FINA556 – Structured Products and Exotic Options

Topic 3 — Convertible bonds and equity-linked hybrid products

3.1 Convertible bonds

3.2 Structured convertibles

3.3 Risk-reward and price sensitivities of convertibles

3.4 Variable annuities products

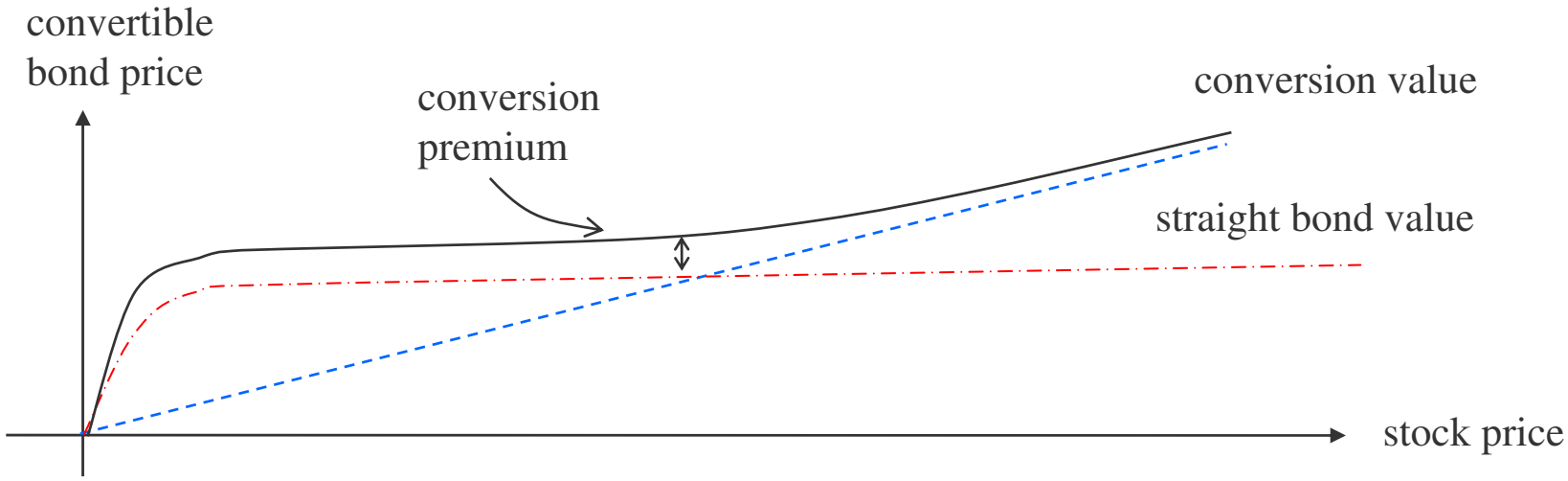
3.1 Convertible bonds

Convertible bonds as an asset class

- Equity and fixed income perspectives
- Convexity ratio
- Embedded features
  - Investor's conversion right: equity participation
  - Issuer's call right: delayed equity financing
  - Investor's put right: principal and coupons protection
- Busted convertibles - seeking investment opportunities in recovery period

Holder's perspective: take advantage of the future potential growth of issuer's company

Issuer's perspective: raise capital at a lower cost by the provision of conversion privilege to bond holders



Equity perspective on convertibles

- To take advantage of the upside potential growth of the underlying stock (participation into equity).

- Swapping the variable stock dividends in return for fixed coupon payments until the earlier of the maturity date and the conversion date.

Fixed income perspective on convertible

- Provides the "bond floor" value.

- Conversion option that allows the investor to exchange the straight bond for fixed number of shares.

Convexity ratio

- Classic "two-thirds upside, one-third downside"

- Convexity ratio is the ratio of upside and downside participation.

  For example, suppose the convertible provides 64% of the upside participation with only 34% of the downside movement, then the convexity ratio is 1.85. That is, the convertible provides 85% more upside participation than downside risk.

Insulation from volatility

The price movements of convertibles are generally far less volatile.

Convertible Bond Price Changes,
August 24, 2001-September 21, 2001

	Common Stock			Convertible		
Company	**Closing Price 8/24/2001**	**Closing Price 9/21/2001**	**% Change**	**Closing Price 8/24/2001**	**Closing Price 9/21/2001**	**% Change**
Avon Products	45.89	44.04	-4.03	51.13	51.25	0.24
Bell Atlandtic/ New Zealand Tel	17.85	14.15	-20.73	100.50	101.25	0.75
Diamond Offshare	27.90	26.35	-5.56	83.50	84.13	0.75
Federal Realty Trust	23.62	20.32	-13.97	93.25	87.13	-6.57
INCO	17.55	12.45	-29.06	99.38	97.25	-2.14
Average			**-14.15**			**-2.78**
DJIA	**10423.20**	**8235.81**	**-20.99**			**-20.99**
S& P 500	**1184.90**	**965.80**	**-18.49**			**-18.49**
NASDAQ	**1916.80**	**1423.19**	**-25.75**			**-25.75**

Call terms

Issuer has the right to call back the bond at a pre-specified call price prior to final maturity, usually with a notice period requirement. Upon call, the holder can either convert the bond or redeem at the call price.

Issuer's perspective on the call right

- To have the flexibility to call if they think they can refinance the debt more cheaply.

- To force bondholders to convert debt into equity, which can reduce debt levels and result a beneficial effect on the balance sheet. The issuer has the flexibility to shift debt into equity to reduce the leverage of the firm. In summary, it is used as a tool by issuer for possible future equity financing – managing the debt / equity balance.

Convertibles as backdoor equity financing

- Delayed equity
  - Convertibles provide a way of selling common stock at a price above the existing market.
  - They are employed as deferred common stock financing.
- The call feature is important since it gives the company the means to shift debt to equity.
- Issuance of convertibles offers a means for the issuer to control the debt/equity ratio.

Call protection

Hard (or absolute):

To protect the bond from being called for a certain period of time. Soft (or provisional): The issuer is allowed to call only when certain conditions are satisfied.

For example, the closing price of stock has been in excess of 150% of the conversion price on any 20 trading days within 30 consecutive days.

Role of call protection

To preserve the value of the equity option for the bondholders. While waiting for the stock price to increase, convertibles typically provide more income than the stock. Without the call protection, this income stream could be called away at any time. Hard call protection with longer time period is more desirable for the investors.

Put feature

Allows the holder to sell back the bond to the issuer in return for a fixed sum. Usually, the put right lasts for a much shorter time period than the maturity date of the bond.

- The holder is compensated for the lesser amount of coupons received in case the equity portion of the convertible has low value.

- It protects the holder against rising interest rates by effectively reducing the year to maturity. With smaller value of duration*, the convertible price becomes less sensitive to interest rate.

* Duration is the weighted average of times of cash flow stream, weighted according to the present value of the cash flow amount. Percentage change in bond price is proportional to negative yield change, where the proportional constant is the duration.

Convertible bond issued by the Bank of East Asia

US$250,000,000

2.00 percent Convertible Bonds due 2003

Issue date	July 19, 1996
Issue price	100 percent of the principal amount of the Bonds, plus accrued interest, if any, from July 19, 1996 (in denominations of US$1,000 each)
Conversion period	From and including September 19, 1996 up to and including July 7, 2003

Conversion feature

Conversion price	HK$31.40 per Share and with a fixed rate of exchange on conversion of HK$7.7405 = US$1.00.
Dilution protection clause	The Conversion Price will be subject to adjustment for, among other things, subdivision or consolidation of the Shares, bonus issues, right issues and other dilutive events.

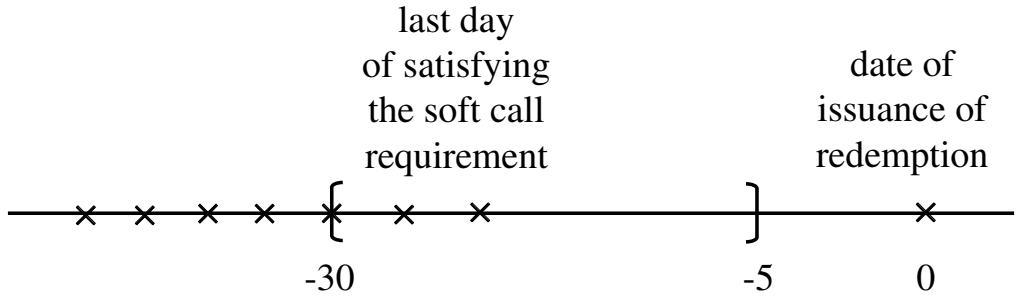
Put feature

Redemption at the option of the bond-holders	On July 19, 2001, the Bonds may be redeemed at the option of the Bondholders in US dollars at the redemption price equal to 127.25 percent of the principal amount of the Bonds, together with accrued interest.

The investors are protected to have 27.25% returns on the bond investment upon early redemption by the issuer.

Call feature

Redemption at the option of the bondholders	On or after July 19, 1998, the Issuer may redeem the Bonds at any time in whole or in part at the principal amount of each Bond, together with accrued interest, if for each of 30 consecutive Trading Days, the last of which Trading Days is not less than five nor more than 30 days prior to the day upon which the notice of redemption is first published, the closing price of the Shares as quoted on the Hong Kong Stock Exchange shall have at least 130 percent of the Conversion Price in effect on such Trading Day.



Soft call protection

The closing price has to be above 130 percent of the conversion price on consecutive 30 trading days.

- On the date of issuance of the notice of redemption (treated as day 0), the Issuer looks back 5 to 30 days (corresponds to $[-30, -5]$ time interval) to check whether the history of the stock price path satisfies the Parisian constraint. That is, the last of the 30 trading days (with closing price above 130% of the conversion price) falls in $[-30, -5]$ time interval.

- From Issuer's perspective, when the Parisian constraint has been satisfied, the Issuer has 5 to 30 days to make the decision on redemption or not.

Casino operator brings ringgit convertible

- Malaysia's only casino operator, Resorts World, has raised M$1.1 billion ($300 million) from a convertible bond that was well received despite offering a negative yield.

- Issuing the *zero-coupon bonds* at par and setting the redemption price at 99%, which results in a yield to maturity of -0.5%. Desire to see bonds convert prompts Resorts World to use rare negative yield structure.

- The *conversion price* was fixed at launch at 10% over yesterday's (September 7, 2006) volume weighted average price of M$11.593, giving an initial conversion price of M$12.75.

- *Call feature* – There is an issuer call after one year, subject to a 120% hurdle, to force conversion in case investors drag their feet.

- *Reset feature* – The reset mechanism has a floor at 90.9% of the original conversion price, which is high compared with the typical reset floor at $80-85\%$. The floor is equal to yesterday's volume weighted average price.

- *Dividend yield and credit spread* – The bonds were priced assuming a credit spread of 40 basis points over the Malaysian interest rate curve, a dividend yield of $2.2\% = 120\%$ of the previous year's, and a stock borrow cost of 5%. Note that the issuer is essentially shorting stock.

Issuer's perspectives

- While common a few years back when interest rates were much lower, negative yields are rarely seen on CBs nowadays but highlights the issuer's desire to have the bonds convert in order to get equity on its balance sheet.

- The bonds have a short maturity of only two years, a conversion premium of only about 10% and two conversion price resets – after the first year and 60 days before maturity – making it all but inevitable that the bonds will convert.

- The issuer is essentially saying that it is happy to sell equity at today's market price, but not lower. The expected appreciation of the ringgit makes the bonds attractive to foreign investors.

Investor's perspectives

- The bond floor was set at 90.7%, which one observer says is "reasonably attractive" given the strong focus on conversion and the implied volatility is 24%. This would mean if no conversion occurs at maturity, the loss in value is about 10%.

- Analysts are, however, optimistic that the company's casino operations will drive earnings growth, and of the 19 analysts that cover the company, according to Bloomberg data, 16 have a "buy" or "overweight" recommendation.

- The share price is up a modest 4.5% this year to Thursday's closing price of M$11.70, which compares with a 6.2% gain in the Kuala Lumpur Composite Index.

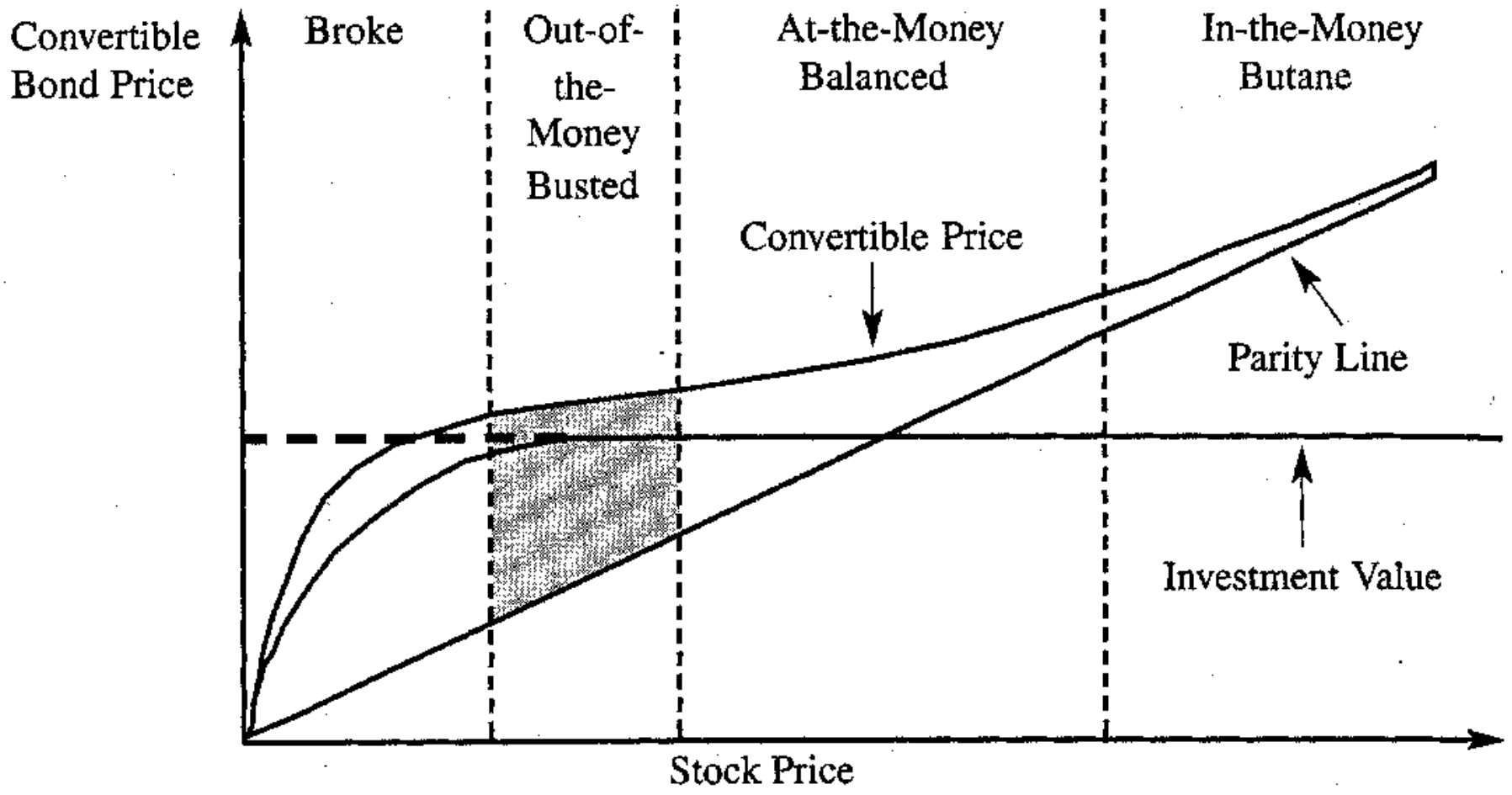
- There is no stock lending available at the moment, although Resorts World, which is a subsidiary of conglomerate Genting, is among a group of stocks that is qualified for short-selling once this becomes available.

Busted convertibles

Busted convertibles are characterized by low equity price sensitivity (low delta), large conversion premium and high yield to maturity.

- delta $< 4\%$
- conversion premium $> 75\%$
- yield more than 10%

Average credit quality of the busted convertibles is BB- versus BB+ for the entire asset class. The convertible trades on its fixed income characteristics.



Secondary Market Behavior: Busted Convertibles

Advantages

- In contrast to junk bonds, the upside potential is not capped. So, it may enjoy unlimited upside potential if the stock price recovers.

- With busted convertibles, the equity warrant (deep out-of-the-money) is often mispriced. Investors are effectively buying high yield debt with a free equity kicker.

- Busted convertibles are more attractive investment than high-yield debts in a modern economy that has shifted from slow growth, cyclical companies to more volatile growth companies.

Disadvantages

- Busted convertibles are often more illiquid. Traditional convertible investors become sellers as equity sensitivity diminishes.

- Convertible securities are generally subordinate to other creditors in the event of a liquidation or bankruptcy.

- The biggest risk is continued credit deterioration.

  Analyzing busted convertibles is a research intensive process involving both equity and credit analysis.

Lattice tree algorithm

One-factor binomial model

- stock price process follows binomial random walk
- interest rates to be deterministic

Two discount rates

1. If the convertible is certain to remain a bond, it is appropriate to use a discount rate corresponding to the creditworthiness of the issuer – risky rate.
2. Suppose the bond is certain to be converted, it is then appropriate to use the riskfree rate.

At maturity, the holder will choose the maximum between the par value and the value of stocks received upon conversion.

How to account for the creditworthiness of the issuer?

The discount rate to be used when we roll back is given by

$$pw_u + (1-p)w_d.$$

Here, p is the probability to a node where the discount rate is wu and $(1-p)$ is probability to a node with w_d. The appropriate discount rate is the weighted average of the discounted rates at the nodes in the next time step.

conv = value of stocks received if conversion takes place

call = call price

roll = value given by the rollback (neither converted nor recalled)

At each node, the optimal strategy of the holder is exemplified by taking the maximum of min($roll, call$) and $conv$.

- The maximum operation reflects the conversion right, which persists with or without recall by the issuer.

- min($roll, call$) gives the bond value when the bond can be called or not called by the issuer.

Dynamic programming procedure:

$$\max(\min(roll, call), conv).$$

Alternative dynamic programming procedure:

$$\min(\max(roll, conv), \max(call, conv))$$

- The term $\max(roll, conv)$ represents the optimal strategy of the holder. Upon recall, the holder chooses to accept the call price or convert into shares. This can be represented by $\max(call, conv)$. The issuer chooses to recall or to abstain from recalling in order to minimize the option value.

The two procedures are seen to be equivalent via the distributive rule over "max" operation [analogous to $5 \times 7 + 5 \times 9 = 5 \times (7 + 9)$]. Here, we take the "max" operation between conv and roll, then between conv and call, and perform the "min" operation afterwards.

Numerical example

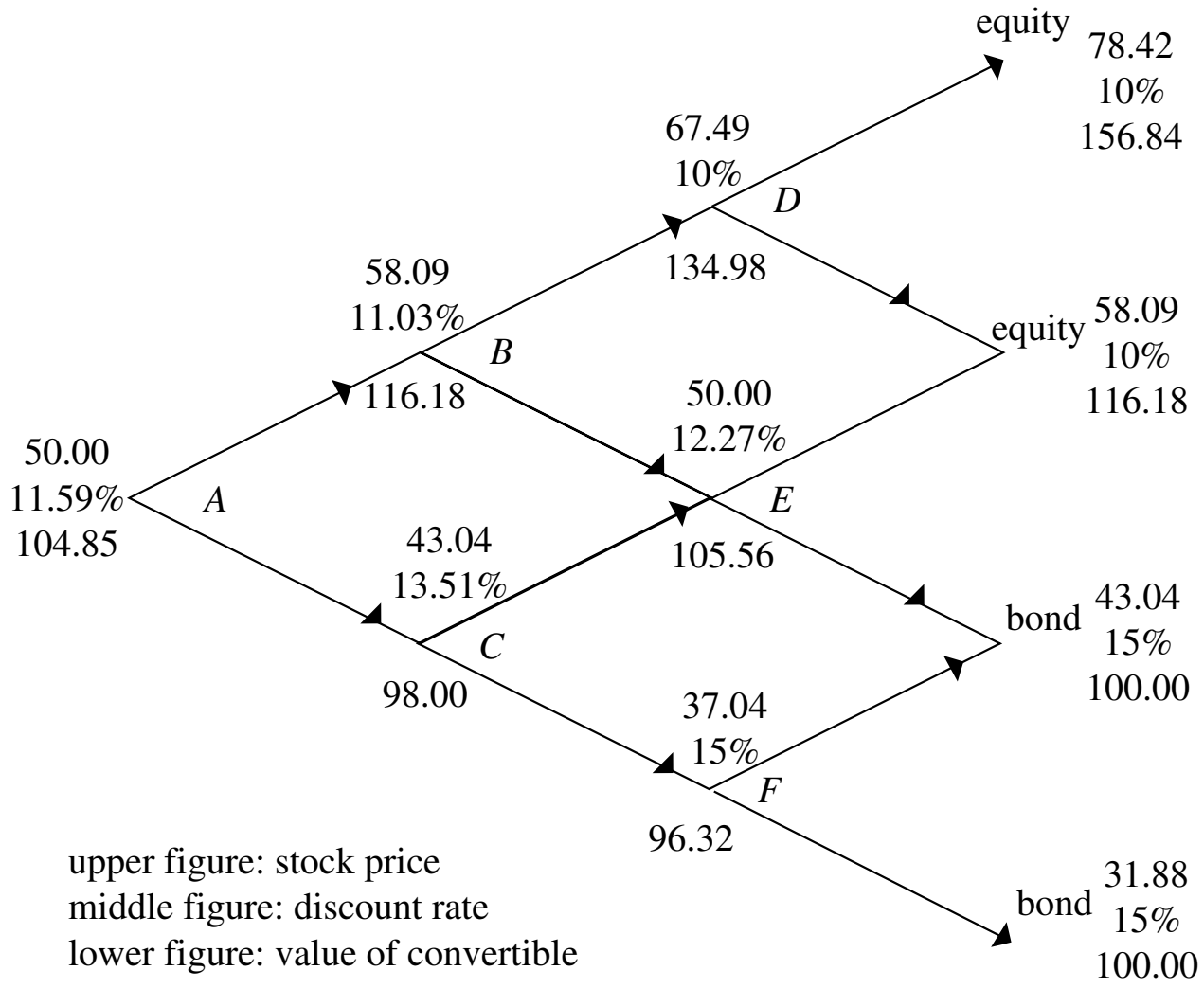
A 9-month discount bond issued XYZ company with a face value of \$100. Assume that it can be exchanged for 2 shares of companys stock at any time during the 9 months.

- It is callable for \$115 at any time.
- Initial stock price $=$ \$50, $\sigma = 30\%$ per annum and no dividend; risk-free yield curve to be flat at 10% per annum.
- Yield curve corresponding to bonds issued by the company to be flat at 15%.
- Tree parameters are: $u = 1.1618, d = 0.8607, p = 0.5467$,

$$R = e^{0.1\Delta t} = 1.0253.$$

- At maturity, the convertible is worth $\max(100, 2S_T)$.

Binomial tree for pricing a risky convertible bond



At node D

Roll back gives the bond value

$$(0.5467 \times 156.84 + 0.4533 \times 116.18)e^{-0.1 \times 0.25} = 134.98.$$

The bondholder is indifferent to conversion or hold, also the issuer is also indifferent as to whether the bond is called; the correct discount rate at node D is 10%.

At node F

The correct discount rate is 15% since the convertible is contain not to be converted if node E is reached.

At node E

The correct discount rate is

$$0.5467 \times 10\% + 0.4533 \times 15\% = 12.27\%.$$

The value of convertible at E

$$(0.5467 \times 116.18 + 0.4533 \times 100)e^{-0.1227 \times 0.25} = 105.56.$$

The bond should be neither converted nor called.

At node B

The discount rate is

$$0.5467 \times 10\% + 0.4533 \times 12.27\% = 11.03\%$$

and value of convertible is

$$(0.5467 \times 134.99 + 0.4533 \times 105.56)e^{-0.1103 \times 0.25} = 118.34.$$

It is optimal to call the bond at node B so that it causes immediate conversion and lead to \$116.18. The discount rate at node B should be taken to be 10%, since optimal conversion by the holder takes place at this node.

At node A

The discount rate is

$$0.5467 \times 10\% + 0.4533 \times 13.5\% = 11.59\%.$$

The convertible value at node A is

$$(0.5467 \times 116.18 + 0.4533 \times 98.00)e^{-0.1159 \times 0.25} = 104.85.$$

Value of the conversion option

If the bond has no conversion option, its value is

$$e^{-0.75 \times 0.15} = 89.36.$$

The value of conversion option $= 104.85 - 89.36 = 15.49$.

3.2 Structured convertibles

- Reverse convertibles
- Mandatory convertible securities
- Exchangeable convertible bonds
- Convertibles with conversion price reset

Reverse convertibles

- Bond aspect

  The payment of the juicy coupon is dependent on the stock prices movement.

- Equity aspect

  Selling a put option on the underlying stocks – obliged to take delivery of a predetermined number of units of the underlying (typically not the stock of the issuer) if the stock price falls below the strike.

- Tradeoff between the juicy coupons received and put option sold.

- Usually, the issuer has the right to call back the bond – upside gain to investors is further capped.

- Shorter life compared to convertibles – typically six months to two years.

24 Month Callable Dual Accrual Cash or Share Security on Wal-Mart Stores, Inc and Intel Corp.

issued by Merrill Lynch

Payment/delivery on the maturity date

- If the settlement prices of BOTH the underlying stocks are higher than or equal to the respective exercise price, each warrant holder will receive 100% of the notional amount per warrant held.

- If either one of the settlement prices is lower than the respective exercise price, each holder will receive per warrant physical delivery of a number of the Worst performing stock equal to

  Notional amount / exercise price of worse performing stock

Forced conversion

This represents an automatic *forced conversion* when the share prices decline (opposite effect to that of a convertible bond).

Issue size:	$10,000,000$ warrants
Minimum subscription:	$100,000$ warrants
Notional Amount:	USD 1 per warrant
Issue Price:	100% of the Notional Amount
Issue Date:	Feb. 23, 2006
Maturity Date:	Feb. 19, 2006

Underlying stocks (uncorrelated)

	Reference price	Exercise price
Wal-Mart Stores Inc.	USD45.48	USD39.5676
Intel Corp	USD20.77	USD18.0699

- Exercise price $= 87\% \times$ reference price

$$\begin{aligned} \text{Terminal payoff} &= \min\left(1, \min\left(\frac{S_1(T)}{S_1^*}, \frac{S_2(T)}{S_2^*}\right)\right) \\ &= 1 - \max\left(1 - \min\left(\frac{S_1(T)}{S_1^*}, \frac{S_2(T)}{S_2^*}\right), 0\right), \end{aligned}$$

where S_1^* and S_2^* are the reference prices of asset 1 and asset 2.

The investor shorts a put on the minimum of two assets.

Additional coupon (accrual feature)

Unless the warrants have been called, over each observation period (3-month period), the holder receives $4.075\% \times n/N$ of notional amount

where $N =$ number of New York Business Days in the period in the applicable Observation Period;

$n =$ number of New York Business Days in the applicable Observation Period on which the closing prices of BOTH the Underlying Stocks are at or above the respective Exercise Price.

This is like an accrual note with the underlying index being the minimum of two share prices.

- The accrual feature can be viewed as *a series of daily binary options* which pay

$$4.075\%/N \times \text{ notional amount}$$

when

$$\min\left(\frac{S_1(T)}{S_1^*}, \frac{S_2(T)}{S_2^*}\right) > 1.$$

Issuer's Call

- On any of the Observation Date, provided that BOTH underlying stocks are greater than or equal to the reference prices, the issuer can call by paying 100% of the Notional Amount.

- This occurs when the value of the embedded put is less than the present value of the enhanced yield over the remaining period.

  This "call" right given to the issuer is like a Bermudan put option.

Overall description

- The investor believes that the prices of BOTH underlying shares at maturity will remain at a level above or equal to their respective Exercise Prices, earning an enhanced yield.

- The warrant pays out a fixed 4.075% coupon for the first quarter.

- The coupon received would depend on the trading path of BOTH underlying stocks due to the accrual feature.

Decomposition into various components

- Reverse convertibles
  = bond (series of binary options under the accrual feature)
  - European put on minimum of two uncorrelated stocks
  - issuer's call (Bermudan put)

Question

Would the investor prefer the two stocks to be positively correlated, uncorrelated or negatively correlated?

1. Market risks – underlying shares

2. Credit risk – default of Merrill Lynch

3. Liquidity risk – will not be listed on any securities exchange and do not expect a trading market with only Merrill Lynch as a possible buyer. There is no guarantee that the secondary price will reflect changes in the underlying price. Treat them as a buy and hold investment.

4. Interest rate risk – bond component: par plus coupons and issuer's call.

Undesirable investment choices

Overweight the sure coupon and underestimate the risks involved

- When the stock market is volatile, the volatility of stock is higher. The put option sold is more valuable.

- Stock returns are not normally distributed. The combination of high volatility, negative skewness (more tendency to stay below the mean) and excess kurtosis (fat tails) creates the potential for very large losses.

Examples of bitter stories in the markets

- In Sept., 2007, investors bought reverse convertibles linked to Countrywide Financial Corp. with a yield of 22%. The share price sank more than 70% by the time the notes matured 6 months in Mar. 2008. Investors lost more than 50% of their money even after interest payments.

- In Oct. 2007, Barclays issued a reverse convertibles linked to Bear Stearns. The yield was 12.3%. Bear Stearns stock sank by more than 90% when the note matures.

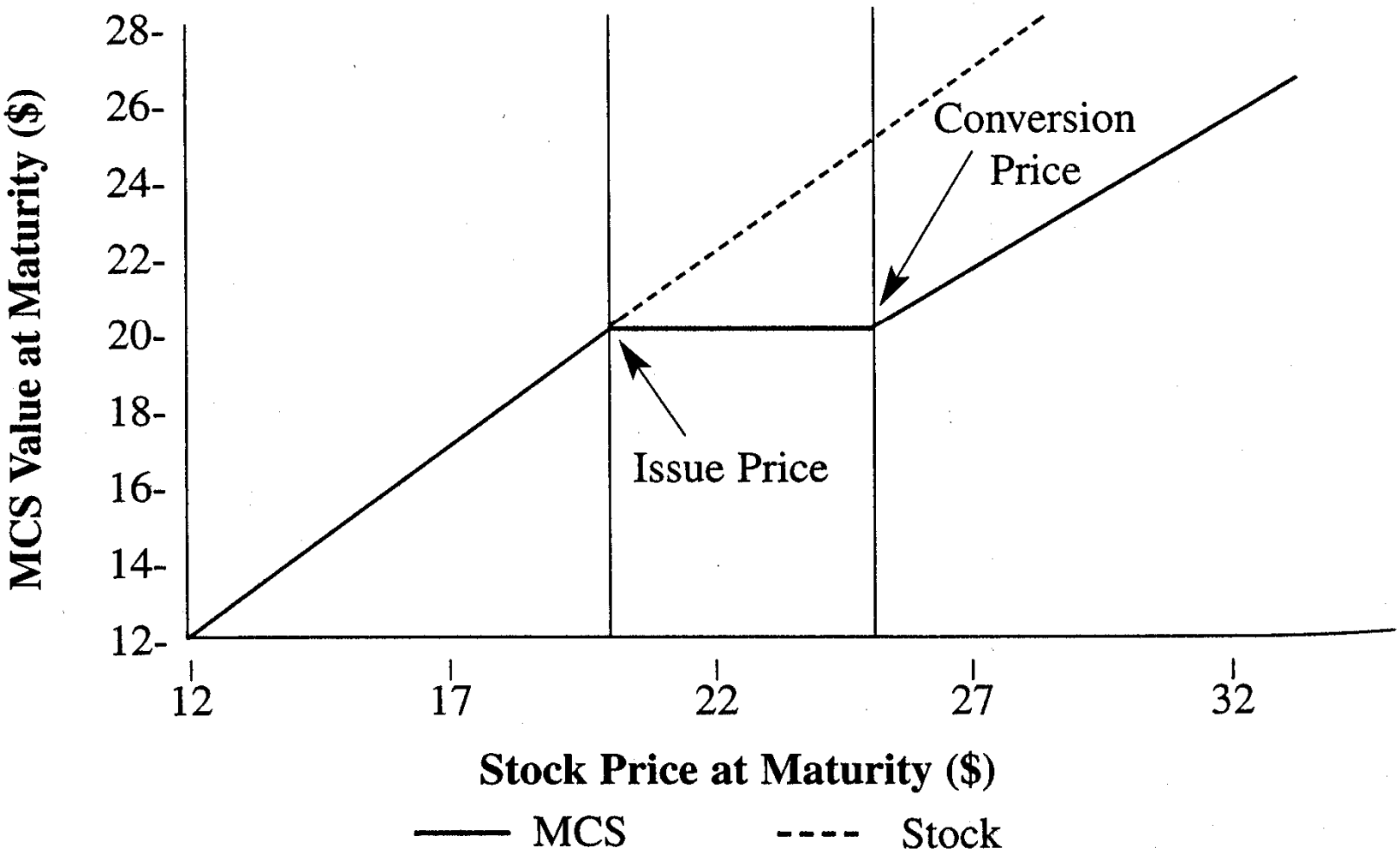
Pricing bias in favor of the issuer

- On 468 issues outstanding in April 2008, a study found an average over pricing of at least 3.4%. The overpricing is positively related to the coupon level, indicating that investors tend to overweight the sure coupon and underestimates the risk involved.

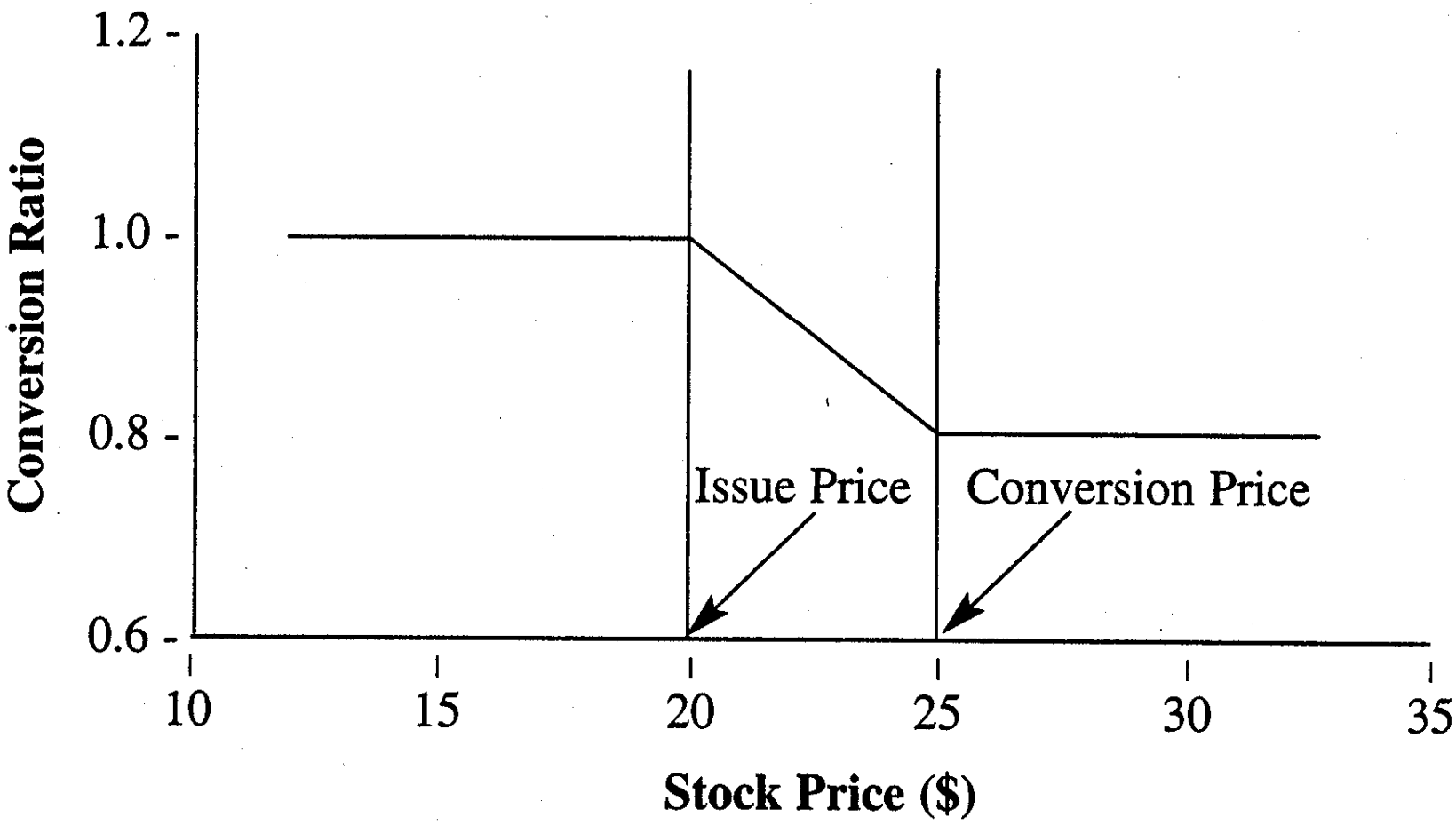
Complexity of financial investments is designed in favor of the issuer, not the buyer.

Mandatory convertible securities (MCS)

- Mandatory convertible into common stock at maturity.
- They are effectively yield-enhanced common stock, and offer no downside protection to the investor apart from their higher yield.
- At issue, the MCS is priced with a so-called conversion premium, which determines the level of the strike price for the long call in the call spread (the upper strike).
- The conversion ratio at maturity changes depending on the price of the stock.



Payoff of an MCS at Maturity



MCS: The Mysterious Changing Conversion Ratio

An MCS consists of the following pieces

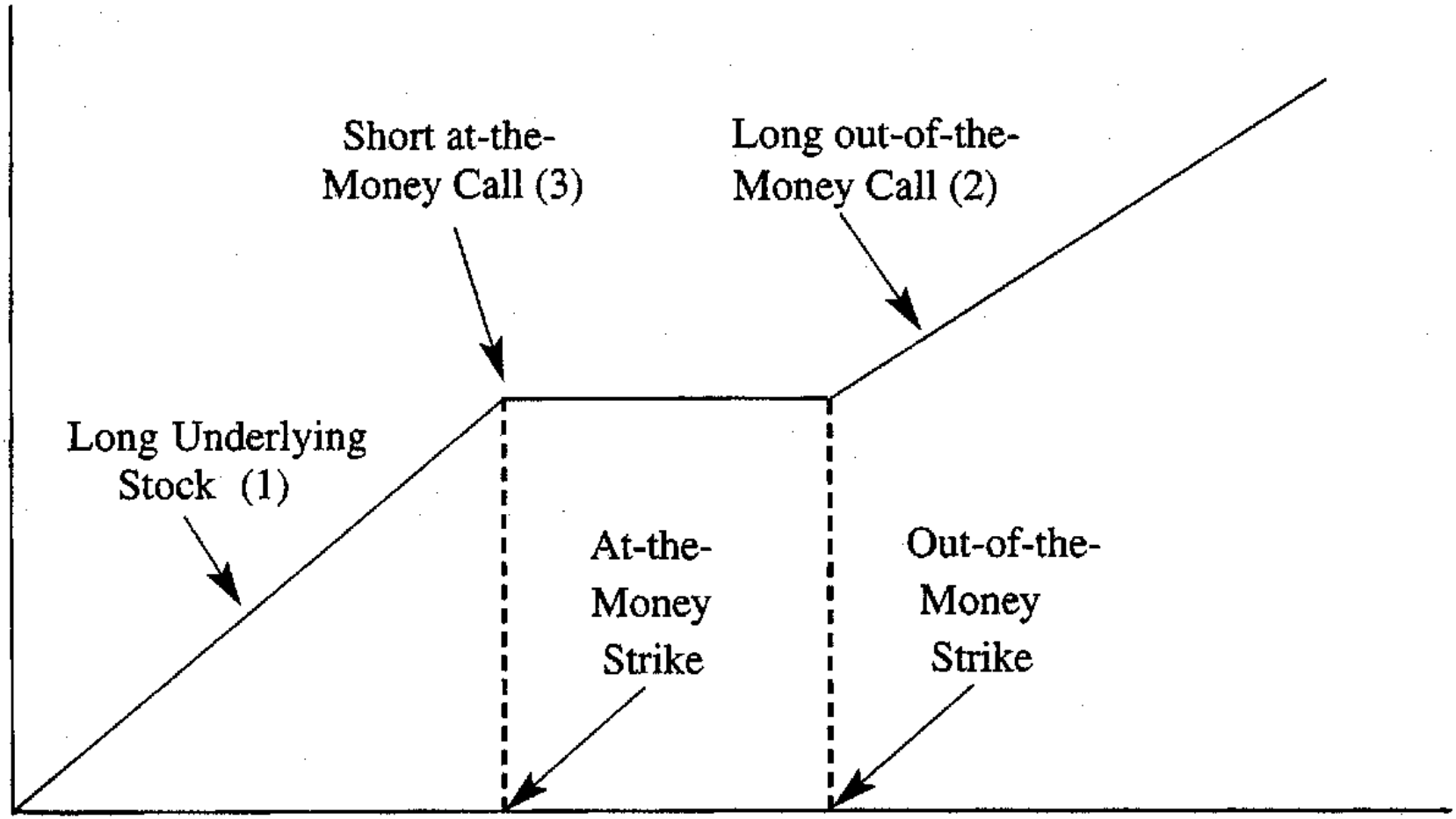
MCS = underlying common stock (stock price $\times$ lower conversion ratio)

$+$ (out-of-the-money call option on the underlying common stock struck at the upper strike price) $\times$ upper conversion ratio

$-$ at-the-money call option on the underlying common stock struck at the lower conversion ratio

- In return, the investor receives a higher dividend.

- Less interest rate sensitive but more equity sensitive compared to convertibles.



The Anatomy of an MCS

Numerical example

Stock price at issue	\$27.875
Upper strike	\$30.750
Lower strike	\$25.000
Valuation	
Long stock value	\$27.875
Long 0.8130 calls struck at \$30.750	\$5.411
Short 1 call struck at \$25	−\$9.228
Present value of net cash flow	+\$4.391
Fair value	\$28.450

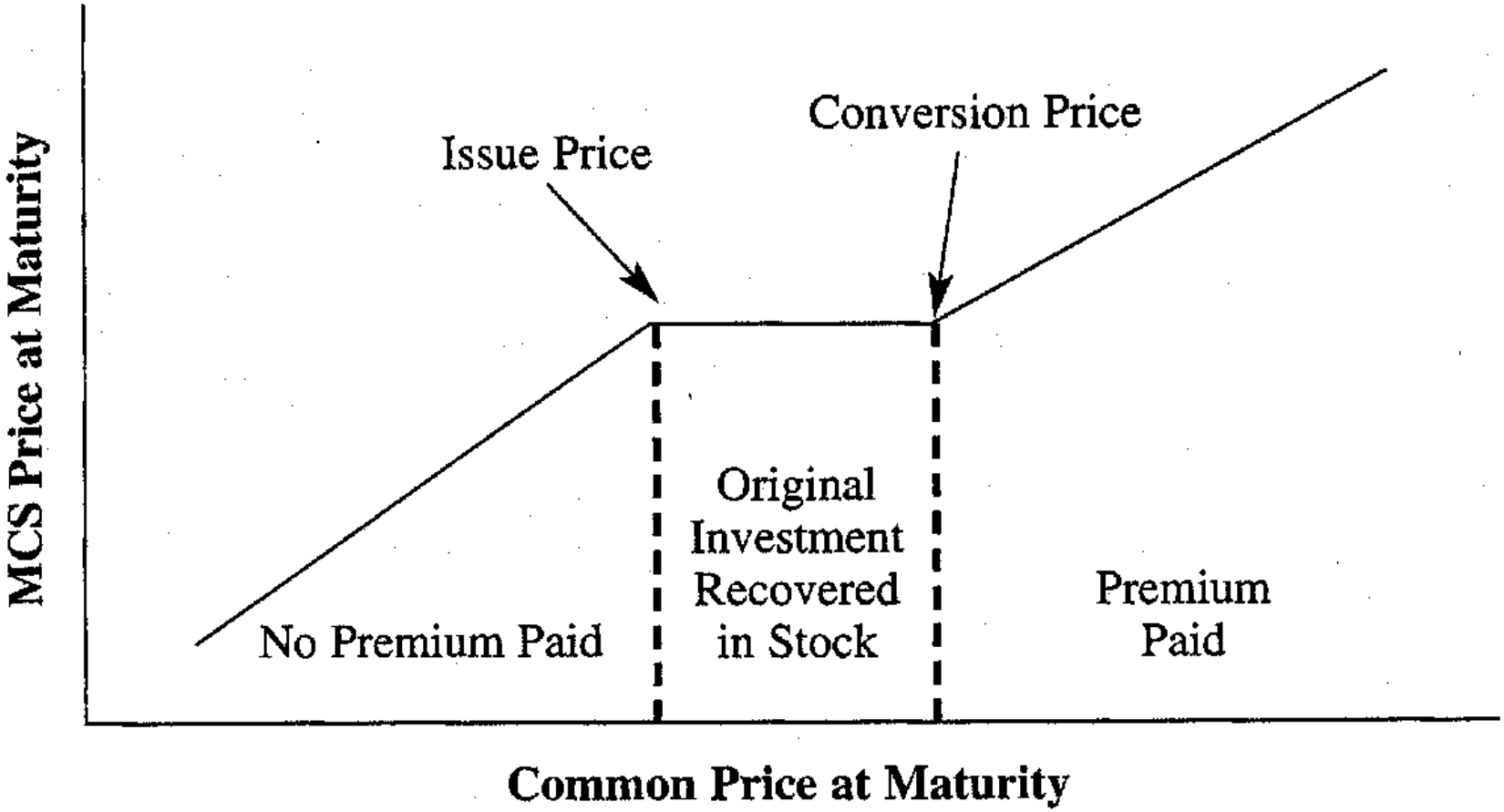
Performance based conversion premium

1. If the stock goes up from the issue price, the participation is at first delayed until the point of the upper strike, and then rises at a reduced rate equal to the upper conversion number.

2. On the downside, participation is one-for-one with the stock.

The investor does not actually pay the conversion premium upfront. The declining ratio represents the conversion premium paid by the investor – paid only when the stock performs well.

Both issuer's and investors' interests are aligned.



MCS: Performance Based Conversion Premium

Parallel debt MCS

- The higher dividend paid by the issuer is not tax deductible.

  To get around the problem: pairing of the equity MCS with a debt security.

  - All the proceeds from the sale of the MCS are invested in US Treasuries with maturities same as that of the MCS.
  - The yield from the Treasuries is supplemented with an additional fee from the issuer to arrive at the stated yield on the MCS.

Parallel debt

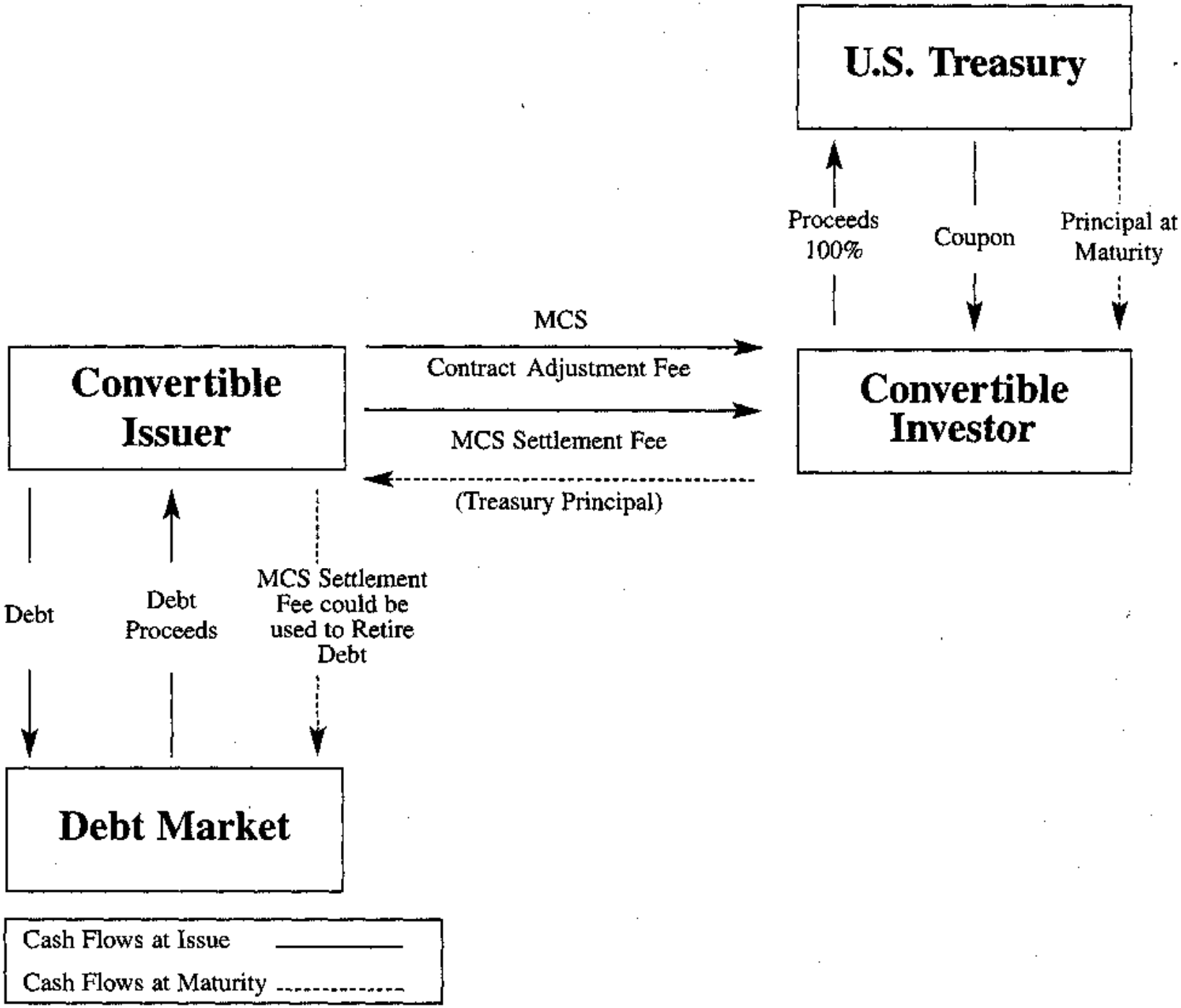
- The issuer enters the public debt market to issue an interest bearing note with a maturity and face amount similar to the terms of MCS.

- At maturity, the investor delivers either cash (the settlement fee) or the maturing Treasury note to satisfy the terms of the purchase contract of the MCS.

- In return, at maturity, the issuer can use these proceeds to retire the corporate debt obligation.

- The Treasuries are owned by the investor – so the investor does not need to bear the default risk of the issuer.

- The investor also enjoys a tax benefit from this structure since that portion of the income received from the Treasury coupon payments is exempted from state and local taxes.



MCS: Parallel Debt Structure

Exchangeable convertible bonds

Issued by one company and converted into the stock of another company

- Pennzoil owned over 18,000,000 shares of Chevron common stock. Using this stock as collateral, Pennzoil issued over $\$902,000,000$ worth of bonds convertible into Chevron shares.

- Advantages:
  - Received the proceeds for selling the issues at a 21% premium over Chevron's current stock price.
  - Pennzoil received $33.4 million annually in dividends from the Chevron shares.
- Disadvantages
  - Forfeit the potential upside growth of the stock price.

Rationale for issuing exchangeables

- The issuer wants to monetize the value of a non-strategic asset in a tax-efficient manner.

- An alternative form of capital raising. The shares in a third company may be held due to aborted takeover.

  – The issuer receives the proceeds of the sale immediately (at a premium to the current share price and may gain advantage from higher volatility of share price prior to aborted takeover), but does not have to pay capital gains tax until the bonds are actually converted several years in the future.

Convertible bonds with reset feature

In most cases, the reset on conversion price is downward and this makes the bond more valuable. For example, the conversion number is reset by dividing the par by the prevailing stock price.

Floor limit

The extent of downward reset cannot be below a certain multiplier of the first conversion price.

Examples of reset convertibles

United Artists Communications issued convertibles that after a fixed period of time, the bonds were evaluated by an independent investment banker. This is to determine the coupon rate that would allow the bonds to trade at 101 plus accrued interest.

Mitsuibishi Bank issued $2 billion of 7-year bond with annual reset of the conversion ratio. It offers investors more shares if the stock price declines, with the goal of keeping the bonds equity value at par.

Popular in Japan in mid-1990's

Japanese banks were considered quite risky as they had large real estates exposures.

To raise capital

a. equity issuance was out of the question since the stock markets were depressed;

b. straight bond issues would have required a high coupon yield.

Reset feature was included in convertible bonds to give investors some sort of insurance against issuer's stock decline

– as sweetener for investors.

Impact on bond price At high stock price (not likely to reset) or low stock price (low equity value) regions, the reset premium is low. The reset premium is significant only at intermediate stock price level.

Nightmare for the issuers

The feature is too sweet for the investors and harmful to the issuer.

- When the stock price drops, the investors are compensated.
- When the stock price rises, the conversion option value becomes more expensive.

These structures have fallen from popularity in recent years.

3.3 Risk-reward and price sensitivities of convertibles

- Over the period for which reliable long-run data are available (since early 1970s), the total return performance of US convertibles has virtually replicated that of the S&P 500, but with significantly lower risk.

- Over the same period, convertibles have significantly outperformed long-term corporate bonds while demonstrating comparable risk.

- Total return for convertible bonds has demonstrated a much higher correlation with the S&P 500 than with the corporate bond market.

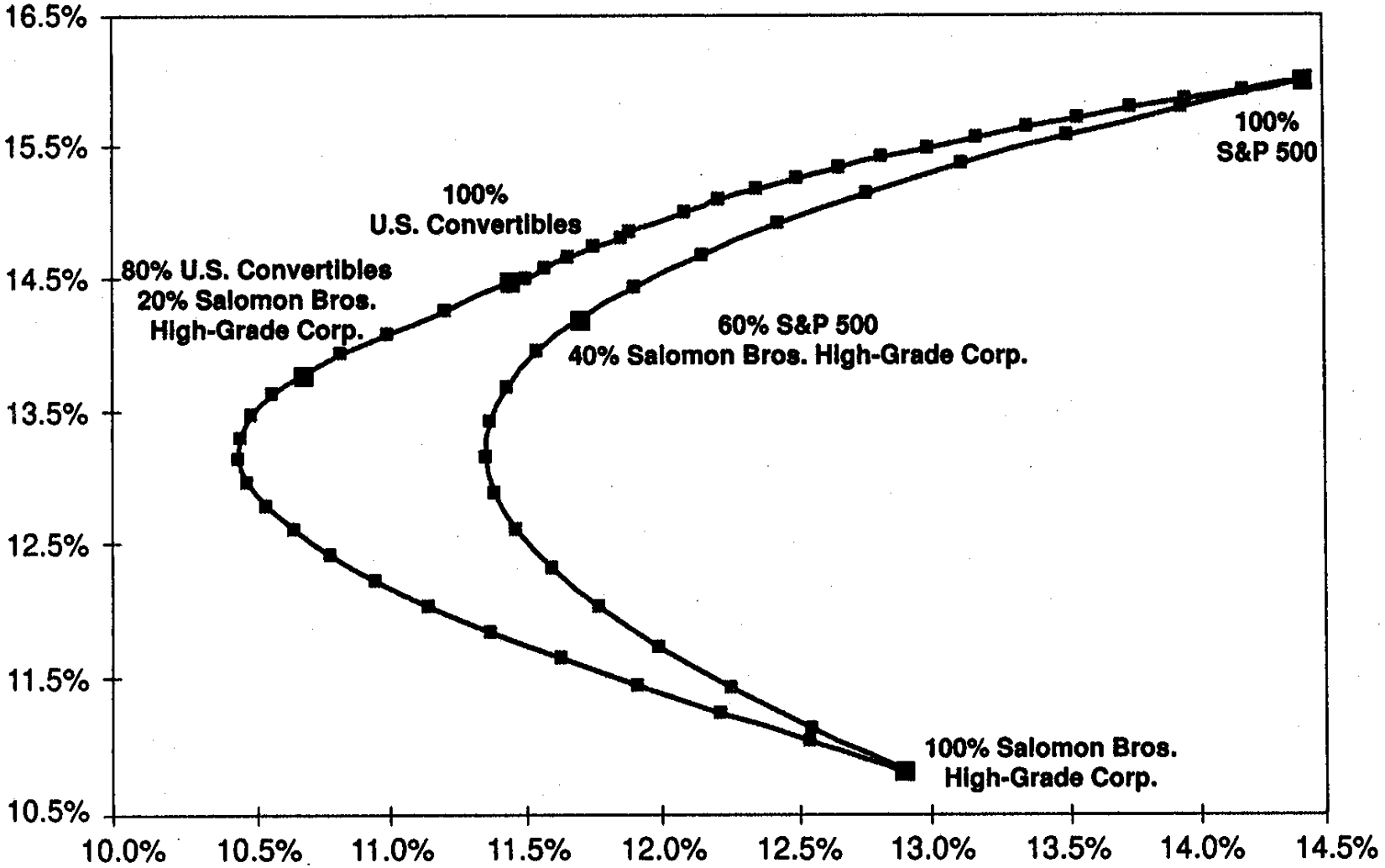
- Convertibles can help maximize performance in both equity and fixed-income portfolios.

Risk-reward relationship for various asset classes of investment

Performance of various asset classes, 1973-1995

	Compound annual return	standard deviation
Convertible bonds	11.70%	12.47%
S&P 500	11.84%	17.27%
Long-term corporate bonds	9.66%	12.44%
Intermediate-term corporate bonds	9.91%	8.93%

Source: Goldman Sachs Global Convertible Research (1996) "Convertibles as an Asset Class."



- Adding convertibles to either bonds or stocks moves the efficient frontier lower in terms of risk and higher in terms of reward.

Valuation of convertible debts

List of parameters

- Coupon rate
- Creditworthiness of the issuer
- Maturity date
- Ratio of conversion price to current stock price
- Volatility of the stock price
- Dividend of the stock price
- Presence of other embedded option features, like callability and puttability
- Prevailing risk free interest rate and volatility of interest rate
- Correlation of the stock price with the interest rate

Interest rate sensitivity

1. The exercise price is a function of the investment value. An increase in interest rates will lower the investment value.

2. However, the exercise price of the embedded call is reduced. A lower exercise price will increase the value of the warrant.

	Basic price	Int rate 1%	Change	Int rate −1%	Change
Investment value	\$847.84	\$812.75	−\$35.09	\$884.74	\$36.90
Warrant value	\$337.66	\$362.58	+\$24.92	\$312.72	−\$24.94
Total	\$1185.50	\$1175.33	−\$10.17	\$1197.47	+\$11.92
Percent change			−1.02%		1.19%

Duration

Duration is the weighted average of the times that the principal and interest payments are made.

$$duration = \frac{\sum_{t=1}^{n} tC_t/(1+i)^t}{\sum_{t=1}^{n} C_t/(1+i)^t}$$

where

t is the time of payment

C_t is the coupon and/or principal payment

i is the market yield.

Duration analysis provides a measure how bond values change with changing interest rates.

Duration analysis applied to convertibles

The approximation for the convertible bonds interest rate sensitivity

$$D^{cv} = D_{adj}\left(1 - \frac{C/I}{2}\right)$$

where $C =$ conversion value and $I =$ investment value.

- The equity component of the convertible bond may dampen the convertible's interest rate sensitivity, depending on the bond's equity participation.

- Convertibles trading high above their investment value will be less sensitive to interest rates.

Duration and coupon

- For non-convertible bonds, the duration decreases as their coupon increases. This is because higher coupon bonds deliver more cash flows near the start of bond's life.

- With convertible feature, the higher coupon rate may lead to lower propensity to convert. The CB then has a longer life, so this leads to higher duration.

  These two effects are counteracting.

Correlation with interest rate

- Consider the impact of an increase on interest rate
  - The future share price is expected to be higher because of higher drift rate.
  - Due to negative correlation between interest rate and share price (say, the S&P 500-stock index has a correlation of about minus 0.5), the share price drops first.

- Negative correlations normally lower the CB value; positive correlations make the CB worth more.

- In some situation, CBs may have price differences in the range of $10 - 15\%$ when correlation moves from 1.0 to -1.0.

3.4 Variable Annuities products

- Insurance companies have created a variety of products that enable their policyholders to participate in bull markets while also providing downside protection in bear markets.

- Variable annuity policies exist to attract investment dollars to insurance companies from the mutual fund investments that they resemble.

- The main benefit from a VA policy is the accumulated investment; additional benefits come from policy "rider" – secondary features attached to an insurance policy.

Variable Annuities

- Policyholders gain equity participation (bull markets) with downside protection (bear markets).

- Offers a range of investment choices, typically mutual funds that invest in stocks, bonds, money market instruments, or some combination of the three. Attract investment dollars to insurance companies from the mutual fund investments that they resemble.

- Accumulated investment and annuity payouts; plus various policy riders as sweeteners to buyers.

Accumulation phase

- During the accumulation phase, the investors make purchase payments, which they can allocate to a number of investment choices.

- Allowed to transfer the money from one investment choice to another without paying tax on the investment income and gains, although the investors may be charged for a fee by the insurance company for transfers.

Payout phase

- Receive purchase payments plus investment income and gains (if any) as

  (i) lump-sum payment,

  (ii) annuities – a stream of payments at regular interval (generally monthly).

- Can choose to have the annuity payments last for a period that is preset (such as 20 years) or the lifetime.

- During the payout phase, holders may choose between receiving payments that are fixed in amount or payments that vary based on the performance of investment.

- Some annuities do not allow to withdraw money from the account once started receiving regular annuity payments.

- If you withdraw money from your account during the early years of the accumulation phase, you may have to pay "surrender charges". You may have to pay a 10% federal tax penalty if you withdraw money before the age of $59_{1/2}$.

Tax-deferred

- Pay no taxes on the income and investment gains from the annuity policy until money is withdrawn.

- Allowed to transfer the money from one investment choice to another within variable annuity without paying tax at the time of the transfer.

- When money is taken out of the variable annuity, the holder will be taxed on the earnings at ordinary income tax rates rather than the lower capital gains rates.

VA as investment vehicles

1. Variable annuities are not suitable for meeting short-term goals since substantial taxes and insurance company charges may apply if you withdraw the money early.

2. Variable annuities also involve investment risks, just as mutual funds do. Capital gains not taxed immediately.

3. Attractive policy riders and guarantees.

Although variable annuities are typically invested in mutual funds, variable annuities differ from mutual funds in several important ways:

- Variable annuities let you receive periodic payments for the rest of your life (or the life of your spouse or any other person you designate).

- The feature offers protection against the possibility that, after you retire, you will outlive your assets.

- The major risk faced by the issuer is the mortality risk

Typical guarantees (policy riders)

(GM = Guaranteed Minimum)

- GM Death Benefit – Guarantees a payment on death that may exceed the account value. Age cutoffs are common - maximum age for purchase and the attained age at which the benefit freezes.

- GM Income Benefit – sold on a deferred annuity, guaranteeing a certain minimum value on annuitization. The holder may decide when to annuitize the contract (usually with 7-10 years waiting period).

Guaranteed Minimum Withdrawal Benefit (GMWB)

- An option to withdraw a certain fixed percentage (seven percent is typical) of the initial deposit every year until the entire principal is returned.

- Assume an investor invests $\$100,000$ in a contract with this feature. This amount is placed in an investment account that behaves like a mutual fund.

- Assuming 7% withdrawal allowance, the policyholder could withdraw $\$7,000$ each year until the total withdrawals reach $\$100,000$. This would take just over 14 years. The policyholder can withdraw the funds irrespective of how the investment account performs.

Numerical example

- Suppose the investment account earns 10% in the first two years but earns returns of -60% in each of the next three years.

Year	Rate earned during the year	Fund before withdrawals	Fund after withdrawals	Amount withdrawn	Guaranteed remaining balance
1	10%	110,000	103,000	7,000	93,000
2	10%	113,300	106,300	7,000	86,000
3	-60%	42,520	35,520	7,000	79,000
4	-60%	14,208	7,208	7,000	72,000
5	-60%	2,883	0	7,000	65,000

- At the end of year five before any withdrawal the value of the fund, \$2,883, is not enough to cover the withdrawal payment of \$7,000.

At this stage the guarantee kicks in: the value of the fund is set to zero and the policyholder's ten remaining withdrawal payments are financed under the insurance company guarantee.

- If the market does really well the policyholder participates in this growth. Suppose the investment account grows at a compound annual rate of ten percent the policyholder would have an account value of $\$183,925$ after fourteen years.

What sort of option our investor has acquired under this rider?

To simplify matters, assume the policyholder starts taking the withdrawal benefit annually from the end of the first year. The package can be viewed as a guaranteed annuity − certain of $\$7,000$ per annum for 14 years plus a 14-year European call option on the investment account.

- Recall that the account is depleted by the periodic withdrawals so there is a chance it will be wiped out.

- The strike price on this call option is zero, because the policyholder is entitled to the full account balance after the fourteen years.

- If the market does well there will be funds left at the end whereas if performance is bad the account balance will have shrunk to zero before the principal is repaid and will remain there.

How the benefit is funded?

- A common way is using a percentage deducting from the account balance.

- For a contract with a seven percent withdrawal allowance, a typical charge is around 40 to 50 basis points. This can be charged against the account value or the guaranteed remaining balance.

- The insurance company receives this fee for providing the guaranteed withdrawal option. So if the fund earned ten percent in the first year and the fee was fifty basis points the effective return to policyholder is 9.5 percent.

- The insurance company receives this fee as long as there is a positive balance in the account. Should the account go to zero the fee income stops.

Numerical example revisited

Suppose the initial lump sum investment of $\$100,000$ is used to purchase 100 units of the mutual fund, so each unit worths $\$1,000$.

- After the first year, the rate of return is 10% so each unit is $\$1,100$. The guaranteed withdrawal of $\$7,000$ represent $\$7,000/\$1,100 = 6.364$ units. The remaining number of units of mutual fund is $100 - 6.364 = 93.636$ units.

- After the second year, there is another rate of return of 10%, so each unit of mutual fund worths $\$1,210$. The withdrawal of $\$7,000$ represents $\$7,000/\$1,210 = 5.785$ units, so the remaining number of units $= 87.851$.

- There is a negative rate of return of 60% in the third year, so each unit of mutual fund worths \$484. The withdrawal of \$7,000 represents \$7,000/\$484 = 14.463 units, so the remaining number of unit = 73.388.

- Depending on the performance of the mutual fund, the total number of units, withdrawal can be less than 100 (if the fund is performing) or otherwise.

  - In the former case, the holder receives the guaranteed total withdrawal amount of \$100,000 (neglecting time value) plus the remaining units of mutual funds held at maturity.

  - If the mutual fund is non-performing, then the total withdrawal amount of \$100,000 is guaranteed (the corresponding total number of units of mutual fund withdrawn would be more than 100).

Guaranteed Annuity Options (GAO)

GAOs are options available to holders of certain pension policies

- The policyholder pays either a single or a regular premium, securing a guaranteed benefit at a specified age (retirement age – not actually coincide with actual retirement).

- The guaranteed benefit is an amount of cash with an option to an annuity at a guaranteed rate.

Guaranteed Annuity Rate

In the United Kingdom, the GAO guarantees a minimum conversion rate of lump sum into annuity.

- Typically, guarantees of $\pounds 111$ annual annuity per $\pounds 1,000$ maturity lump sum have been offered for male policyholders, and around $\pounds 91$ annuity per $\pounds 1,000$ maturity lump sum for females.

- The conversion rate is known as the guaranteed annuity rate (GAR).

1997 survey in United Kingdom

Annuity ($\pounds$ p.a. per $\pounds 1,000$)	Number of contracts
114 or more	5
108 – 114	46
102 – 108	12
97 – 102	24
93 – 97	10
93 or less	9
	106

Interpreted as a call option with variable notional

- Let g be the guaranteed annuity rate (e.g., $g = 1/9$ for a rate of 111 annuity per 1000 lump sum), and let $a_x(t)$ be the market price at t of a whole-life annuity of 1 per year payable immediately to a life aged x. The value of the separate fund account at t is F_t.

- The payoff under the GAO at the maturity of the separate fund account, $t = n$ (which is the annuity vesting date), for a life age 65 at vesting, is

$$\max(gF_n a_{65}(n) - F_n, 0).$$

  This option is in-the-money when $a_{65}(t)$ is greater than $1/g$ and out-of-the-money otherwise.

Liabilities faced by insurers

- These guarantees were popular in UK retirement savings contracts issued in the 1970's and 1980's when the long term interest rates were high.

- At that time, the options were very far out of the money and insurance companies apparently assumed that interest rates would remain high and the guarantees would never become active. In the 1990's, long term interest rates began to fall.

- Another two factors that cause more losses

  1. Strong stock market performance;

  2. Improvement of mortality.

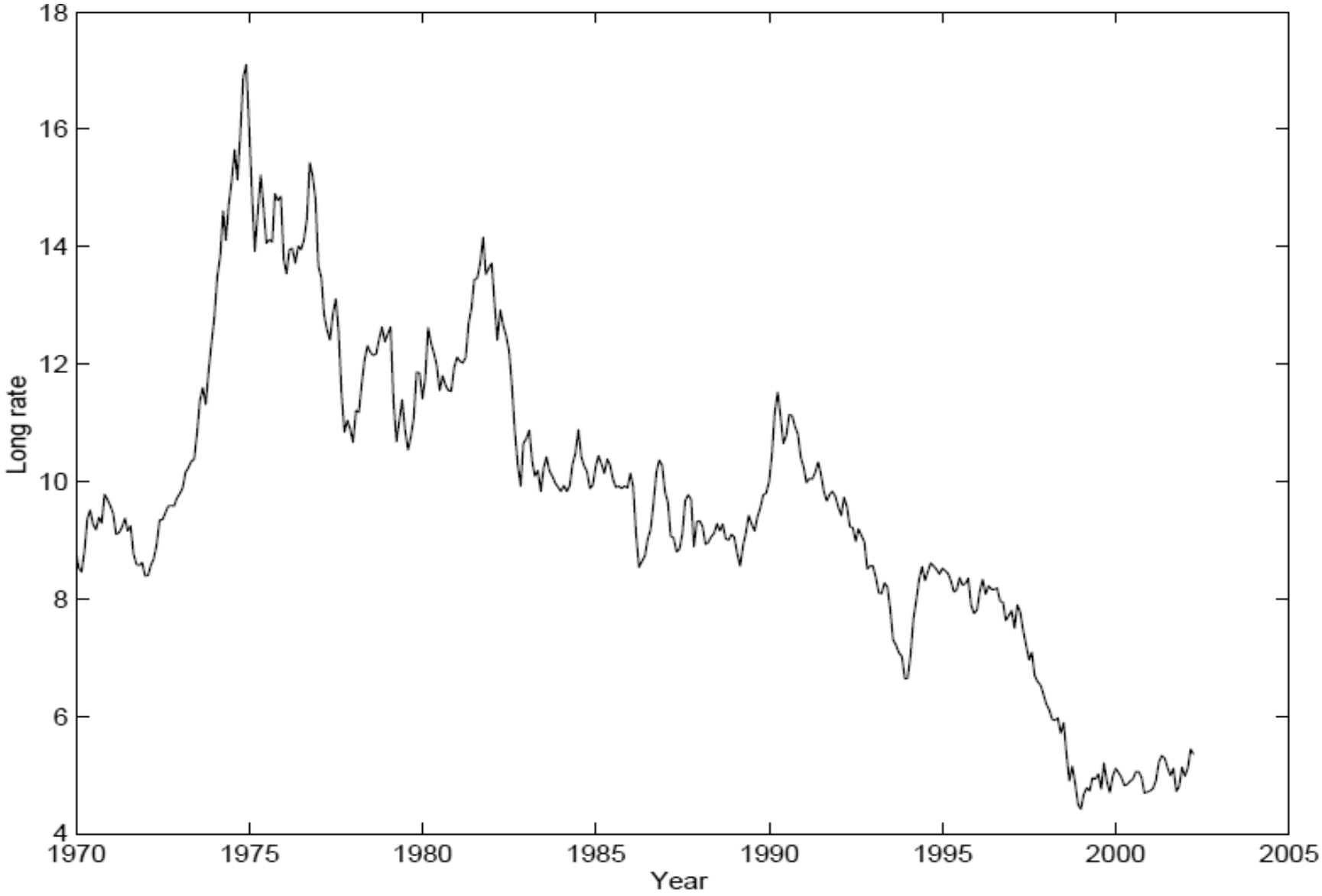


Figure 1: UK Interest Rates 1970-2002

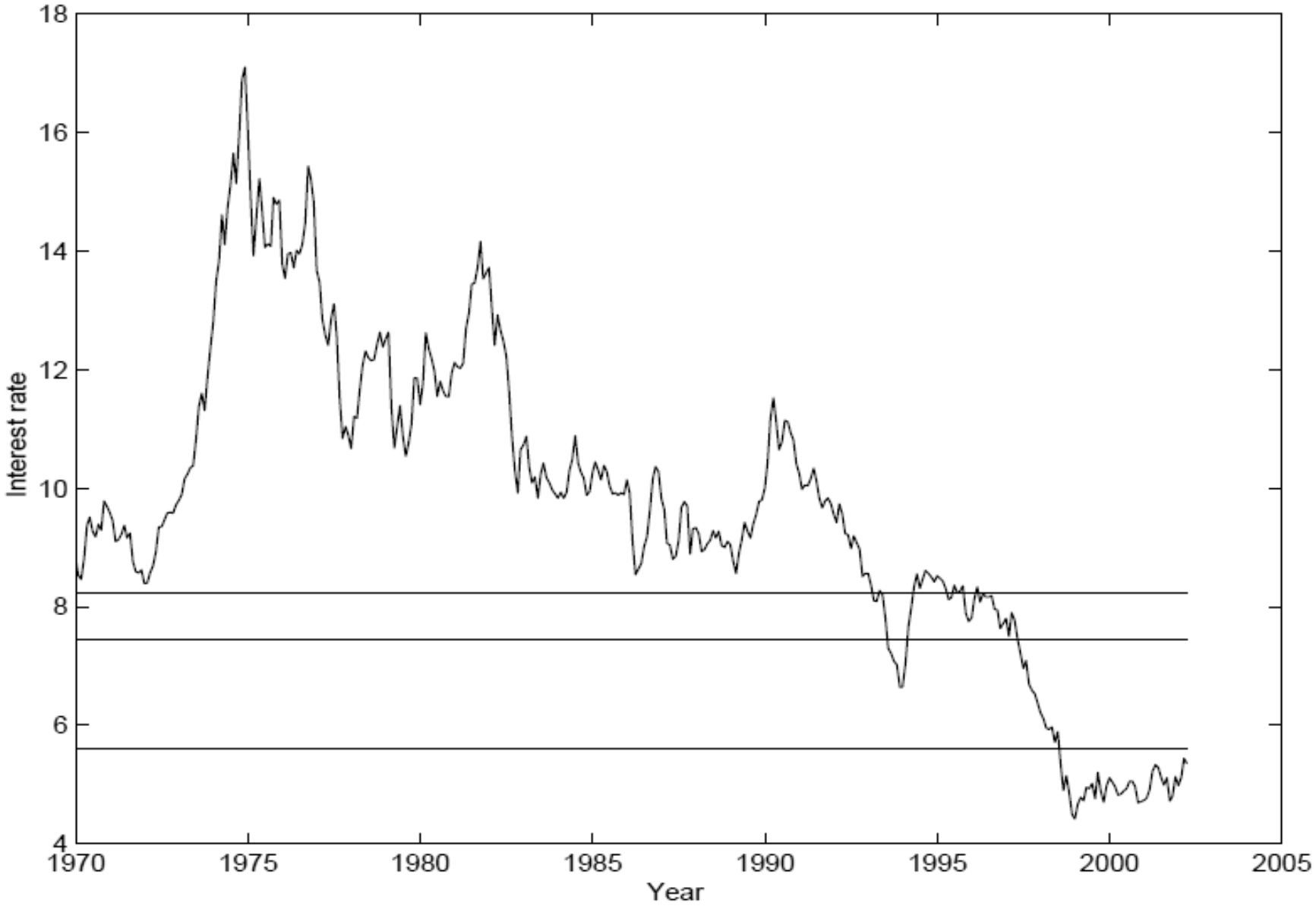


Figure 2: Interest rates levels that will trigger the guarantee, based on various mortality assumptions.

Participating life insurance policies

Guarantees and options available to holders of pension policies

- Annual rate of return guarantee
  - interest is credited to the policy account balance according to some smoothing surplus distribution mechanism
- Bonus option
  - specified claim to a fraction of any excess return (surplus) generated by the investments
- Surrender option
  - right to put back to the issuer

Accounting rules

- Policy account balance, $P(t)-$ book value

  The funds that are set aside to cover the contract liabilities.

- Market value of the asset base backing the contract, $A(t)$

- Undistributed reserve or buffer, $B(t)$

  Protect the policy reserve from unfavorable fluctuations in the asset base.

  Accounting rule gives

$$A(t) = B(t) + P(t)$$

Interest rate credit mechanism

$$Rp(t) = \max\{Rg, \alpha[B(t-1)/P(t-1) - \gamma]\}$$

- The investor is always guaranteed returns at the rate Rg.
- γ is the target level of the buffer ratio $B(t)/P(t)$
- α is the distribution ratio – distribute portion of the buffet if the investment is performing.

Guaranteed equity bonds (GEBs)

- The issuer (usually an insurance company) guarantees a stated interest rate and some protection from loss of initial capital, and provides an opportunity to earn additional interest based on the performance of an equity market index (say, Standard and Poor's 500 Composite Stock Price Index).

- GEBs credit interest using a formula based on changes in the index to which it is linked. It enables investors to achieve potential capital appreciation by participating in the positive performance of the index but also provide investors with a guaranteed minimum return of their investment at maturity.

Term

The index term is the period over which index-linked interest is calculated. Interest is credited to the investor at the end of a term.

Participation Rate

The participation rate decides how much of the increase int he index will be used to calculate index-linked interest. For example, if the calculated change in the index is 9% and the participation rate is 70%, the index-linked interest rate for the contract will be 6.3%($9\% \times 70\% = 6.3\%$).

- The company usually guarantees the participation rate for a specific period, from one year to the entire term.
- When that period is over, the company sets a new participation rate for the next period.
- Some contracts guarantee that the participation rate will never be set lower than a specified minimum or higher than a specified maximum.

Cap Rate

Some contracts may put an upper limit, or cap, on the index-linked interest rate. This is the maximum rate of interest the contract will earn.

Floor

The floor is the minimum index-linked interest rate that will be paid. The most common floor is 0%. A 0% floor assures that even if the index decreases in value, the index-linked interest that can earn will be zero and not negative.

Guaranteed interest compounding

Some contracts pay simple interest during an index term. That means index-linked interest is added to the original premium amount but does not compound during the term. Others pay compound interest during a term, which means that index-linked inteest that has already been credited also earns interest in the future.

Dividends

Depending on the index used, stock dividends usually are not included in the index value. For example, the S & P 500 is a stock price index and only considers the prices of stocks. It does not recognize any dividends paid on those stocks.

Early withdrawal

In most cases, investors cannot take all or part of the money out of contract at any time during the term. There will be a cost and the index-linked interest on the amount withdrawn will not be paid.

Indexing method

The approach used to measure the amount of change, if any, in the index.

Point-to-Point

The contingent claim $C(t)$ in year t for one unit of GEB can be represented as followed

$$C(t) = \max(\min(1 + \alpha R_t, (1 + \zeta)^t), (1 + g)^t), \quad R_t = \frac{S(t)}{S(0)} - 1.$$

- While subject to the maximum cap rate ζ that can be earned under this design, the first random term allows the investors to have a participation rate α in any potential upside gain int he equity market.

- In the event of an adverse market environment, the downside risk is constrained to the minimum guarantee floor component, that is, $(1 + g)^t$.

- The presence of the cap rate, although placing an upper bound on the rate of return of the contract, culd reduce the cost of such design substantially.

Annual reset

The payoff in year t for one unit of GEB is given by

$$C(t) = \max\left(\prod_{S=1}^{t} \max(\min(1+\alpha R, 1+\zeta), 1), (1+g)^t\right)$$

where the random variable R_S again measures the appreciation of the referenced index fund in year S.

- R_S is solely determined by the index levels at the beginning and the end of year S:

$$R_S = \frac{S(s)}{S(s-1)} - 1.$$

- The interest is credited each year for the annual reset GEBs. The credited interest cannot be lost even if the index subsequently goes down.
- The index level used to determine the appreciation of the index is reset annually. This 'lock in' feature can be extremely valuable, particularly in a more volatile market.