1. Consider an asset with price $\tilde{S}_T$ at time $T$. Show that an investor who puts down a cash amount $G_{t,T}$ that equals the time-$t$ futures price of the asset with maturity date $T$ and together performs certain dynamic strategy of long holding futures on successive dates is equivalent to paying the time-$t$ spot price of a security that has time-$T$ payoff

$$\frac{\tilde{S}_T}{B_{t,t+1}B_{t+1,t+2} \cdots B_{T-1,T}}.$$ 

Here, $B_{t,t+1}$ denotes time-$t$ spot price of the discount bond maturing on the next date. You are required to describe the corresponding dynamic strategy in this question. Note that quantities with “tilde” at top indicate stochastic variables. [6]

2. Explain why a swaption can be regarded as an option to exchange a fixed rate bond for the notional principal of the swap. What is the type of option (call or put) when the swaption buyer is the fixed rate payer in the underlying swap upon exercising the swaption? Under what condition that the buyer should choose to exercise the swaption at maturity. [4]

3. Consider the following asset swap

The pricing issue is the determination of the asset swap spread $s^A(0)$. The whole asset swap package is sold at par at initiation. Let $A(0)$ denote the unit annuity stream that pays $1$ per annum with the same tenor as the interest rate swap. Let $\bar{C}(0)$ and $C(0)$ denote the time-$0$ price of the risky corporate bond and the riskfree counterpart, respectively. Show that

$$s^A(0) = \frac{C(0) - \bar{C}(0)}{A(0)}.$$ [5]

**Hint:** A riskfree par floater is a riskfree bond that pays the floating rate LIBOR periodically. Its value at initiation is simply equal to par.
4. Consider a single-dividend paying American call option, whose price function is denoted by $C(S, \tau; X, D)$. Here, $S$ is the underlying spot price, $X$ is the strike of the American call and $D$ is the finite dividend amount, and $\tau$ is the time to expiry of the option.

(a) Show that a necessary condition for the optimal early exercise of the American call option is given by

$$D > X[1 - B(\tau)],$$

where $B(\tau)$ is the price of a unit par discount bond with time to expiry $\tau$.

(b) Discuss in details the necessary and sufficient condition for the optimal early exercise of the American call option.

*Hint*: Consider the condition on the asset price at time immediately prior to the ex-dividend date.

5. (a) Given the following discounted payoff matrix

$$S^*(1; \Omega) = \begin{pmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 1 & 3 & 4 \end{pmatrix}$$

and the current discounted price vector $S^*(0) = (1, 2, 4)$.

Show that the Law of One Price does not hold by presenting a counter example. How do you modify the initial price of the second risky asset [last entry in $S^*(0)$] such that the Law of One Price holds under the modified securities model?

(b) Construct a securities model with two risky securities and riskfree security under 3 possible states such that it admits arbitrage opportunities but dominant strategies do not exist.

(c) Consider the following securities model

$$S^*(1; \Omega) = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 2 & 4 \\ 1 & 3 & 5 \end{pmatrix}, \quad S^*(0) = (1, 2, 4).$$

Does risk neutral measure exist? If not, explain why. If yes, find the set of risk neutral measures.

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