FINA690K — Structured Products and Exotic Options

Solution to Homework Two

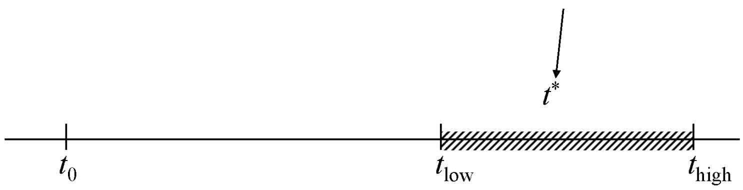
1. We recognize that there will be two coupons before delivery: one in 6 months and one just prior to delivery. Hence using the present value form and a 6-month compounding convention, we have immediately

$$\$9,260 = \frac{F + \$400}{(1.045)^2} + \frac{\$400}{1.045}.$$

This can be solved to give

$$F = \$9,260(1.045)^2 - \$400 - \$400(1.045) = \$9,294.15.$$

2. The currency forward has the embedded early exercise right where the buyer can exercise the purchase of the underlying currency at any time t^* during a specified time interval $[t_{low}, t_{high}]$.



Since the writer can hedge the exposure by long holding one unit of the underlying currency, the value of this American currency forward at initiation time t_0 is zero if the forward price F is set according to the time of purchase t^* as

$$F = Se^{(r_d - r_f)(t^* - t_0)}$$

where S = domestic currency price of one unit of foreign currency, r_d, r_f = domestic and foreign interest rate, respectively.

Now, suppose the forward price is set at fixed value: either $Se^{(r_d-r_f)(t_{low}-t_0)}$ or $Se^{(r_d-r_f)(t_{high}-t_0)}$. Assume that the company chooses the delivery date least favorable to the bank. If the foreign interest rate is higher than the domestic interest rate, then

(1) The earliest delivery date will be assumed when the company has a long position.

(2) The latest delivery date will be assumed when the company has a short position.

If the foreign interest rate is lower than the domestic interest rate then

(1) The latest delivery date will be assumed when the company has a long position.

(2) The earliest delivery date will be assumed when the company has a short position.

3. The two-year swap rate implies that a two-year LIBOR bond with a coupon of 11% sells for par. If R_2 is the two-year zero rate

$$11e^{-0.10\times 1.0} + 111e^{-R_2 \times 2.0} = 100$$

so that $R_2 = 0.1046$. The three-year swap rate implies that a three-year LIBOR bond with a coupon of 12% sells for par. If R_3 is the three-year zero rate

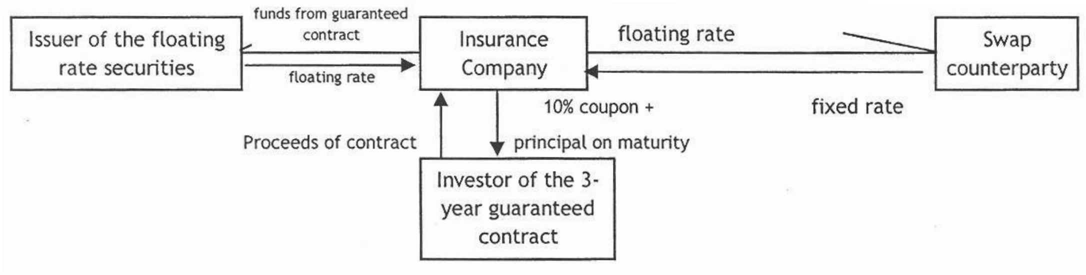
$$12e^{-0.10\times 1.0} + 12e^{-0.1046\times 2.0} + 112e^{-R_3\times 3.0} = 100$$

so that $R_3 = 0.1146$. The two and three year rates are therefore 10.46% and 11.46% with continuous compounding.

4. This 3-year guarantee contract can be replicated by a 3-year fixed rate coupon rate of 10%. By issuing this contract, the life insurance company is thus exposed to the risk of decreasing interest rates lower than 10% within the contract period.

   To hedge its risk on this contract, the insurance company can invest the funds in a floating rate security at t_0 with maturity of 3 years paying annual floating rate coupon. On maturity, the principal/par can be used to settle principal of the guaranteed contract.

   At the same time, it also enters into a 3-year interest rate swap in which it pays a floating rate (at the same or lower rate of floating security, as financed from the floating rate security it invests) and receives a fixed rate of 10% or more (as to finance its interest payment on 3-year guaranteed investment contract) for financing and hedging purposes.



6. The CDS buyer would pay $0.5 \times 0.0060 \times 300,000,000 = \$900,000$ at times $0.5, 1.0, 1.5, 2.0, 2.5, 3.0, 3.5, 4.0$ years. An accrual payment of \$300,000 would be required at the time of the default. The payout to the CDS buyer at the time of the default is $0.6 \times 300,000,000 = \$180,000,000$.

7. In this case, $A(t_i) = 0.04$ and $e(t_i) = 0$ for all i. Also, $v(t_1) = 0.9418, v(t_2) = 0.8869, v(t_3) = 0.8353$, and $v(t_4) = 0.7866$ while $u(t_1) = 0.9561, u(t_2) = 1.8565, u(t_3) = 2.7045$, and $u(t_4) = 3.5031$. Also, $p_1 = 0.01, p_2 = 0.015, p_3 = 0.02, p_4 = 0.025, \pi = 0.93$, and $1 - \widehat{R} - A(t_i)\widehat{R} = 0.792$

$$s = \frac{0.792(0.01 \times 0.9418 + 0.015 \times 0.8869 + 0.02 \times 0.8353 + 0.025 \times 0.7866)}{0.01 \times 0.9561 + 0.015 \times 1.8565 + 0.02 \times 2.7045 + 0.025 \times 3.5031 + 0.93 \times 3.5031}$$

   or 0.0136. The credit default swap spread is 136 basis points. This means that $0.5 \times 1.36 = 0.68\%$ of the notional principal would be paid every six months. If the swap were a binary swap we would not multiply by 0.792 in the numerator and the swap spread would be 172 basis points.

8. M = Principal = \$100

   n = mortgage term in months $= 15 \times 12 = 180$

   i = monthly interest rate = 10%/12

   P = The monthly mortgage payment

$$P = \frac{Mi(1+i)^n}{(1+i)^n - 1} = \frac{100\left(\frac{0.1}{12}\right)\left(1+\frac{0.1}{12}\right)^{180}}{\left(1+\frac{0.1}{12}\right)^{180} - 1} = 1.0746$$

   After 1 year, the outstanding balance is equal to the present value of the monthly payment for a term of 168 months

$OB_{12} = P\left[\frac{1-(1+i)^{-(n-12)}}{i}\right] = (1.0746)\left[\frac{1-\left(1+\frac{0.1}{12}\right)^{-168}}{\frac{0.1}{12}}\right]$

$OB_{12} = 1.0746 \times 90.236 = 96.968$

The present value of the monthly payment discounted at the current rate of 8% is

$MV_{12} = P\left[\frac{1-(1+j)^{-(n-12)}}{j}\right] = (1.0746)\left[\frac{1-\left(1+\frac{0.08}{12}\right)^{-168}}{\frac{0.08}{12}}\right] = 108.40.$

If the mortgage is refinanced,

- the present value of the savings in interest $= 108.402 - 96.968 = 11.433$
- the refinancing cost $= 2\%(96.968) = 1.9393$ so the immediate gain from refinancing is $11.433 - 1.9393 = 9.4940$

If the mortgage is refinanced after 5 years instead,

$OB_{60} = P\left[\frac{1-(1+i)^{-(n-60)}}{i}\right] = (1.0746)\left[\frac{1-\left(1+\frac{0.1}{12}\right)^{-120}}{\frac{0.1}{12}}\right]$

$OB_{60} = 1.0746 \times 75.671 = 81.371.$

The present value of the monthly payment discounted at the current rate of 8% is

$MV_{60} = P\left[\frac{1-(1+j)^{-(n-60)}}{j}\right] = (1.0746)\left[\frac{1-\left(1+\frac{0.08}{12}\right)^{-120}}{\frac{0.08}{12}}\right] = 88.571.$

If the mortgage is refinanced,

- the present value of the savings in interest $= 88.570 - 81.316 = 7.254$.
- the refinancing cost $= 2\%(81.3166) = 1.626$, so the immediate gain from refinancing is $7.254 - 1.626 = 5.628$.

9. M = Original principal = \$100

   n = Original mortgage term in months $= 30 \times 12 = 360$

   P = monthly payment of the original mortgage

   i = monthly interest rate of the original mortgage

   OB_{240} = Outstanding balance after 240 months = \$50.23.

   By using the usual formulas, we calculate P and OB_{240} as follows,

$$P = \frac{Mi(1+i)^n}{(1+i)^n - 1} = \frac{100i(1+i)^{360}}{(1+i)^{360}-1}$$

   $OB_{240} = P\left[\frac{1-(1+i)^{-(n-240)}}{i}\right] = P\left[\frac{1-(1+i)^{-120}}{i}\right]$

   So, $OB_{240} = \left[\frac{100i(1+i)^{360}}{(1+i)^{360}-1}\right]\left[\frac{1-(1+i)^{-120}}{i}\right]$

   $\left[\frac{100i(1+i)^{360}}{(1+i)^{360}-1}\right]\left[\frac{1-(1+i)^{-120}}{i}\right] = 50.23.$

   By solving this equation by using Excel goal seek function, we have $i = 4.888\%$.

   As the current interest rate of 6% is higher than the original interest rate of 4.888%, the mortgage should not be refinanced. John would have to pay more interest in addition to the refinancing cost if he refinanced the existing loan.

10. *Gain to the bank*

- To off-load the bank's investment exposure to the mortgage assets; to transform illiquid assets into marketable securities – better use of balance sheet
- May profit from the spread between the mortgage pool and the load to MBS investors, benefit from the fees charged from servicing the loans.

Gains to investors in mortgage backed securities

- New class of investment assets available that fit investors' attitudes for risk and returns
- Take on prepayment risk and default risk for a higher yield

Net gain to the financial system

- Improve liquidity and efficiency of mortgage loans market; helps revitalize the mortgage market by giving a "market price" to mortgage loans
- Bank can re-capitalize and investors can bet their desired form of income flows at a above-market rate