1. Suppose \( u(w) = \ln(w) \). Show that the inverse function \( I(i) = i^{-1} \), the Lagrange multiplier \( \lambda = v^{-1} \), the optimal attainable wealth is \( W = vL^{-1}B_1 \), and the optimal objective value is \( \ln(v) - E[\ln(L/B_1)] \). Compute these expressions and solve for the optimal trading strategy in the case where \( N = 1, K = 2, r = 1/9, S_0 = 5, S_1(\omega_1) = 20/3, S_1(\omega_2) = 40/9 \), and \( P(\omega_1) = 3/5 \).

2. Suppose \( u(w) = \gamma^{-1}w^\gamma \), where \( -\infty < \gamma < 1 \) and \( \gamma \neq 0 \). Show that the inverse function \( I(i) = i^{-1/(1-\gamma)} \), the Lagrange multiplier
\[
\lambda = v^{-(1-\gamma)}\{E[(L/B_1)^{-\gamma/(1-\gamma)}]\}^{(1-\gamma)}
\]
the optimal attainable wealth
\[
W = \frac{v(L/B_1)^{-1/(1-\gamma)}}{E[(L/B_1)^{-\gamma/(1-\gamma)}]}
\]
and the optimal objective value \( E[u(W)] = \lambda u' \). Compute these expressions and solve for the optimal trading strategy in the case where the underlying model is as in Problem 2.

3. Derive formulas for \( \lambda, C_0 \), and \( C_1 \) for the consumption investment problem in the case where the utility function is:
   (a) \( u(c) = -\exp(-c) \).
   (b) \( u(c) = \gamma^{-1}c^\gamma \), where \( -\infty < \gamma < 1 \) and \( \gamma \neq 0 \).

4. Suppose we allow the customer to have income or endowment \( \bar{E} \) at time \( t = 1 \), where \( \bar{E} \) is a specified random variable. Consider the optimization problem:
\[
\begin{align*}
\text{maximize} \quad & u(C_0) + E[u(C_1)] \\
\text{subject to} \quad & C_0 + H_0B_0 + \sum_{n=1}^{N} H_nS_n(0) = v \\
& C_1 - H_0B_1 - \sum_{n=1}^{N} H_nS_n(1) = \bar{E} \\
& C_0 \geq 0 \quad C_1 \geq 0 \quad H \in \mathbb{R}^{N+1}
\end{align*}
\]
The pair \( (v, \bar{E}) \) is sometimes called the endowment process for the consumer. Show that the consumption-investment plan \( (C, H) \) is admissible if and only if
\[
C_0 + E_Q[C_1 - \bar{E}/B_1] = v
\]
for every risk neutral probability measure \( Q \).
5. Assume a one-period model. The aggregate consumption at time 0 is 8 units. There are three states at time 1, \( \{\omega_1, \omega_2, \omega_3\} \). All agents have homogeneous beliefs, and the probability of each state is 1/3. (This is the \( P \) measure.) The aggregate consumption in these states is 
\[
C(\omega_1) = 64, \quad C(\omega_2) = 27, \quad C(\omega_3) = 125.
\]
The representative agent’s utility function is of the form 
\[
v(c_0, C_1) = c_0^{1/3} + C_1^{1/3}.
\]
Suppose the three Arrow-Debreu securities are traded in this model. Compute the prices of these three securities. A traded asset exists that pays 1% of aggregate consumption at time 1 in each state. Find the price of this asset at time 0.

6. There are \( K = 3 \) states and \( N = 3 \) securities with the payouts
\[
\begin{align*}
d_1(\omega_1) &= 24, \quad d_2(\omega_1) = 44, \quad d_3(\omega_1) = 12 \\
d_1(\omega_2) &= 20, \quad d_2(\omega_2) = 44, \quad d_3(\omega_2) = 12
\end{align*}
\]
The prices of these securities are 
\[
p_1 = 35, \quad p_2 = 40, \quad \text{and} \quad p_3 = 12.
\]
(a) Find the set of all the attainable consumption processes.
(b) Is the consumption process 
\[
c(0) = 10, \quad c(T, \omega_1) = 6, \quad c(T, \omega_2) = 5, \quad c(T, \omega_3) = 12
\]
attainable? Find the initial endowment and the trading strategy that attain it.
(c) Is the consumption process 
\[
c(0) = 0, \quad c(T, \omega_1) = 9, \quad c(T, \omega_2) = 1, \quad c(T, \omega_3) = 17
\]
attainable? Find the initial endowment and the trading strategy that attain it.
(d) Does the given price system permit arbitrage strategies?