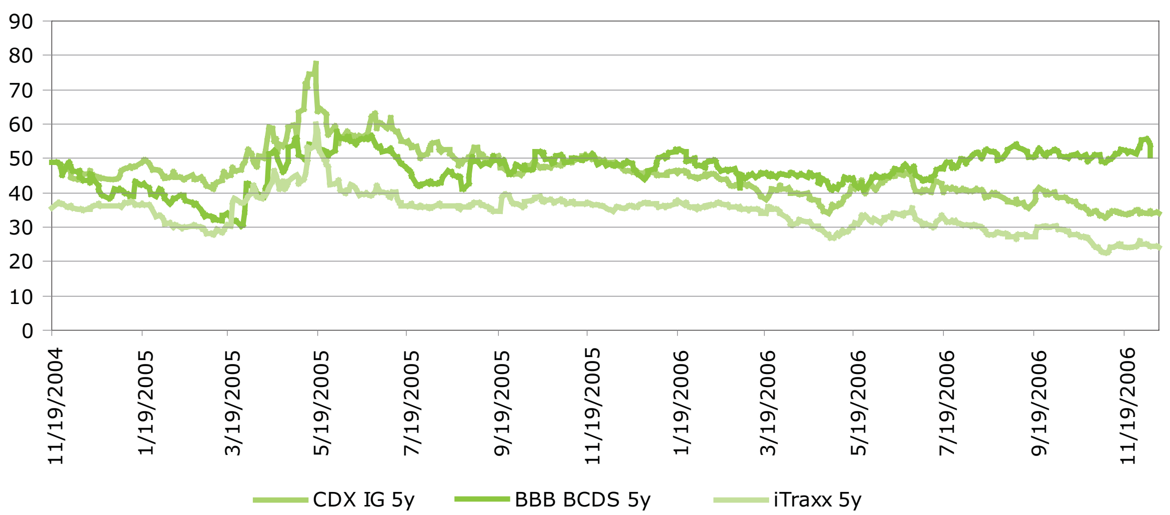
BOND-IMPLIED CDS (BCDS) SPREADS – LEHMAN BROTHERS

In a series of publications (see Mashal and Naldi (2005) and Berd et al. (2004)), Lehman Brothers discusses BCDS spreads. Essentially, corporate bond pricing is used to fit (or strip out) term structures of survival probabilities. These survival probabilities are then used in the standard CDS pricing equation, resulting in BCDS spreads. This implementation ensures that BCDS spreads are directly comparable with CDS spreads and therefore suitable for CPDO analysis. We have monthly data for "A," BBB and BB indices for maturities of three, five and seven years over the period May 1994 to November 2006. Similar bi-monthly data is available from May 1997 and daily data from January 2002.

COMPARATIVE ANALYSIS

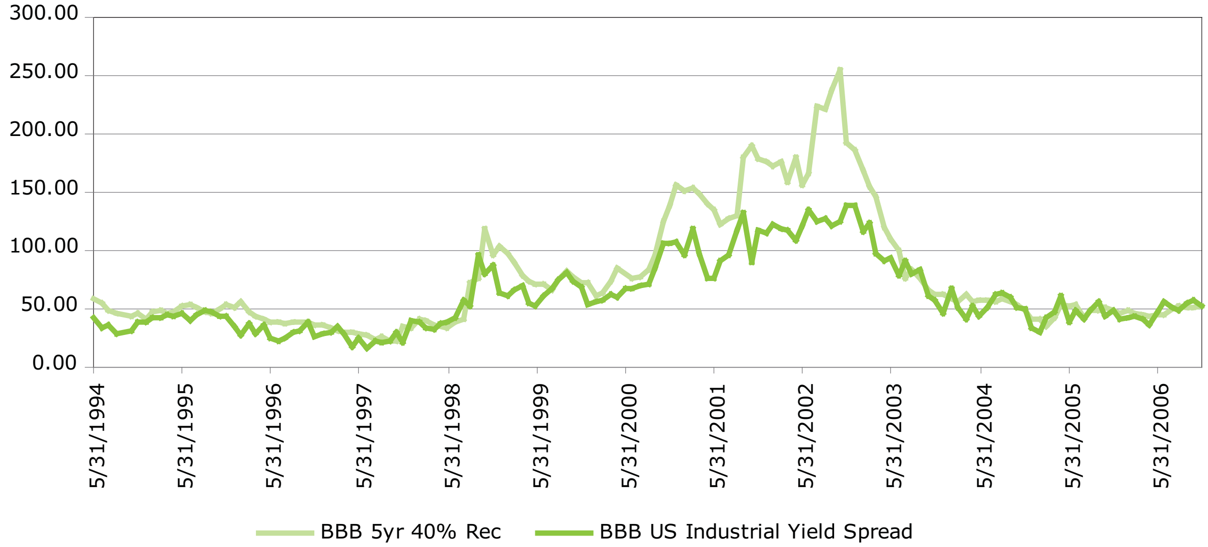
While a full comparison of various bond measures is beyond the scope of this discussion, a good understanding can be gained from Berd et al. (2004) and O'Kane and Sen (2004). Intuitively, the fact that BCDS spreads are based on standard CDS conventions and directly comparable with CDS quotes (in contrast to option-adjusted spreads (OAS) or yield spreads) makes BCDS spreads a good choice for further analysis. Appendix 1 Figure 1 plots five-year CDX, iTraxx and BBB BCDS spreads from November 2004 to January 2007. It is apparent that during the correlation crises of May 2005, caused by the downgrade of Ford Motor Company and General Motors Corporation, the CDS market appeared to have reacted stronger than the bond market. Furthermore, since August 2006, CDS spreads have continued to tighten compared with BBB bond-implied spreads, which may be caused, to some degree, by technical factors (such as the hype surrounding CPDOs).

Appendix 1 Figure 1: Comparison of BCDS Spreads with iTraxx and CDX Spreads



Source: Markit, Lehman Brothers.

Appendix 1 Figure 2: Bond Yield Spread Versus BCDS



Source: Lehman Brothers, Bloomberg.

Appendix 1 Figure 2 shows a comparison of BBB BCDS spreads to BBB yield spreads computed from Bloomberg U.S. Industrial fair-value spreads and risk-free, benchmark five-year swap rates. The figure reveals a high correlation and also that yield spreads were 100 bp tighter than BCDS spreads in 2002. This difference can be explained, as BCDS spreads (just like actual CDS spreads) are based on the concept of default arrival (and its probability), while nominal spreads are simply differences between internal rates of returns.

Typically, when there is a sharp decrease in observable bond prices, default probabilities (and BCDS spreads) react more significantly than bond yields, which are still discounting all of the promised cash flows as if they were going to be received by investors with certainty. We therefore believe that BCDS spreads provide a suitable proxy for credit spread analysis in the absence of sufficient CDS data.

Appendix 2: CPDO Cash Flow Mechanics

Evaluating a CPDO requires a series of distinct steps in order to compute the strategy NAV, shortfall and hence required leverage. In broad terms, a CPDO algorithm can be described by the following steps, assuming the algorithm has been executed up to time T_{g-1}, and we are now at time T_g:

(1) Simulate spreads and rates for time T_g (survival curves and discount factors).

(2) Generate default events for period $[T_{g-1}, T_g]$.

(3) Book incurred defaults.

(4) Update risky exposure due to default events in preceding period.

(5) Calculate value of liabilities and assets, PV_L and PV_A.

(6) Calculate MTM.

(7) Calculate cash balance (add rate and spread income and subtract fees, default losses and coupons).

(8) Calculate NAV.

(9) Calculate target leverage.

(10) Calculate implied leverage.

(11) Adjust leverage.

(12) Update risky exposure due to change of leverage.

(13) Update cash balance (e.g., realised MTM losses or gains as a result of adjusting notional on rebalancing or roll dates).

(14) Update effective contractual spread due to a leverage-up event.

In the following section, we will outline most of these steps in further detail, starting with the present value liabilities and assets (step 5), assuming spreads are simulated according to the models outlined in Section 2.

PRESENT VALUE (PV) OF LIABILITIES

The PV of liabilities at time T_g is obtained by discounting coupon payments $C(\cdot)$, fees $F(\cdot)$ and note notional N at the risk-free discount rate, taking into consideration the actual day count fraction $DCF(T_g, T_{g+1})$ between periods T_g and T_{g+1}:

$$PV_L(T_g) = N \sum_{i \in \{c:c>g\}}^{\sup c} C(T_i)\, DCF(T_{i-1}, T_i)\, B(T_g, T_i)$$

$$+N \sum_{i \in \{c:c>g\}}^{\sup c} F(T_i)\, DCF(T_{i-1}, T_i)\, B(T_g, T_i)$$

$$+N\, B(T_g, T_{\sup g}).$$

Here $B(t,T)$ denotes the present value of a risk-free zero-coupon bond (ZCB) with maturity T as seen at time t and can be given by

$$B(t,T) = E^Q\left[\exp\left(-\int_t^T r_s ds\right)\right]$$

where $E^Q(\cdot)$ denotes the expectation under the risk-neutral (pricing) measure and $r(t)$ the spot interest rate/short rate.[1]

PRESENT VALUE OF ASSETS

In order to compute the PV of assets, a risky discount factor or survival probability needs to be computed. A simple approach is to assume that the term structure of the index spread corresponds to a constant hazard rate λ. This assumption implies that the corresponding survival curve is $Q(t) = e^{-\lambda t}$. Considering a CDS expiring at time T in a continuous setting, one obtains

$$PV_{premium} = s_{par} \int_0^T B(0,u) Q(0,u) du$$

$$PV_{protection} = (1-R) \int_0^T \lambda B(0,u) Q(0,u) du \;.$$

[1] Depending on the model chosen for the short-rate process r(t), B(t,T) may be derived in closed form. For example, the CIR model (Cox et al. (1985)) is a popular, mean-reverting short-rate model for interest rates: $dr = (\eta - \gamma r)dt + \sqrt{\alpha r}\, dZ_r$, where η / γ denotes the steady-state or long term mean of r, $1/\gamma$ mean reversion speed, α the interest rate volatility, and Z_r the usual Wiener process. It can be shown that this model allows one to compute the price of a ZCB as $P(t,T) = G(t,T) e^{-H(t,T) r(t)}$, where $H(t,T) = \frac{2(e^{k(T-t)} - 1)}{(k+\gamma)(e^{k(T-t)} - 1) + 2k}$, $G(t,T) = \left[\frac{2k e^{(\gamma+k)(T-t)/2}}{(k+\gamma)(e^{k(T-t)} - 1) + 2k}\right]^{\frac{2\eta}{\gamma\alpha}}$ and $k = \sqrt{\gamma^2 + 2\lambda^2}$.

This leads to the relationship $s_{par} = \lambda(1-R)$. As a result, the risky discount factor or survival probability is given by

$$Q(T_g, T_{g'}) = \exp\left\{-\frac{s_{CDS}(T_g)(T_{g'} - T_g)}{1-R}\right\}.$$

The PV of the assets consisting of a short CDS exposure is then given by

$$PV_A(T_g) = s_{CDS}(T_g) N \sum_{i \in \{\bar{c}:\bar{c}>g\}}^{\sup \bar{c}} DCF(T_{i-1}, T_i) Q(T_g, T_i) B(T_g, T_i)$$

$$= s_{CDS}(T_g) N \, DV01_A(T_g, T_{\sup \bar{c}})$$, where

$DV01_A(T_g, T_{\sup \bar{c}})$ denotes the present value of a risky coupon stream between times T_g and $T_{\sup \bar{c}}$ (risky duration), and $s_{CDS}(T_g)$ the contractual CDS spread that applies to time T_g.

RISKY EXPOSURE

The risky exposure is defined as the total notional of the short CDS position:

$RiskExp(T_g) = Lev(T_g)\, N$

with initial condition:

$RiskExp(T_0) = Lev(T_0)\, N$

MTM OF RISKY CDS INDEX POSITION

A MTM gain or loss is realised at any rebalancing event or roll date when protection is bought back on the off-the-run index and sold on the new, on-the-run index. For the roll date, this MTM is given by

$$MtM(T_g) = (s_{CDS}(T_g) - \hat{s}_{CDS}(T_g) - \frac{ba}{2} 1_{\{g=roll\}})\, RiskExp(T_g) \times$$
$$\times \sum_{i \in \{\bar{c}:\bar{c}>g\}}^{\sup \bar{c}} DCF(T_{i-1}, T_i) Q(T_g, T_i) B(T_g, T_i)$$
$$= (s_{CDS}(T_g) - \hat{s}_{CDS}(T_g) - \frac{ba}{2} 1_{\{g=roll\}})\, RiskExp(T_g)\, DV01_A$$

where $\hat{s}_{CDS}(T_g)$ denotes the spot spread at time T_g, ba the bid-offer spread on the index, and $1_{\{g=roll\}}$ the indicator function equal to 1 if T_g denotes a roll date. The MTM is computed accordingly in between roll dates when the CDS position is rebalanced (caused by changes in leverage).

VALUE OF CASH ACCOUNT

The cash account is credited with the following:

- Accrued interest in the LIBOR account.
- Credit premiums from the risk holding (CDS index portfolio).
- Realised MTM gains on any roll or rebalancing date.

The cash account is debited with the following:

- Note coupon payments.
- Losses from defaults.
- Realised MTM losses on any roll or rebalancing date.
- Fees.

The balance of the cash account is given by

$$Cash(T_g) = (1 + LIBOR(T_{g-1}))Cash(T_{g-1}) + s_{CDS}(T_g) RiskExp(T_g)$$
$$+ MtM(T_{g-1}) 1_{\{roll/rebalance\}} - L_{def}(T_g) - F(T_g) - C(T_g)$$

Here L_{def} denotes credit losses from defaults, F, and C fees and coupons as introduce above, and $Cash(T_0) = (1 - F(T_0))N$ incorporates an upfront fee $F(T_0)$.

STRATEGY NAV

The NAV is the net value of the asset position. This means that it equals the present value of the CDS position plus the value of the cash account:

$NAV(T_g) = MtM(T_g) + Cash(T_g)$.

LEVERAGE FACTOR

The target leverage Lev_{tgt} is defined as the leverage that makes the expected income equal to the difference in expected future costs (PV of liabilities) and total asset value (PV of assets), also denoted as the shortfall. More precisely, it is the leverage that makes the present value of the assets equal to the difference in the present value of the liabilities and the NAV. The setup of CPDOs varies. For example, a more aggressive leverage strategy may be employed by means of a gearing factor; this means that the CPDO may accelerate the "catching-up" of the shortfall. A strategy may also include a cushion (here introduced as fraction cush of the notional). This means that the target leverage will be such that asset value will be higher than what is required to cover the shortfall at all times (up to the maximum allowed contractual leverage).

More formally, we obtain from

$$Lev_{tgt}(T_g) PV_A(T_g) := Shortfall(T_g) = (PV_L(T_g) - NAV(T_g))\, gear + cush\, N$$

the target leverage implemented by the CPDO strategy:

$$Lev_{tgt}(T_g) = \frac{(PV_L(T_g) - NAV(T_g))\, gear + cush\, N}{PV_A(T_g)}$$

The implied leverage[2] at any point in time is

$$Lev(T_g) = \frac{RiskyExp(T_g)}{N}$$

The rebalancing procedure is controlled by the rebalancing factor (*rf*) given as a percentage of the target leverage. If the absolute value of the current leverage and the target leverage is bigger than *rf*, rebalancing will occur. More precisely we have

$$Lev(T_g) = Lev_{tgt}(T_g)\left(1_{\{Lev(T_{g-1})<(1-rf)Lev_{tgt}(T_g)\}} + 1_{\{Lev(T_{g-1})>(1+rf)Lev_{tgt}(T_g)\}}\right)$$
$$+Lev(T_{g-1})1_{\{Lev(T_{g-1})<(1+rf)Lev_{tgt}(T_g)\}}1_{\{Lev(T_{g-1})>(1-rf)Lev_{tgt}(T_g)\}}$$

Essentially, if the current leverage, denoted by $Lev(T_{g-1})$, lies within the bounds given by the rebalancing factor $Lev(T_{g-1}) \in \left[(1-rf)Lev_{tgt}(T_g),(1+rf)Lev_{tgt}(T_g)\right]$, no rebalancing action is required and the new leverage $Lev(T_g) = Lev(T_{g-1})$.
On the contrary, if $Lev(T_{g-1}) \notin \left[(1-rf)Lev_{tgt}(T_g),(1+rf)Lev_{tgt}(T_g)\right]$, the new leverage is set at the target leverage, i.e., $Lev(T_g) = Lev_{tgt}(T_g)$.

After determining the new leverage, the new risky exposure is given by $RiskExp(T_g) = Lev(T_g)\,N$ as previously defined.

SELLING ADDITIONAL INDEX PROTECTION (INCREASING LEVERAGE)

If the CPDO leverage mechanism determines that the portfolio should be de-levered, the CPDO buys back protection on a fraction of the holding, at which point the proceeds are realised in the cash account in the form of realised MTM gains or losses (depending on the spot spread $\hat{s}_{CDS}(T_g)$ at time T_g compared with the contractual spread $s_{CDS}(T_g)$).

After levering up, however, the new contractual spread income equals the old contractual spread income on the old risky exposure plus the spot spread income on the additional risky exposure, and this defines the new effective contractual spread at time T_g (required in the subsequent computation of the PV of assets). Starting from

$$NLev(T_g)s_{CDS}(T_g) = NLev(T_{g-1})s_{CDS}(T_{g-1}) + N(Lev(T_g) - Lev(T_{g-1}))\hat{s}_{CDS}(T_g)$$

we obtain

$$s_{CDS}(T_g) = \frac{Lev(T_{g-1})s_{CDS}(T_{g-1}) + (Lev(T_g) - Lev(T_{g-1}))\hat{s}_{CDS}(T_g)}{Lev(T_{g+})}$$
$$= w(T_g)s_{CDS}(T_{g-1}) + (1-w(T_g))\hat{s}_{CDS}(T_g)$$, where

$$w(T_g) := \frac{Lev(T_{g-1})}{Lev(T_g)} = \frac{RiskyExp(T_{g-1})}{RiskyExp(T_g)}$$

denotes the percentage weighted holding on which we generate the previous contractual spread.

[2] Implied from the current position.

Appendix 3: Cumulative Corporate Default Probabilities

Cumulative Corporate Default Probabilities

Maturity (Years)	AAA	AA	A	BBB	BB	B	CCC
1	0.017%	0.047%	0.073%	0.304%	2.206%	5.299%	46.789%
2	0.043%	0.113%	0.172%	0.695%	4.386%	10.554%	60.798%
3	0.078%	0.190%	0.294%	1.145%	6.438%	15.186%	66.091%
4	0.123%	0.277%	0.439%	1.635%	8.341%	19.155%	68.785%
5	0.177%	0.373%	0.607%	2.154%	10.096%	22.540%	70.541%
6	0.241%	0.480%	0.796%	2.693%	11.712%	25.435%	71.861%
7	0.315%	0.597%	1.008%	3.246%	13.198%	27.925%	72.924%
8	0.399%	0.727%	1.240%	3.807%	14.567%	30.080%	73.809%
9	0.493%	0.868%	1.492%	4.374%	15.828%	31.959%	74.561%
10	0.597%	1.022%	1.762%	4.943%	16.994%	33.608%	75.209%

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