FINA556 – Structured Product and Exotic Options

Topic 1 – Overview of basic structured products

1.1 Markets for structured products

1.2 Examples of structured notes and equity-linked products

1.3 Accumulators

1.4 Exotic forms of forward contracts

Appendix: Various types of interest rates

1.1 Markets for structured products

What are structured products?

- Financial instruments that are designed to facilitate certain highly customized risk-return objectives.

- In its simplest form, this can be accomplished by taking a traditional security, like an investment-grade bond, and replacing the usual payment features with non-traditional payoffs derived, say, from the performance of one or more underlying assets or indices (equity-linked payoff).

- Widely accessible to retail investors in the same way like that of stocks, bonds, exchange-traded funds (ETFs) and mutual funds.

- *U.S. Securities and Exchange Commission (SEC) Rule 434* defines structured securities as

  "Securities whose cash flow characteristics depend upon one or more indices or that have embedded forwards or options or securities where an investor's investment return and the issuer's payment obligations are contingent on, or highly sensitive to, changes in the value of underlying assets, indices, interest rates or cash flows."

Embedded optionality and customized exposure

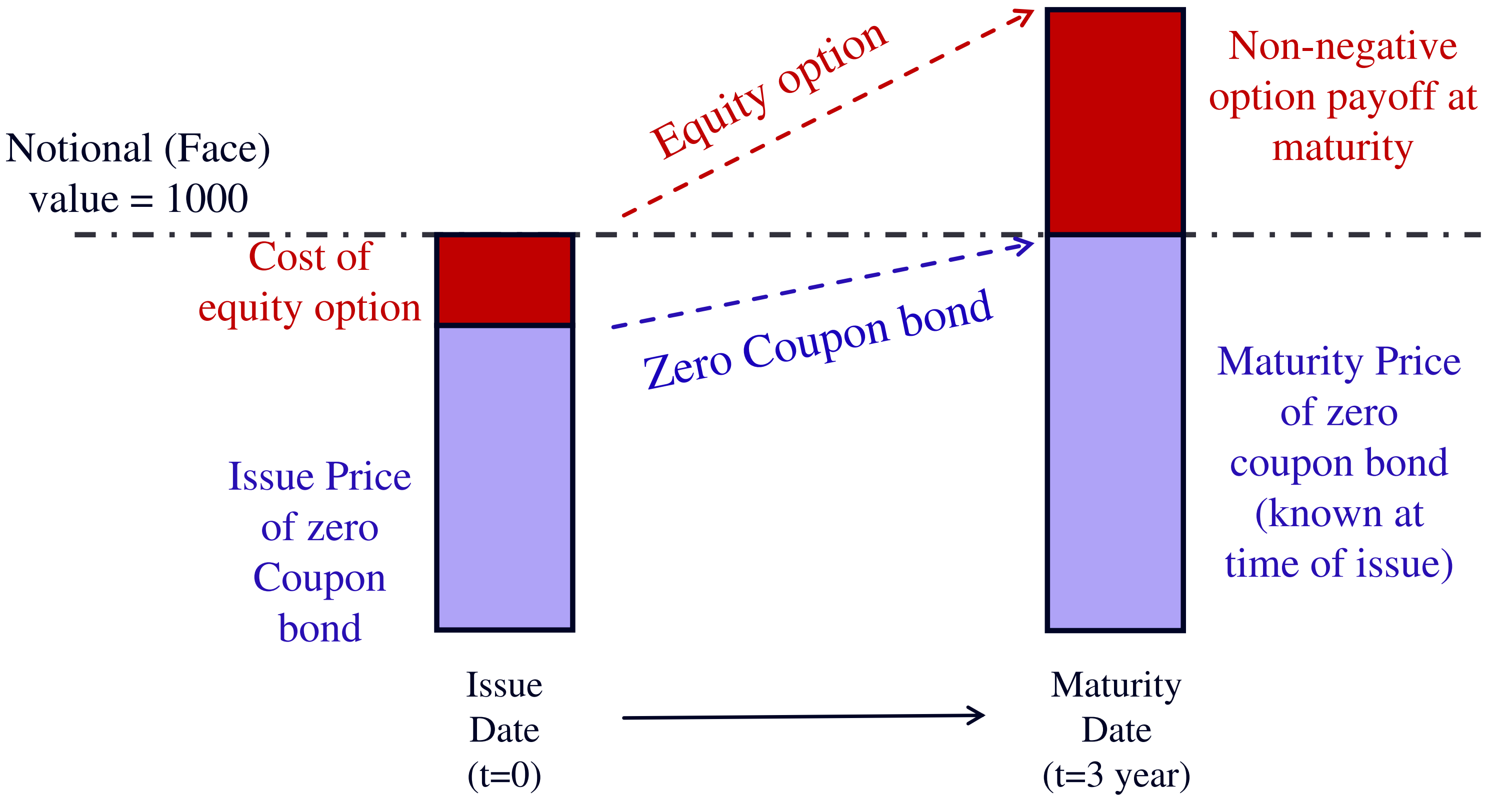
- With embedded optionality features such as leveraged upside participation or downside buffers. Allow investors to participate to a disproportionately high degree in the performance of the underlying.

- Offer investors the chance of a higher yield than they would earn on a direct investment in the underlying. Guarantee the investor a certain minimum repayment (usually 100%) of the invested amount at the end of its term. The upside potential of equity participation may be either capped or unlimited.

- Offer customized exposure to otherwise hard-to-reach asset classes. This makes structured products useful as a complement to traditional components of diversified portfolios. Combine a certain number of defined individual securities (a basket) or an index into a single security.

Structured note

Combination of a zero-coupon bond and a call option on an underlying equity instrument



Principal protection

- On the issue date, pay the face amount of $1,000.
- This note is fully principal-protected, getting a minimum of $1,000 back at maturity no matter what happens to the underlying asset.
- Performance component: the underlying, priced as a European call option, will have intrinsic value at maturity if the underlying asset's value on that date is higher than its value when issued. If not, the option expires worthless and you get nothing in excess of your $1,000 return of principal.
- In essence, the interest earned during the life of the structured note is used as the option premium paid to acquire the embedded equity option.

Structured equity

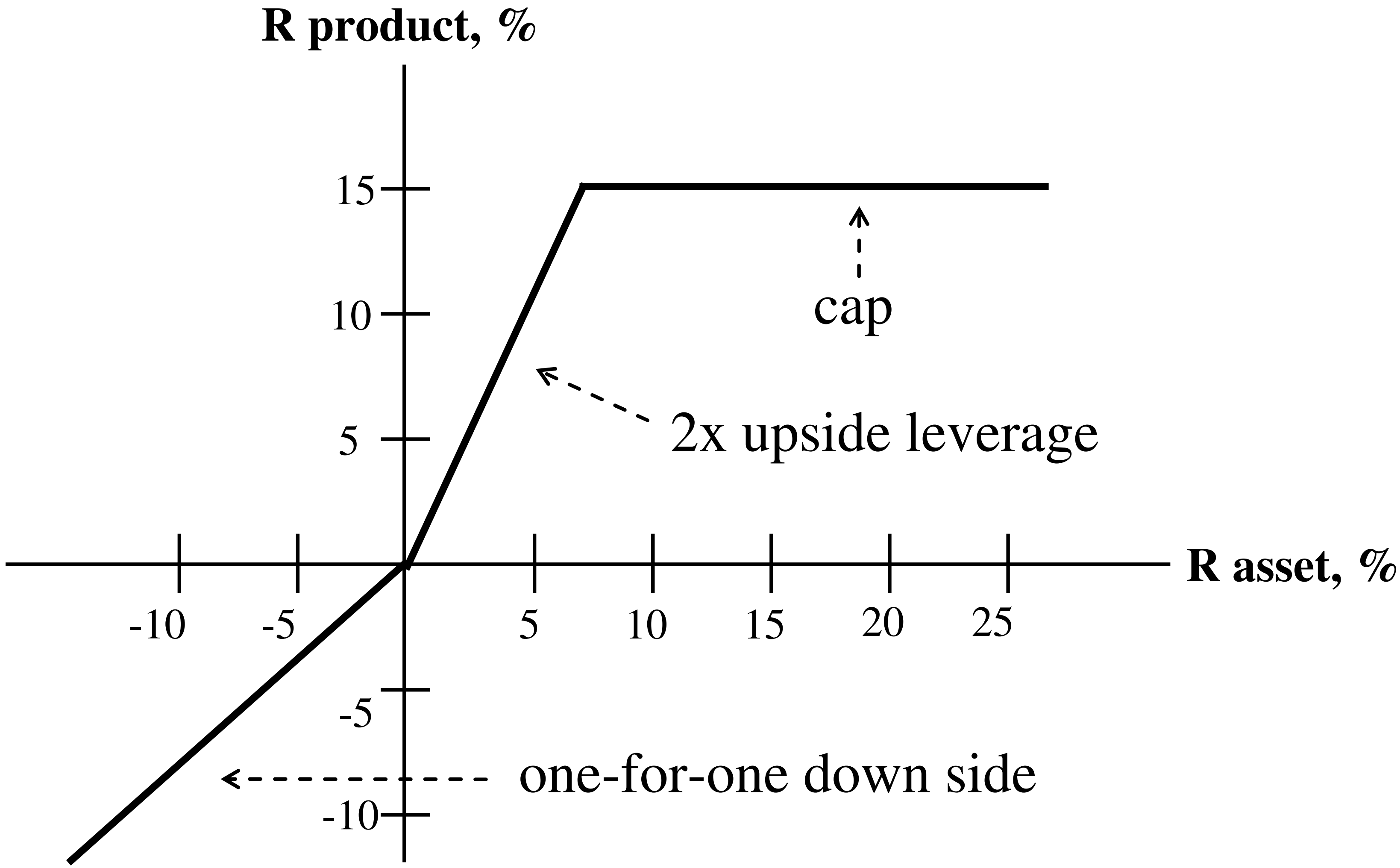
Enhancement of mildly bullish view

- Adopt a strategy that is consistent with one who expects positive but generally weak performance and is looking for an enhanced return above what she thinks the market will produce.

- Substantial upside gain is capped at 15% while mild gain is leveraged up by a factor of 2.

- The structure can be decomposed as

$$\begin{array}{ll} & \text{one unit of the underlying asset} \\ + & \text{one unit of call option with zero strike} \\ - & \text{two units of call option with strike corresponding to 15\% gain} \end{array}$$


R product, %
15
10
5
-5
-10
R asset, %
-10
-5
5
10
15
20
25
cap
2x upside leverage
one-for-one down side

Exotic option features

- Rainbow note - offers exposure to more than one underlying asset. For example, from three relatively low-correlated assets: Russell 3000 Index of U.S. stocks, MSCI Pacific ex-Japan index, and Dow-AIG commodity futures index.

- Asian feature - the value of the underlying asset is based an average of values taken over the note's term.

- Lookback feature - realized maximum or minimum value over a period (no regret).

Risks faced by investors

Counterparty risk – credit risk of the issuer

- The products themselves are legally considered to be the issuing financial institution's liabilities.

- They are typically not issued through the bankruptcy-remote third party vehicles in the way like the asset-backed securities.

- A typical example of counterparty risk is the Lehman Brothers mini-bonds.

What about liquidity?

- A relative lack of liquidity due to the highly customized nature of the investment.

- The full extent of returns from the complex performance features is often not realized until maturity - tend to be more of a buy-and-hold investment decision rather than a means of getting in and out of a position with speed and efficiency.

Pricing transparency

- Highly customized payoff, making it harder to compare the net-of-pricing attractiveness of alternative structured products offerings.

- Apparently, there is no explicit fee or other expense to the investor. On the flip side, this means that the investor cannot know for sure what the implicit costs are.

1.2 Examples of structured notes and equity-linked products

Target redemption note

7.5% USD Target Redemption Index Linked Deposit (issued by the Bank of East Asia, 2004)

Selling points - Enjoy potentially higher returns with Index Linked Deposit

- 100% principal protection plus 7.5% guaranteed coupon return over a maximum of 5-year investment period.
- 1st year annual coupon is guaranteed at 3.5% (relatively juicy), payable semi-annually.

- The remaining coupon rate of 1% will be based on the LIBOR movement. The inverse floater formula is

$$\max\{7\% - 2\times \text{ 6-month LIBOR (in arrears)},0\}.$$

  However, the total coupon received cannot shoot beyond the target accumulation rate of 7.5%. If the coupon payment accrued during the deposit period is less than the target rate, then the remaining amount will be paid at maturity.

Early termination

Once the accumulated coupon payment has reached the target rate, the deposit will be terminated automatically. This is why this is called the target redemption note.

Market background

The US Fed policymakers voted unanimously to keep the Fed Fund Rate unchanged at 1% on 28 October 2003, the lowest level in the past 45 years. They had indicated that the interest rate would remain at a low level for a considerable period.

Nightmare to investors

The 6-month LIBOR rises beyond 3.5% one year afterwards and never come down again. The deposit is held for 5 years until maturity so that the annual return for the deposit is only 1.5% per annum.

Target redemption notes on multi-stocks

- 10-year fund that is 100% capital guaranteed. Pay a juicy fixed coupon of 10% in the first year.
- For Year Two, the coupon payment is referenced to the average performance of the 6 worst stocks in a basket of 24 blue-chip stocks.

$$\max\{0, 10\% + 0.5 \times \text{ average performance of the 6 worst stock}\}.$$

- From Year Three onwards, the investor gets the better of the previous years coupon or the payout formula.
- Once the aggregate coupon payments reaches or exceeds 20%, the fund terminates with full payment of the coupon for that year.
- Worst scenario: 10-year fund with the total coupon of 20%.

Autocall Structured Deposit

Issuer:	Maybank (a Malaysia bank)
Issue date:	April 3, 2009

- Investors have the potential to enjoy yield enhancement by participating in the recovery of a portfolio of stocks linked to 6 China's infrastructure stocks.

- The investors have the view that China's equity market will recover in the medium to long term.

- Principal received upon maturity only. If it is redeemed or sold prior to maturity, the investor may face fees or costs. Normally, it is hard to sell in the secondary markets.

Linked to a basket of 6 stocks

Chinas infrastructure – largest beneficiary of the Government's massive economic stimulus plan

- China Construction Bank
- China Life Insurance
- China Communications Construction Company
- China Telecom
- Petrochina
- Sinopec

Coupon formula

Guaranteed coupon: 5% at the end of the first Year

End of Year 2, if performance of worst off stock in basket $> 10\%$, MASD is auto-called and will payout 6%; otherwise continue.

End of Year 3, if performance of worst off stock in basket $> 10\%$, MASD is auto-called and will payout 12%; otherwise continue.

End of Year 4, if performance of worst off stock in basket $> 10\%$, MASD is auto-called and will payout 18%; otherwise continue.

End of Year 5, if performance of worst off stock in basket $> 10\%$, MASD is auto-called and will payout 24%; otherwise continue.

End of Year 6, if performance of worst off stock in basket $> 10\%$, MASD is auto-called and will payout 30%; otherwise continue.

End of Year 7, if performance of worst off stock in basket $> 10\%$, MASD is auto-called and will payout 36%; otherwise 0%.

Example

If auto-called at the end of Year 4, total coupon collected $= (5+18)\% = 23\%$.

Good investment or otherwise?

- Potential risk

  If one of the 6 stocks never perform, then the investor can get back only 5% coupon plus the principal at the end of 7 years.

- Negative aspects

  - The duration of 7 years may be too long.
  - Exposure to 6 stocks is too much.
  - The coupons are not too juicy.

1.3 Accumulators

- Entails the investor entering into a commitment to purchase a fixed number of shares per day at a pre-agreed price (the "Accumulator Price"). This Price is set (typically 10-20%) below the market price of the shares at initiation. This is portrayed as the "discount" to the market price of the shares.

- The contract is for a fixed period, typically 3 to 12 months.

Key driver

- Rapid appreciation of some Asian currencies against the dollar or the price of commodities. They are marketed as "currency enhancement" or "cost reduction" programs.

Example

Citic Pacific entered into an Australian dollar accumulator as hedges "with a view to minimizing the currency exposure of the company's iron ore mining project in Australia". The company benefits from a strengthening in the A$ above A$1 = US$0.87.

Citic Pacific's（中信泰富）bitter story

- Citic Pacific signed an accumulator that not only set the highest gains but failed to include a floor for losses. The Australian dollar's value was rising when the contract was signed.

- After July, 2008, the AUD's value against the USD declined, sliding as low as 1 to 0.65. This slide contributed to HK$800 million loss when the company terminated leveraged foreign exchange contracts between July 1 and October 17.

- The firm also said its highest, mark-to-market loss could reach HK$14.7 billion. Some analysts say if the AUD falls to 1 to 0.50 USD, the mark-to-market loss would rise to HK$26 billion.

- Citic Pacific shares fell 80% on the Hong Kong exchange to HK$5.06 a share on October 24, compared with HK$28.20 a share July 2.

- The company was driven by a "mixture of greed and a gambling mentality" to use the accumulator. Why not simply buy the less risky currency futures?

Cap on upside gain

If the market price of the shares rises above a pre-specified level ("Knock-Out price") then the obligation to purchase shares ceases. This Price is set (typically 2% to 5%) above the market price of the shares at initiation.

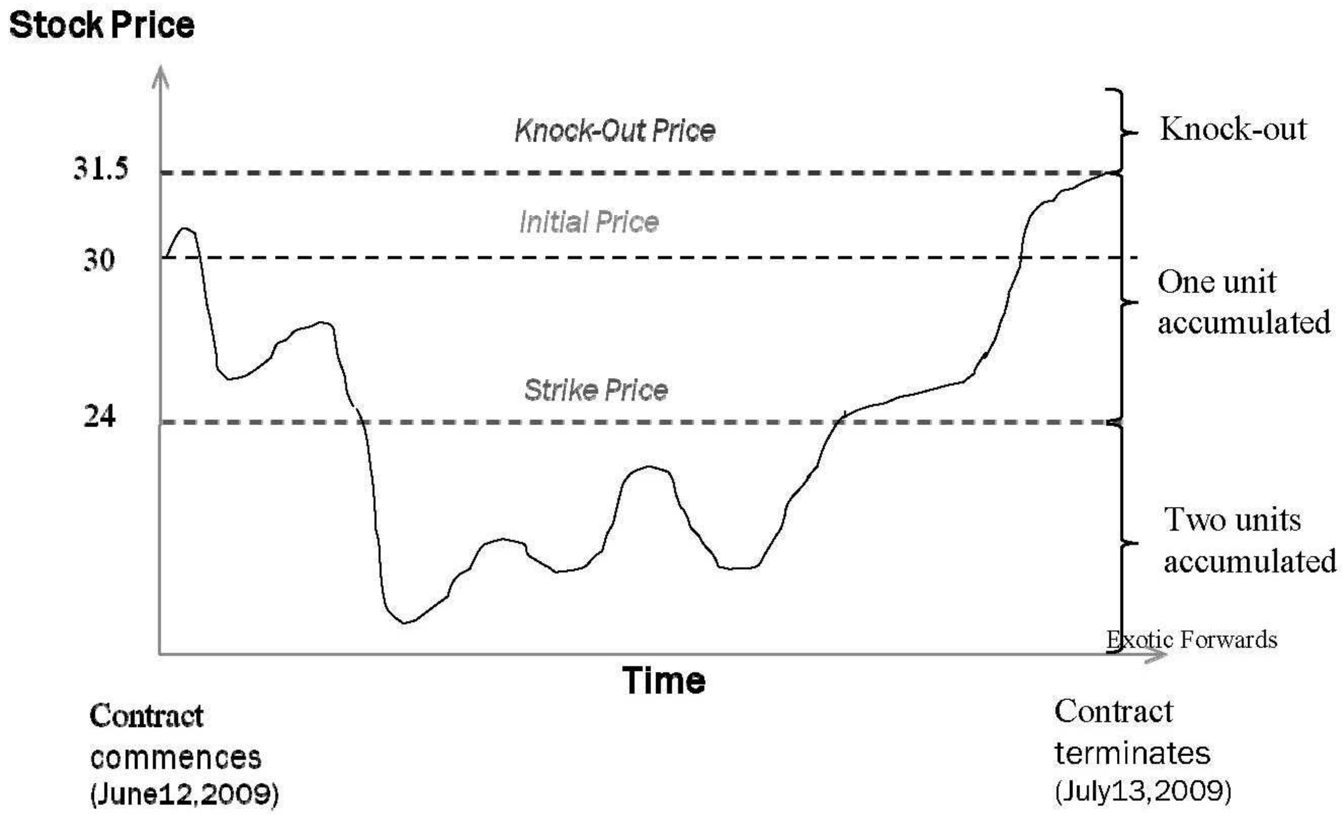
Intensifying downside losses ("I will kill you later")

If the market price falls below the Accumulator Price (10-20% below the market price at initiation), then the investor would be obligated to purchase more shares. This is called the Step-Up feature. The Step-Up factor can be 2 or up to 5.

- Margin is required to minimize counterparty risks. The investor generally benefits where the share prices remain relatively stable preferably, between the Knock Out Price and the Accumulator Price.

Example of an accumulator on China Life Insurance Company

- *Stock Price Movement of China Life Insurance Company Limited (June 12, 2009 - July 13, 2009)*



SGD-Equity Accumulator Structure

Underlying Shares:	SEMBCORP INDUSTRIES LTD
Start Date:	05 November 2007
Final accumulation Date:	03 November 2008
Maturity Date:	06 November 2008 (subject to adjustment if a Knock-Out Event has occurred)
Strike Price:	$4.7824

Knock-Out Price:	\$6.1425
Knock-Out Event:	A Knock-Out Event occurs if the official closing price of the Underlying Share on any Scheduled Trading Day is greater than or equal to the Knock-Out Price. Under such event, there will be no further daily accumulation of Shares from that day onward. The aggregate number of shares accumulated will be settled on the Early Termination Date, which is the third business day following the occurrence of Early Termination Event.

Shares Accumulation:	On each Scheduled Trading Day prior to the occurrence of Early Termination Event, the number of shares accumulated will be 1,000 when Official Closing Price for the day is higher than the Strike Price 2,000 when Official Closing Price for the day is lower than the Strike Price
Monthly Settlement Date:	The Shares accumulated for each Accumulation Period will be delivered to the investor on the third business day following the end of each monthly Accumulation Period
Total Number of Shares:	Up to the maximum of 500,000 shares (since the total number of days of accumulation is 250)

Accumulation Period and Delivery Schedule

12 accumulation periods in total

Accumulation Period	Number of days	Delivery Date
05 Nov 07 to 03 Dec 07	20	06 Dec 07
04 Dec 07 to 02 Jan 08	19	07 Jan 08
03 Jan 08 to 04 Feb 08	23	11 Feb 08
05 Feb 08 to 03 Mar 08	18	06 Mar 08
04 Mar 08 to 02 Apr 08	21	07 Apr 08
03 Apr 08 to 02 May 08	21	06 May 08

Accumulation Period	Number of days	Delivery Date
05 May 08 to 02 Jun 08	20	05 Jun 07
03 Jun 08 to 02 Jul 08	22	07 Jul 08
03 Jul 08 to 04 Aug 08	23	07 Aug 08
05 Aug 08 to 02 Sep 08	21	05 Sep 08
03 Sep 08 to 02 Oct 08	21	07 Oct 08
03 Oct 08 to 03 Nov 08	21	06 Nov 08

Decomposition of an accumulator into barrier options

- Without the "intensifying loss" feature, the product is like a portfolio of forward contracts with the knock-out feature. Purchases are conditional on survival until the date of transactions of shares.

- The "intensifying loss" feature can be considered as a portfolio of forward contracts with the "excursion time" feature. The accumulated amount of shares depends on the total excursion time of the stock price below the strike price, again conditional on survival until the date of transactions of share.

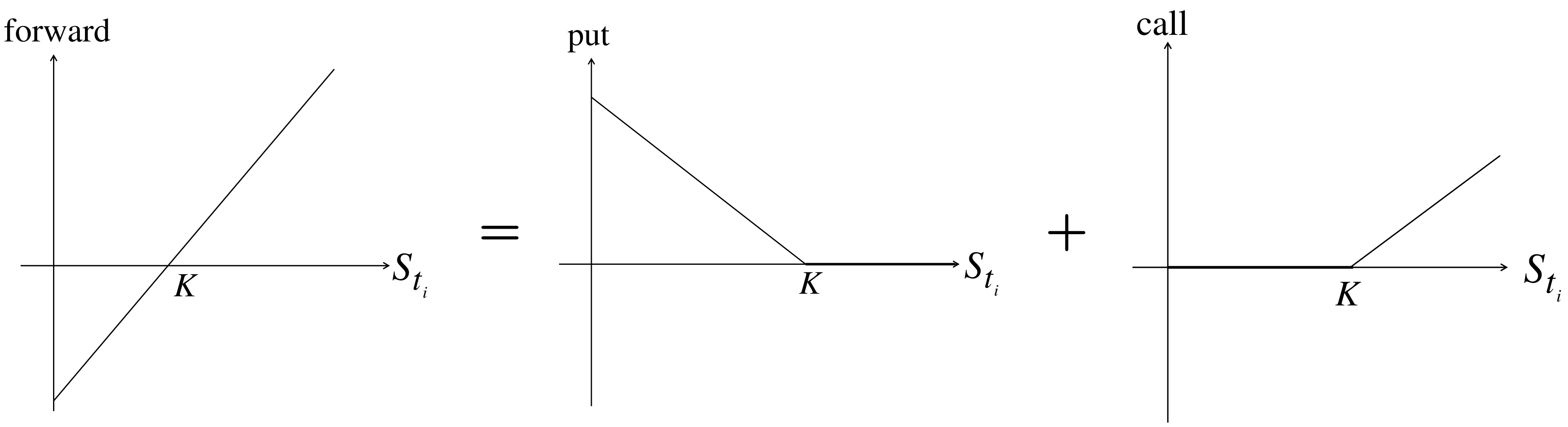
It is necessary to count the number of days that the stock price stays below the strike price, conditional on "no knock-out". Essentially, the holder sells 250 barrier put options (strike price equals $4.7824 and upper knock-out level at $6.1425) whose expiry dates fall on the 250 observation dates.

Decomposition under immediate settlement

The payoff on date t_i is given by

$$\begin{cases} 0 & \text{if } \max_{0\le\tau\le t_i} S_\tau \ge H \\ S_{t_i} - K & \text{if } \max_{0\le\tau\le t_i} S_\tau < H \text{ and } S_{t_i} \ge K \\ 2(S_{t_i} - K) & \text{if } \max_{0\le\tau\le t_i} S_i < H \text{ and } S_{t_i} < K, \end{cases}$$

where $K =$ strike price and $H =$ upper knock-out level.



Let

$n =$ total number of observation dates

$c_{uo} =$ up-and-out barrier call option

$p_{uo} =$ up-and-out barrier put option

Fair value of an accumulator
$= \sum_{i=1}^{n} c_{uo}(t_i; K, H) - 2p_{uo}(t_i; K, H)$.

- The upper knock-out barrier can be monitored discretely at the end of all observation dates or continuously throughout the term of the contract.

Decomposition under delayed settlement

Let $T_i, i = 1, 2, \cdots, n$, denote the settlement date of the quantities fixed at observation date t_i. While there are 250 observation dates, there are only 12 settlement dates.

$$\begin{aligned} & \text{Fair value of an accumulator} \\ = \; & \sum_{i=1}^{n} c_{uo}^F(t_i; K, H, T_i) - 2p_{uo}^F(t_i; K, H, T_i), \end{aligned}$$

where $c_{uo}^F(t_i; K, H, T_i)$ is the price of a barrier call option with strike K on forward contract with purchase price K and maturity date T_i. The option has an up-and-out barrier H and expires at date t_i.

The t_i-maturity call with strike price K on a T_i-maturity forward with delivery price K means the holder takes the long position of the forward when $S_{t_i} > K$. The underlying asset will be delivered to the holder at T_i by paying the purchase price K.

Numerical example

One-year tenor, 21 trading days in each month, $n = 252, H = \$105$. The initial stock price S_0 is \$100, quantity bought on each day is either 1 or 2 depending on outside the down-region or otherwise.

- Since the accumulator parameters are designed so that it has a near zero-cost structure, the fair price for the sample accumulator is small.

FAIR VALUES OF ACCUMULATOR CONTRACTS

Volatility (σ)	Discounted Purhcase Price K				Zero-structure discounted price
	78	84	90	96	
10%	2639.5	1821.5	978.4	24.2	96.14
15%	1785.8	1108.4	369.8	-499.5	92.70
20%	1217.4	604.0	-82.2	-883.4	89.32
25%	790.0	211.6	-437.1	-1180.8	86.04
30%	445.2	-109.3	-727.2	-1423.3	82.86
35%	155.2	-380.6	-972.4	-1629.2	79.80
40%	-95.2	-615.9	-1185.4	-1809.6	76.84

For example, assume that $S_0 = 100, H = 105, r = 0.03, q = 0.00, \sigma = 20\%$. For a zero-cost accumulator with monthly settlement to be fairly priced, a fair discounted purchase price is shown to be 89.32.

Implied Volatility

Options' implied volatility is the volatility implied by the market price of the options based on a pricing model. In other words, given a particular pricing model, it is the volatility that yields a theoretical option value equal to the market price.

IMPLIED VOLATILITIES OF ACCUMULATOR CONTRACTS

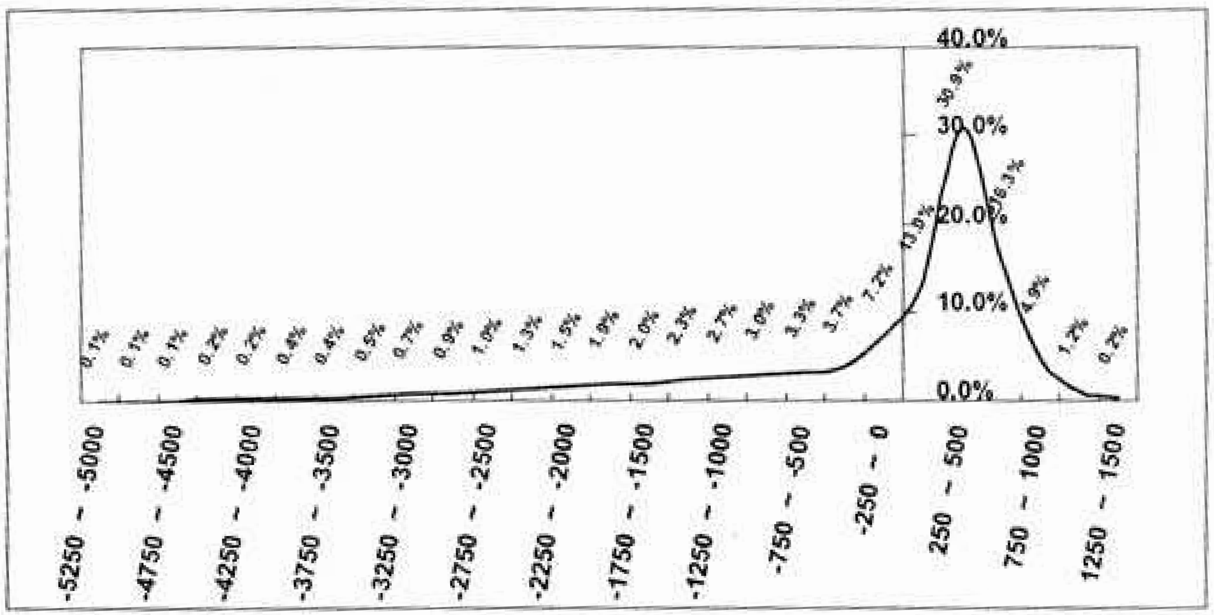
Barrier	Discounted Purchase Price *K*				
level	80	84	88	92	96
107	**36.06%**	**29.55%**	23.30%	17.26%	11.31%
105	**34.63%**	**28.16%**	21.97%	16.00%	10.16%
103	**33.10%**	**26.68%**	20.54%	14.63%	8.91%

- Since the accumulator is a zero-cost structure, we find the volatility that makes the fair price equal to zero.

- Suppose an investor anticipates a volatility of 25% in the future one year. This investor will find the barrier-strike combination in the upper left corner (bold area in Table) favorable because implied volatilities in those cells have implied volatility larger than 25%.

Value at risk analysis

- Profit/loss distribution is highly asymmetric.



Probability distribution of profit and loss of the sample accumulator

- It has a long left tail meaning that extreme loss is possible. The total loss can run higher than the notional value of the contract.

- Extreme profit is unlikely as the distribution has a short right tail. This is because the contract will be knock-out once the stock price breaches the upper barrier H.

- For the sample accumulator contract analyzed, the lower 5-percentile is $-\$2424.50$. This means that at the finish of the contract, there is a 5% chance to run a loss more than \$2424.50.

- For the seller of the contract, we can estimate his/her corresponding loss using the same confidence level 0.95. Computation result shows that the value at risk at contract finish is \$841.01 with 95% confidence. Since the difference between the two values at risk, we can conclude that the seller runs a much smaller risk than the buyer.

Greek values

GREEKS OF ACCUMULATOR CONTRACTS

Spot price S		88	92	96	100	104
Immediate settlement	Delta	290.12	211.95	137.19	65.98	-3.54
	Gamma	-18.49	-19.40	-18.13	-17.55	-16.23
	Vega	-12139	-12507	-11182	-8072	-2966
Delay settlement	Delta	288.05	209.63	134.88	63.48	-6.47
	Gamma	-18.67	-19.35	-18.16	-17.62	-16.37
	Vega	-12201	-12554	-11220	-8100	-2978

Gamma is the sensitivity of delta to stock price.

Vega is the sensitivity of contract value to volatility.

- Delta, gamma, and vega are all sizable because an accumulator contract is composed of many option contracts with different expiration dates.

- There is an asymmetry in the delta and vega values. When the spot price is low (say $S = 88$), the magnitude of delta and vega values are much larger than those when the spot price is high (say $S = 104$).

- Delta values are decreasing function of S because gamma values remain at a negative level. Delta has a magnitude of 288.05 (discrete settlement) when $S = 88$, but its magnitude drops to -6.47 when $S = 104$. This means that losing buyers will be more vulnerable to price changes than winning buyers.

- Vega has a magnitude of 12201 when $S = 88$, but drops to a magnitude of 2978 when $S = 104$ meaning that compared to winning buyers, losing buyers are more vulnerable to volatility changes as well. This may be one reason why some buyers of the contract become very desperate when the market turns south in recent months.
- This asymmetry is consistent with the finding that the value at risk of the buyer is several times that of the seller.

Recent market trends for structured products

- Equity-linked notes with principal-protected payout structures in all asset classes during 2008 and 2009 have been so profound that the 39% of products in 2008 that were principal protected has swelled to 51% of notional issued in 2009.

- The return of the reverse convertibles and accumulators as popular choices.

- Accumulator contracts worth $5 billion are outstanding in Hong Kong so far in September, 2009. By comparison, in April 2008, Hong Kong regulators estimated that $23 billion in accumulators were outstanding.

1.3 Exotic forms of forward contracts

Forward contract 遠期合約

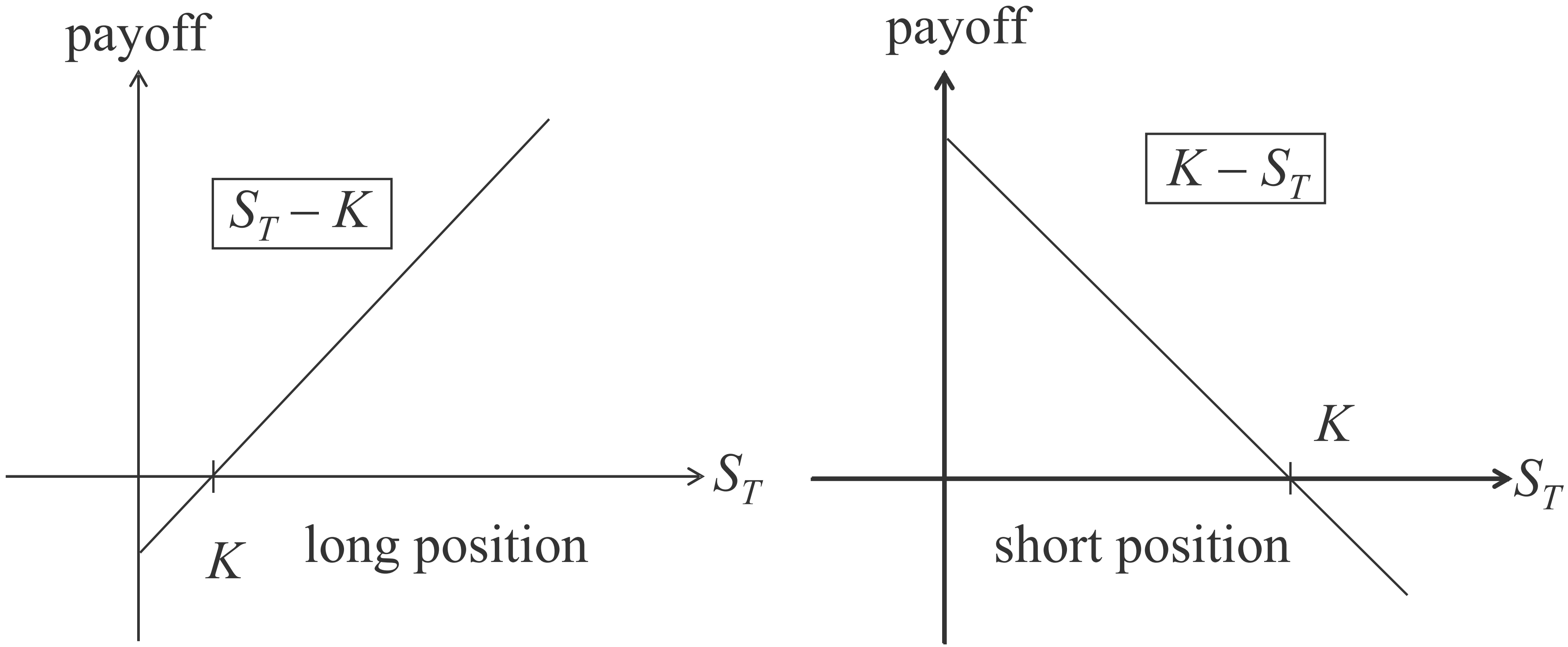
The buyer of the forward contract agrees to pay the delivery price K dollars at future time T to purchase a commodity whose value at time T is S_T. The pricing question is how to set K?

Objective of the buyer:

To hedge against the price fluctuation of the underlying commodity.

- Intension of a purchase to be decided earlier, actual transaction to be done later.
- The forward contract needs to specify the delivery price, amount, quality, delivery date, means of delivery, etc.

Terminal payoff from a forward contract



$K =$ delivery price, $S_T =$ asset price at maturity

Zero-sum game between the writer (short position) and buyer (long position).

Potential default of either party (counterparty risk): writer or buyer.

Is the forward price related to the expected price of the commodity on the delivery date? No!

$$\begin{aligned} & \text{Forward price} \\ = \; & \text{spot price} + \underbrace{\text{cost of fund} + \text{storage cost}}_{\text{cost of carry}} \end{aligned}$$

- Cost of fund is the interest accrued over the period of the forward contract.

- Cost of carry is the total cost incurred to acquire and hold the underlying asset, say, including the cost of fund and storage cost.

- Dividends paid to the holder of the asset are treated as negative contribution to the cost of carry.

Numerical example on arbitrage

- spot price of oil is US$19
- quoted 1-year forward price of oil is US$25
- 1-year US dollar interest rate is 5% pa
- storage cost of oil is 2% per annum, paid at maturity

Any arbitrage opportunity? *Yes*

Sell the forward and expect to receive US$25 one year later. Borrow $19 now to acquire oil, pay back $\$19(1+0.05) = \$19.95$ a year later. Also, one needs to spend $\$0.38 = \$19 \times 2\%$ as the storage cost.

$$
\begin{aligned}
& \text{total cost of replication (dollar value at maturity)} \\
= \; & \text{spot price} + \text{cost of fund} + \text{storage cost} \\
= \; & \$20.33 < \$25 \text{ to be received.}
\end{aligned}
$$

Close out all positions by delivering the oil to honor the forward. At maturity of the forward contract, guaranteed riskless profit $= \$4.67$.

Value and price of a forward contract

Let $f(S, \tau)$ = value of forward, $F(S, \tau)$ = forward price,

$$\begin{aligned} \tau &= \text{time to expiration}, \\ S &= \text{spot price of the underlying asset}. \end{aligned}$$

Further, we let

$$B(\tau) = \text{value of an unit par discount bond with time to maturity } \tau$$

- If the interest rate r is constant and interests are compounded continuously, then $B(\tau) = e^{-r\tau}$.

- Assuming no dividend to be paid by the underlying asset and no storage cost.

We construct a "static" replication of the forward contract by a portfolio of the underlying asset and bond.

Portfolio A: long one forward and a discount bond with par value K

Portfolio B: one unit of the underlying asset

Both portfolios become one unit of asset at maturity. Let $\Pi_A(t)$ denote the value of Portfolio A at time t. Note that $\Pi_A(T) = \Pi_B(T)$. By "no-arbitrage" argument*, we must have $\Pi_A(t) = \Pi_B(t)$. The forward value is given by

$$f = S - KB(\tau).$$

The forward price is defined to be the delivery price which makes $f = 0$, so $K = S/B(\tau)$. Hence, the forward price is given by

$$F(S, \tau) = S/B(\tau).$$

*Suppose $\Pi_A(t) > \Pi_B(t)$, then an arbitrage can be taken by selling Portfolio A and buying Portfolio B. An upfront positive cash flow is resulted at time t but the portfolio values are offset at maturity T.

Discrete dividend paying asset

$D =$ present value of all dividends received from holding the asset during the life of the forward contract.

We modify Portfolio B to contain one unit of the asset plus borrowing of D dollars. The loan of D dollars will be repaid by the dividends received by holding the asset. We then have

$$f + KB(\tau) = S - D$$

so that

$$f = S - [D + KB(\tau)].$$

Setting $f = 0$ to solve for K, we obtain $F = (S - D)/B(\tau)$.

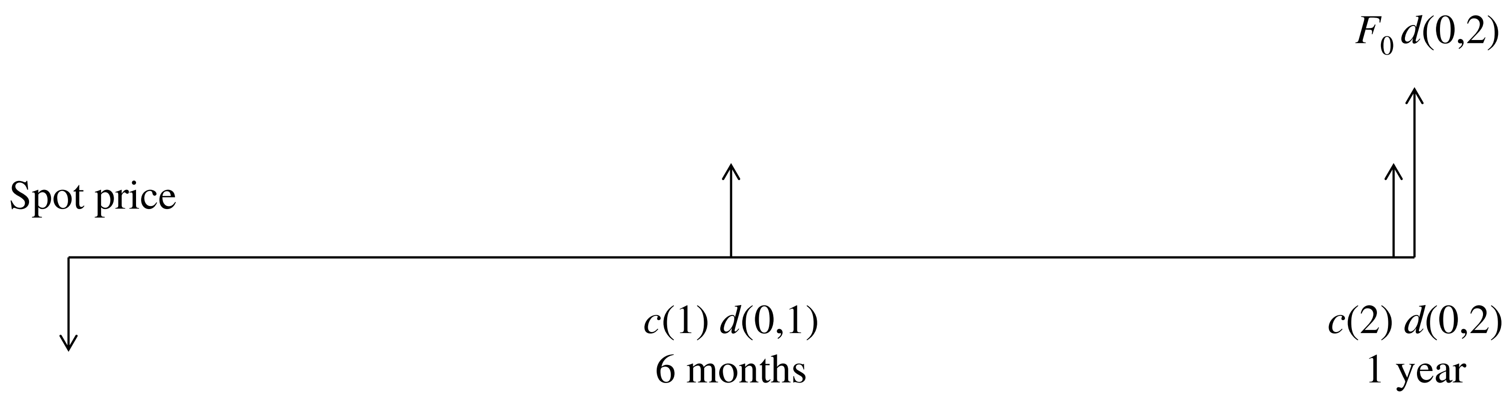
The "net" asset value is reduced by the amount D due to the anticipation of the dividends. Unlike holding the asset, the holder of the forward will not receive the dividends. As a fair deal, he should pay a lower delivery price at forward's maturity.

Example — Bond forward

- A 10-year bond is currently selling for \$920.
- Currently, hold a forward contract on this bond that has a delivery date in 1 year and a delivery price of \$940.
- The bond pays coupons of \$80 every 6 months, with one due 6 months from now and another just before maturity of the forward.
- The current interest rates for 6 months and 1 year (compounded semi-annually) are 7% and 8%, respectively (annual rates compounded every 6 months).
- What is the current value of the forward?

Let $d(0,k)$ denote the discount factor over the $(0,k)$ semi-annual period. All cash flows are transferred to their future value at time 2. The current forward price of the bond

$$\begin{aligned} F_0 &= \frac{\text{spot price}}{d(0,2)} - \frac{c(1)d(0,1)}{d(0,2)} - \frac{c(2)d(0,2)}{d(0,2)} \\ &= 920(1.04)^2 - \frac{80(1.04)^2}{1.035} - \frac{80(1.04)^2}{(1.04)^2} = 831.47. \end{aligned}$$



The value of the forward contract $= \frac{831.47-940}{(1.04)^2} = -100.34.$

Cost of carry

Additional costs to hold the commodities, like storage, insurance, deterioration, etc. These can be considered as negative dividends. Treating U as $-D$, we obtain

$$F = (S + U)e^{r\tau},$$

$U =$ present value of total cost incurred during the remaining life of the forward to hold the asset.

Suppose the costs are paid continuously, we have

$$F = Se^{(r+u)\tau},$$

where $u =$ cost per annum as a proportion of the spot price.

In general, $F = Se^{b\tau}$, where b is the cost of carry per annum. Let q denote the continuous dividend *yield* per annum paid by the asset. With both continuous holding cost and dividend yield, the cost of carry $b = r + u - q$.

Example — Sugar with storage cost

- The current price of sugar is 12 cents per pound. How to find the forward price of sugar to be delivered in 5 months?
- The cost of carry of sugar is 0.1 cents per pound per month, to be paid at the beginning of the month. The interest rate = 9% per annum = 0.0075 per month.

$$\begin{aligned} F &= (1.0075)^5(0.12) \\ &\quad + 0.001(1.0075 + 1.0075^2 + 1.0075^3 + 1.0075^4 + 1.0075^5) \\ &= 0.1295 = 12.95\text{cents}. \end{aligned}$$

Currency forward

The underlying is the exchange rate X, which is the domestic currency price of one unit of foreign currency.

$$\begin{aligned} r_d &= \text{constant domestic interest rate} \\ r_f &= \text{constant foreign interest rate} \end{aligned}$$

Portfolio A: Hold one currency forward with delivery price K and a domestic bond of par K maturing on the delivery date of forward.

Portfolio B: Hold a foreign bond of unit par maturing on the delivery date of forward.

Let $\Pi_A(t)$ and $\Pi_A(T)$ denote the value of Portfolio A at time t and T, respectively.

Remark

Exchange rate appears when we convert the price of the foreign bond (tradeable asset) from the foreign currency world into the domestic currency world.

On the delivery date, the holder of the currency forward has to pay K domestic dollars to buy one unit of foreign currency. Hence, $\Pi_A(T) = \Pi_B(T)$, where T is the delivery date.

Using the law of one price (two securities with the same terminal payoff must have the same price at the current time), $\Pi_A(t) = \Pi_B(t)$ must be observed at the current time t.

- Violation of "law of one price" means "existence of arbitrage", so absence of arbitrage $\Longrightarrow$ law of one price.

Note that

$$B_d(\tau) = e^{-r_d\tau}, B_f(\tau) = e^{-r_f\tau},$$

where $\tau = T - t$ is the time to expiry. Let f be the value of the currency forward in domestic currency,

$$f + KB_d(\tau) = XB_f(\tau),$$

where $XB_f(\tau)$ is the value of the foreign bond in domestic currency.

By setting $f = 0$,

$$K = \frac{XB_f(\tau)}{B_d(\tau)} = Xe^{(r_d - r_f)\tau}.$$

We may recognize r_d as the cost of fund and r_f as the dividend yield. This result is the well known *Interest Rate Parity Relation*.

Example

Suppose that the 2-year interest rates in Australia and the United States are 5% and 7%, respectively. The spot exchange rate $= 0.6200$ USD per AUD. By the Interest Rate Parity Relation, the 2-year forward exchange rate should be

$$0.62e^{(0.07-0.05)\times 2} = 0.6453.$$

Suppose first that the 2-year forward exchange rate is less than the above forward rate, say, 0.6300.

How to take arbitrage profit?

- Borrow $1,000$ AUD at 5% per annum for 2 years, convert to 620 USD and invest the USD at 7% (continuously compounded).

- Enter into a forward contract to buy $1,105.17$ AUD for $1,105.17 \times 0.63 = 696.26$ USD.

The 620 USD that are invested at 7% grow to $620e^{0.07\times 2} = 713.7$ USD in 2 years. Of this, 696.26 USD are used to purchase $1,105.17$ AUD under the terms of the forward contract.

This strategy gives rise to a riskfree profit of

$$713.17 - 696.26 = 16.91 \text{ USD}.$$

Geared forward (GF)

The forward with geared rate is a series of outright forward transactions.

Motivation: Enhances the rates against forward outright.

- Expressing the market view of the buyer – FX rate can be improved if the spot rate moves in certain conditions.
- The settlement depends on four parameters defined by the trade:
  - monthly spot fixing
  - strike rate
  - geared rate
  - knock-out FX rate

Example

Currency	EUR/USD
Notional	EUR 10 million per month
Tenor	12 months
Spot Rate	f
Strike Rate	k
Knock-out Rate (continuous)	k_0
Knock-out event	At any time during option period if spot trades at or above k_0, all options will be knock-out.

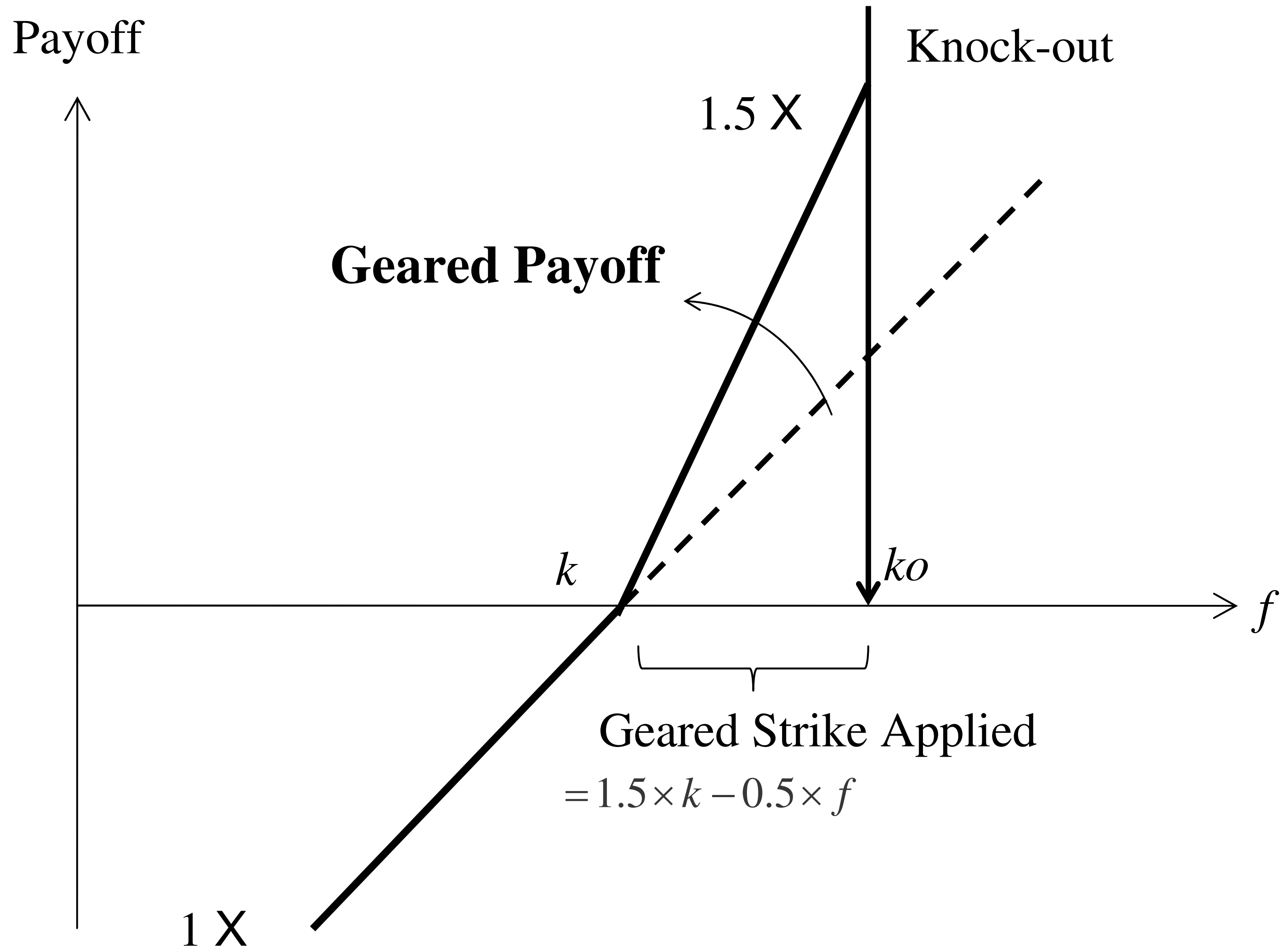
Expiry payoff

- If $k \leq f < k_0$, client buys EUR 10 million at the geared rate of $1.5k-0.5f$ so that the corresponding payoff is $f-(1.5k-0.5f) = 1.5(f-k)$. Apparently, the strike is reduced from k to $1.5k-0.5f$.
- If $f < k$, client buys EUR 10 million at k

  Correpsonding payoff $= f - k$

There is an extra gain in payoff of amount $0.5(f-k)$ when $k < f < k_0$. This resembles the payoff of 0.5 units of a call option with strike at k but subject to knock-out at the upper strike k_0.

Payoff Pattern



Multiplicity Forward (MF)

- The Multiplicity Forward is a hedging strategy designed to generate attractive strike rates for a cash flow with multiple regular fixings.

- An attractive Strike Rate is guaranteed for the first month.

- Thereafter, Strike Rates for subsequent months are determined based on a predetermined "multiplicity formula".

$$\begin{aligned}\text{Strike Rate} &= \text{Strike Rate of preceding month}\\ &\quad \times \text{Max}\,[1, \text{Spot Fixing/Multiplicity Rate}]\end{aligned}$$

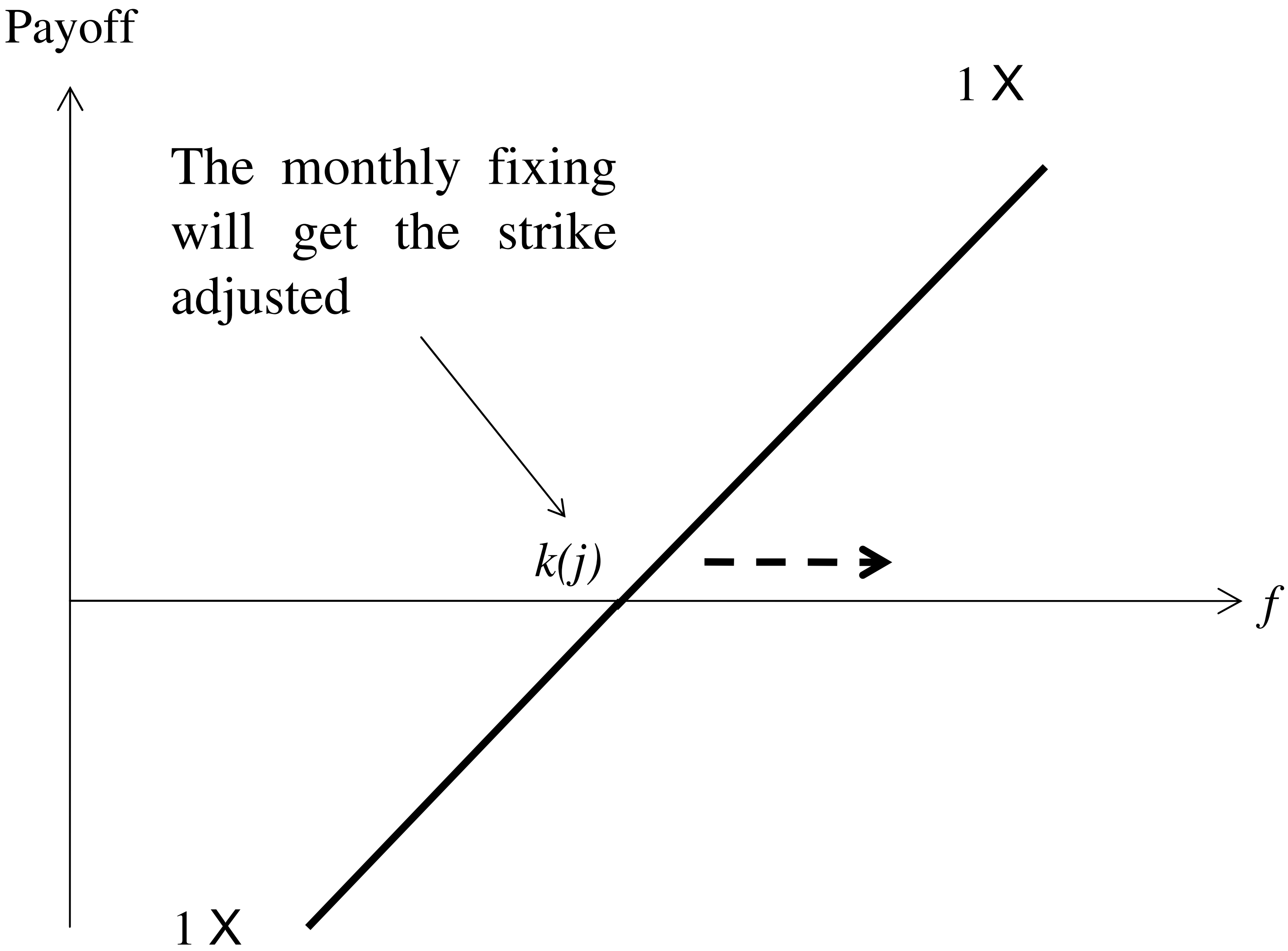
  Strike Rate is very attractive at first, but will be adjusted against the Investor.

Example

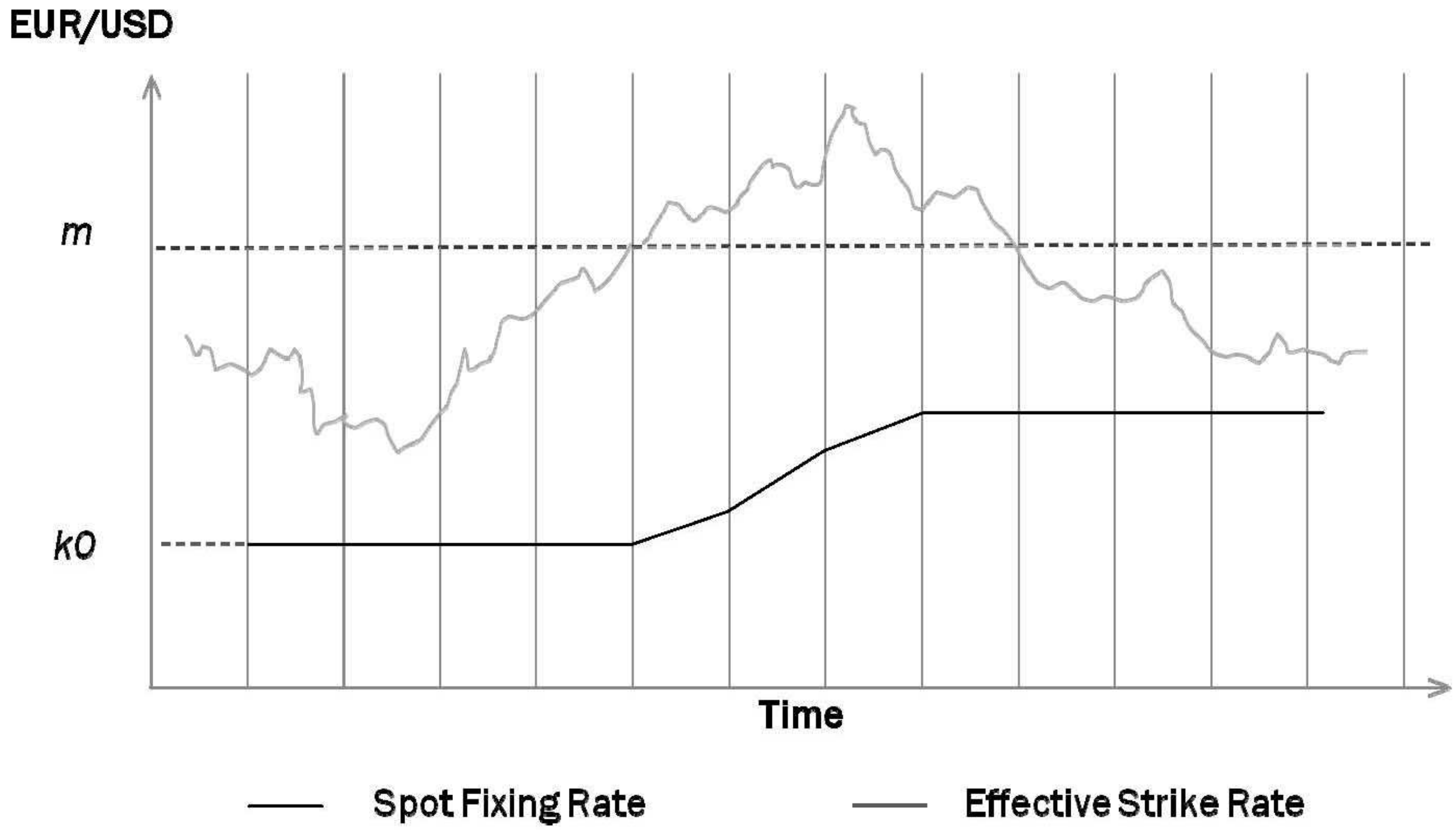
Currency	EUR/USD
Expiry/tenor	Monthly for one year
Notional	EUR 10 million per month
Spot Fixing Rate of Month j	$f(j)$
Initial Strike Rate	k_0
Multiplicity Rate	m
Strike Rate of Month j	$k(j)$

$$k(j) = \begin{cases} k_0 & \text{if } j = 1 \\ k(j-1)\max\left[1, \frac{f(j)}{m}\right] & \text{if } j > 1 \end{cases}$$

MF Payoff Pattern for a Single Settlement



Multiplicity Forward Mechanism



The upward adjustments of strike at subsequent fixing dates are cumulative.

Some variations of Multiplicity Forward

- *Short position*
  - For short position, the Initial Strike Rate is set to be high (high strike is favorable for investors shorting a forward)
  - Multiplicity Rate is set to be lower than the spot rate at initiation
  - Accordingly the subsequent Strike Rate formula is:

$$\begin{aligned}\text{Strike Rate} &= \text{Strike Rate of preceding month} \\ &\quad \times \text{Min}\,[1, \text{Spot Fixing/Multiplicity Rate}]\end{aligned}$$

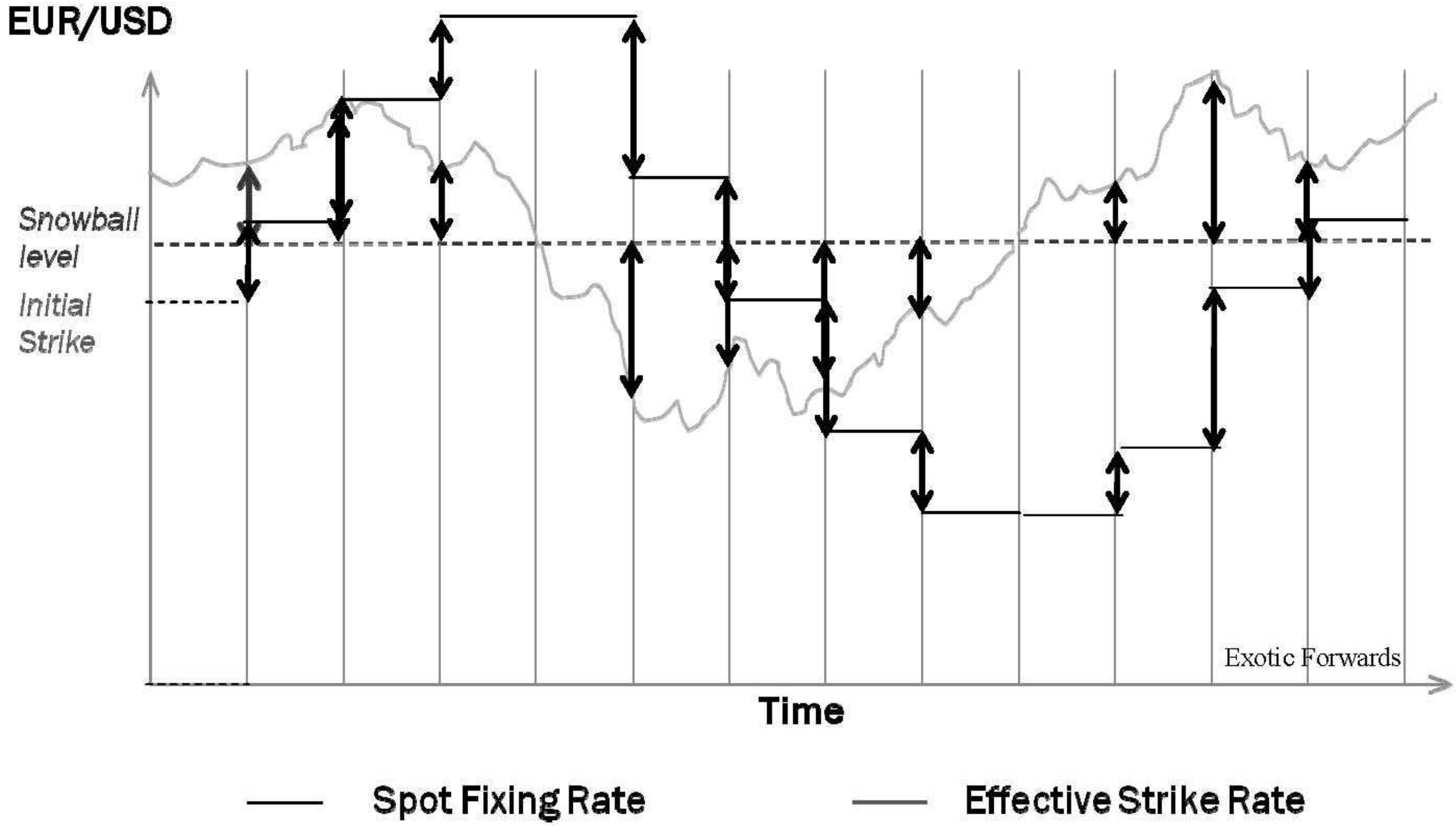
- *Intensifying downside losses*
  - The Notional may not be fixed at one amount. For example, accept doubles Notional in case of unfavorable market movement, compensated by a more attractive initial strike.

Snowball Forward contract (for long position)

Currency	EUR/USD
Expiry/tenor	Monthly for one year
Notional	EUR 1 million vs. 2 million
Spot Fixing Rate of Month j	$f(j)$
Initial Strike Rate	k_0
Snowball level	m, where $k_0 < m < f(0)$
Strike Rate of Month j	$k(j)$

$k(j) = k0$ for the first three months, afterwards it is adjusted by the difference between $f(j)$ and m. The initial strike rates are attractive but the gains may be undermined by subsequent increase on the strike rate based on the snowball formula.

Snowball Strike adjustment mechanism



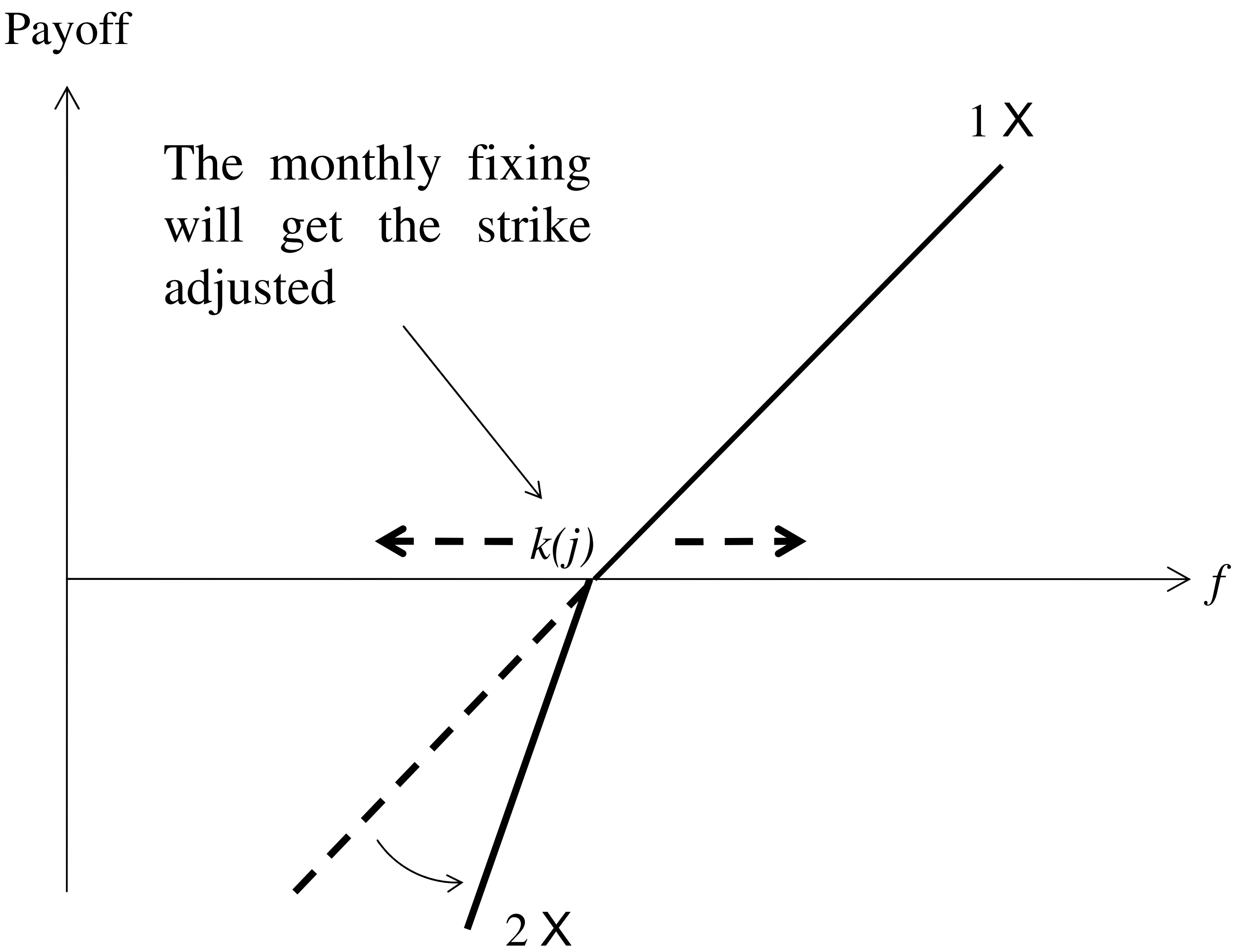
Step-up feature (for long position):

For Month j,

- Notional = EUR 1 million, $f(j) \geq k(j)$ (in-the-money)
- Notional = EUR 2 million, $f(j) < k(j)$ (out-of-the-money)

Again, this intensifies the downside losses.

Payoff for a single settlement (for long position)



Snowball Forward with the Callable Feature

- Callable Feature:

  The issuer has the right to call the structure on any of the early termination dates.

- If the structure is deep in-the-money, it may be called by the issuer.

- The investor should be compensated by a more attractive initial strike.

Snowball Forward with Extendable Feature

- Extendable Feature:

  In this case, the structure is extended on every fixing date, if EUR/USD fixings are at or above the Extend Level.

- If the structure is NOT extended, there will be no further obligation between the Issuer and the Investor.

Summary

- Callable – Option of the Issuer

- Extendable – Pre-specified termination rule

Example

- Extendable Dates:
  - Monthly, starting from Month 3, on every Fixing date
- Extend Level:
  - 1.3000 – for months 3-5
  - 1.2800 – for months 6-8
  - 1.2600 – for months 9-11

Month	Snowball Level	Monthly Fixing Rate	Strike	Extend Level	Extended
1		1.3100	1.2550		
2		1.2954	1.2550		
3		1.3080	1.2550	1.3000	Yes
4	1.2690	1.3100	1.2960	1.3000	Yes
5	1.2690	12.950	1.3220	1.3000	No
6					
7					

The structure is terminated in Month 5 since the Fixing Rate is below the Extend Level.

The adjusted strike after Month 4 $= 1.2550 + (1.3100 - 1.2690) = 1.2960$.

Target Snowball Boosted KO Forward

- Combination of Target Knock Out and a Snowball Forward.
- The KO Event is triggered when the accumulated Monthly ITM (in-the-money) Intrinsic Value is equal to or greater than some preset threshold value.
- Monthly ITM Intrinsic Value:

$$\text{Max}[f(j) - k(j), 0]$$

Month	Snowball Level	Monthly Fixing Rate	Strike	Monthly ITM Intrinsic value	Accumulated Monthly ITM Intrinsic Value
1		1.2700	1.2000	0.07	0.07
2		1.2600	1.2000	0.06	0.13
3		1.2400	1.2000	0.04	0.17
4	1.2650	1.2600	1.1950	0.065	0.235
5	1.2650	1.2500	1.1800	0.07	0.305
6					
7					

Knock-out event occurs at Month 5 when the Accumulated Monthly ITM Intrinsic Value is equal to or greater than 0.25.

EUR/USD ESCALATOR FORWARD

Terms and Conditions:

Fixing Dates	:	6 Monthly Fixing Dates, as per Schedule hereto
Settlement Dates	:	6 Monthly Settlement Dates, as per Schedule hereto
Notional Amounts	:	EUR $20,000,000$ per Settlement
Periodic Exchange	:	With respect to each Settlement Date, and subject to Knockout Event and Puttable Option: Party B sells EUR $20,000,000$ against USD at Strike to Party A
Spot Reference	:	1.2390

Strike	:	Settlement 1: 1.2650 Settlement 2 to Settlement 6: Strike of previous settlement + Max [1.2400 – EURUSD_Fix, 0]
EURUSD_Fix	:	The mid side of the EUR/USD foreign exchange rate, expressed as USD per EUR 1.00, as determined by the Calculation Agent with reference to Reuters page 'TKFE', at 3.00 pm Tokyo time on each Fixing Date. If the rate is not available, it will be determined by the Calculation Agent acting in good faith and in a commercially reasonable manner.
Knockout Level	:	1.3100

Knockout Event	:	Knockout Event is deemed to have occurred when the Strike is greater than knockout Level on any Fixing Date. For the avoidance of doubt, in the event that Strike is greater than Knockout Level on any Fixing Date, the Monthly Exchange corresponding to the relevant Fixing Date and all the Monthly Exchanges with a Fixing Date after a Knockout Events has occurred (the "Terminated Transactions") shall terminate at no cost and be of no further effect and no further amounts shall be payable by either party in respect of the Terminated Transactions.

Puttable Option	:	Party B shall have the right but not the obligation to cancel this trade on each Fixing Date by paying a one time payment of EUR $10,000,000$ ("Puttable Amount") to Party A. For the avoidance of doubt, in the event that Party B exercises the Puttable Option, Party B would pay Party A the amount of EUR $10,000,000$ on the corresponding Settlement Date. However, the Periodic Exchange corresponding to that Fixing Date and all Periodic Exchanges with a Fixing Date after a Puttable Option has been exercised (the "Terminated Transactions") shall terminate and be of no further effect and no further amounts shall be payable by either party in respect of the Terminated Transactions.

Schedule:

Settlement	Fixing Dates	Settlement Dates
1	24-May-10	26-May-10
2	24-Jun-10	28-Jun-10
3	22-Jul-10	26-Jul-10
4	24-Aug-10	26-Aug-10
5	22-Sep-10	27-Sep-10
6	22-Oct-10	26-Oct-10

Scenario analysis

Scenario 1 – spot moves according to where the currency forwards imply

No	Fix	Strike	Notional (EUR)	P/L (USD)	Accumulated P/L (USD)
1	1.2420	1.2650	20,000,000	460,000	460,000
2	1.2423	1.2650	20,000,000	454,000	914,000
3	1.2425	1.2650	20,000,000	450,000	1,364,000
4	1.2428	1.2650	20,000,000	444,000	1,808,000
5	1.2431	1.2650	20,000,000	438,000	2,246,000
6	1.2434	1.2650	20,000,000	432,000	2,678,000

Scenario 2 – spot moves against Party B (seller of EURO), spot moves up by 0.02 *on each Fixing*

No	Fix	Strike	Notional (EUR)	P/L (USD)	Accumulated P/L (USD)
1	1.2420	1.2650	$20,000,000$	$460,000$	$460,000$
2	1.2620	1.2650	$20,000,000$	$60,000$	$520,000$
3	1.2820	1.2650	$20,000,000$	$(340,000)$	$180,000$
4	1.3020	1.2650	$20,000,000$	$(740,000)$	$(560,000)$
5	1.3220	1.2650	$20,000,000$	$(1,140,000)$	$(1,700,000)$
6	1.3420	1.2650	$20,000,000$	$(1,540,000)$	$(3,240,000)$

Scenario 3 – spot moves in favour to Party B, spot moves down by 0.04 *on each Fixing*

No	Fix	Strike	Notional (EUR)	P/L (USD)	Accumulated P/L (USD)
1	1.2420	1.2650	20,000,000	460,000	460,000
2	1.2020	1.3030	20,000,000	2,020,000	2,480,000
3	1.1620	1.3810	Knockout Event Occurs		
4	1.1220	1.4990			
5	1.0820	1.6570			
6	1.0420	1.8550			

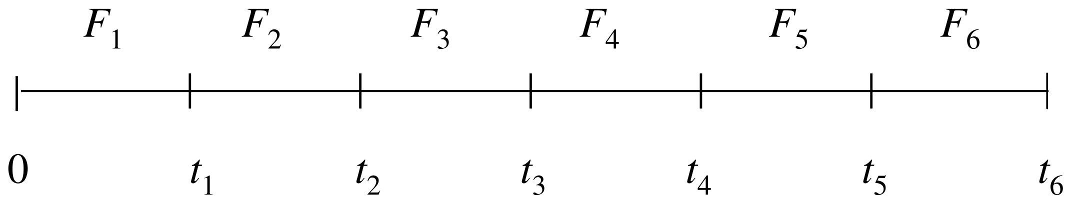
When Fix drops, Strike increases so the gain to Party B is intensified. However, the gains may be terminated due to the occurrence of Knockout Event.

Scenario 4 – spot moves strongly against Party B, spot moves up by 0.10 *on each Fixing, and Party B exercises the Puttable Option on the 4th fixing*

No	Fix	Strike	Notional (EUR)	P/L (USD)	Accumulated P/L (USD)
1	1.2420	1.2650	20,000,000	460,000	460,000
2	1.3420	1.2650	20,000,000	(1,540,000)	(1,080,000)
3	1.4420	1.2650	20,000,000	(3,540,000)	(4,620,000)
4	1.5420	1.2650	Party B exercises the Puttable Option		
5	1.6420	1.2650			
6	1.7420	1.2650			

Currency forward with the flexibilities on notionals

- Consider a 6-month currency forward contract. The exchange rate over each one-month period is preset to assume some constant value.



  The holder can exercise parts of the notional at certain preset fixing dates during the life of the forward, but she has to exercise the whole notional amount by the maturity date of the currency forward.

For example, assume the total notional is $\$100,000$ HKD, and the exchange rate from HKD into Euro is set as $F_1 = 10.1, F_2 = 10.2, F_3 = 10.3, F_4 = 10.4, F_5 = 10.5, F_6 = 10.6$. Suppose the holder exercises $\$10,000$ at the end of the first month, $\$20,000$ at the end of the second month, and nothing in the next 3 months. She is required to exercise $\$70,000$ at the end of the sixth month.

Observations and queries

Since the value of the currency forward together with the optionality feature per unit notional is independent of the notional size, one may argue that the holder should either do nothing or exercise the whole notional amount on one of the fixing dates. The flexibility on notional is sort of fictitious.

- What should be the optimal exercise policies adopted by the holder?
- How does the seller set the predetermined exchange rates so that the value of this flexible notional currency forward is zero at initiation?

Appendix

Various types of interest rates

Reference: Chapter 4 "Interest Rates" in Hull's text

Discrete compounding versus continuous compounding

An amount A is invested for n years at an interest rate of R per annum.

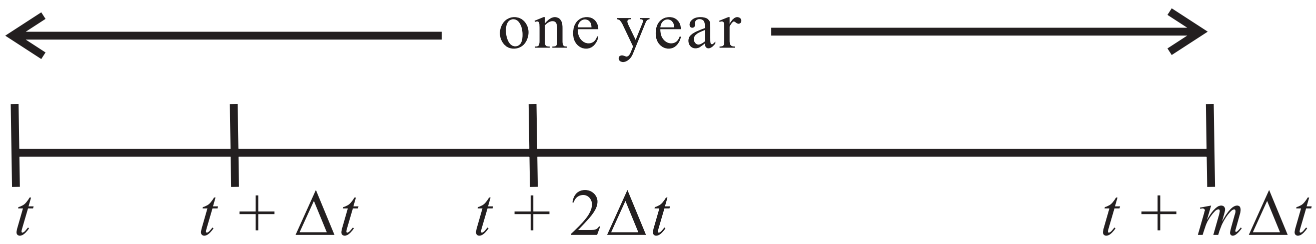
- Compounded once per annum

$$\text{terminal value } = A(1+R)^n.$$

- Compounded m times per annum, e.g. $m = 12$ means compounded monthly

$$\text{terminal value } = A\left(1+\frac{R}{m}\right)^{mn}.$$

Compounded continuously



divided into m compounding intervals, then take $m \to \infty$

$M_t =$ bank account value at time t;

interest amount collected over the small time interval $\Delta t = rM_t\Delta t$.

$M_{t+\Delta t} = M_t + \Delta M_t = (1 + r\Delta t)M_t$

growth factor over $\Delta t = \frac{M_{t+\Delta t}}{M_t} = 1 + r\Delta t$.

Continuous compounding: $dM_t = rM_t\,dt$.

Assuming constant interest rate, we have

$$\int_{M_0}^{M} \frac{dM}{M} = \int_0^t r\,dt$$
$$M_t = M_0 e^{rt}.$$

Numerical example

$A = \$100, n = 1, R = 0.1$

Compound annually: terminal amount $= \$100(1 + 0.1 \times 1) = \$110$.

Compound continuously: terminal amount $= \$100 e^{0.1\times 1} = \$110.52$.

There is slight difference between $1 + r\Delta t$ and $e^{r\Delta t}$.

Mathematical aspect:

$$\begin{aligned} & \text{Growth factor over } \Delta t \text{ under continuous compounding} \\ = \;& e^{r\Delta t} \approx 1 + r\Delta t + \frac{r^2 \Delta t^2}{2} + \frac{r^3 \Delta t^3}{6} + \cdots . \end{aligned}$$

In this numerical example

$$\begin{aligned} e^{0.1\times 1} &\approx 1 + 0.1 \times 1 + \frac{(0.1 \times 1)^2}{2} + \frac{(0.1 \times 1)^3}{6} + \cdots \\ &\approx 1 + 0.1 + 0.005 + 0.000167 + \cdots \approx 1.1052. \end{aligned}$$

Zero rates

- The n-year zero-coupon interest rate is the rate of interest earned on an investment that starts today and lasts for n years. [Also known as the n-year spot rate]

  Example

  Suppose a 5-year zero rate with continuous compounding is quoted as 5% per annum. That is, given $100, if invested for 5 years, it grows to

  $$100 \times e^{0.05 \times 5} = 128.40.$$

- *Discount bond price*

  The zero rate can be interpreted as the spot interest rate earned on a zero-coupon bond.

Example

The price of a 3-month (quarter of a year) unit par zero-coupon bond at the spot rate of 8% per annum is

$$e^{-0.08/4} = 0.9802.$$

We may consider 0.9802 as the 3-month discount factor at the spot rate of 8% per annum.

	Money market account	*discount bond*
time 0	\$1	\$0.9802
3-month later	\$1.0202	\$1

$$\text{Growth factor} = \frac{1}{\text{discount factor}} = \frac{1}{0.9802} = 1.0202.$$

Pricing of coupon-bearing bonds

Treasury zero rates

Maturity (years)	Zero rate (%) (continuously compounded)
0.5	5.0
1.0	5.8
1.5	6.4
2.0	6.8

2-year Treasury bond with a principal of \$100 provides coupons at the rate of 6% per annum semiannually.

$$\begin{aligned}\text{Fair bond price} &= 3e^{-0.05\times 0.5} + 3e^{-0.058\times 1.0} \\ &\quad + 3e^{-0.064\times 1.5} + 103e^{-0.068\times 2.0} \\ &= 98.39.\end{aligned}$$

Bond yield (inverse problem)

A bond's yield is the discount rate that, when applied to all cash flows, gives a bond price equal to its market price.

Given the bond price = 98.39 of the two-year coupon bond, find the yield y of the bond

$$3e^{-y\times 0.5} + 3e^{-y\times 1.0} + 3e^{-y\times 1.5} + 103e^{-y\times 2} = 98.39$$

giving $y = 6.76\%$.

All transactions involve some combination of the following types of risk:

Market Risk is the risk that the value of a Transaction will be adversely affected by fluctuations in the level or volatility of or correlation or relationship between one or more market prices, rates or indices or other market factors or by illiquidity in the market for the Transaction or in a related market.

- Leveraged Transactions will entail a higher degree of risk as the losses arising from a small market movement will be multiplied and you may be required to provide substantial margin at short notice to meet your obligations. Failure to meet such obligations may result in us having to liquidate your position at a loss for which you would be liable.

- While we will seek to observe "stop loss" and "stop limit" orders, market conditions may prevent us from executing any "stop loss" or "stop limit" orders which may have been previously agreed.

Credit Risk is the risk that we may, under certain circumstances, fail to perform our obligations to you when due.

Funding Risk is the risk that, as a result of mismatches or delays in the timing of cash flows due from or to you under Transactions or related hedging, trading, collateral or other transactions, you will not have adequate cash available to fund current obligations.

Liquidity Risk is the risk that due to prevailing market conditions it may not be possible to liquidate, nor to assess a fair value of your position. In addition, you should be aware that the operation of exchange rules or any power or system failure affecting electronic trading facilities may, in certain circumstance, impair or prevent us from liquidating or executing your Transactions, thus increasing the likelihood of loss.

Operational Risk is the risk of loss to you arising from inadequacies in, or failure of, any internal procedures and controls for monitoring and quantifying the risks and contractual obligations associated with Transactions.

Options Risk Option Transactions may carry a high degree of risk. The purchaser of an option may offset or exercise the option or allow the option to expire; if the purchased option expires worthless the purchaser will suffer a total loss of his investment which will consist of the option premium plus transaction costs. Selling ("writing" or "granting") an option generally entails considerably greater risk than purchasing an option; although the premium received by the seller is fixed, the seller may sustain a loss well in excess of that amount if the seller is not "covered" or hedged.