Total return swap

- Exchange the total economic performance of a specific asset for another cash flow.

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<table>
<thead>
<tr>
<th>Total return payer</th>
<th>total return of asset</th>
<th>Total return receiver</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>LIBOR + 5^{TRS}</td>
<td></td>
</tr>
</tbody>
</table>
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Total return comprises the sum of interests, fees and any change-in-value payments with respect to the reference asset.

A commercial bank can hedge all credit risk on a bond/loan it has originated. The counterparty can gain access to the bond/loan on an off-balance sheet basis, without bearing the cost of originating, buying and administering the loan. The TRS terminates upon the default of the underlying asset.
The payments received by the total return receiver are:

1. The coupon $\bar{c}$ of the bond (if there were one since the last payment date $T_{i-1}$).

2. The price appreciation $(\bar{C}(T_i) - \bar{C}(T_{i-1}))^+$ of the underlying bond $C$ since the last payment (if there were any).

3. The recovery value of the bond (if there were default).
The payments made by the total return receiver are:

1. A regular fee of LIBOR $+s^{TRS}$.

2. The price depreciation $(\overline{C}(T_{i-1}) - \overline{C}(T_i))^+$ of bond $C$ since the last payment (if there were any).

3. The par value of the bond $C$ (if there were a default in the meantime).

The coupon payments are netted and swap’s termination date is earlier than bond’s maturity.
Some essential features

1. The receiver is synthetically long the reference asset without having to fund the investment up front. He has almost the same payoff stream as if he had invested in risky bond directly and funded this investment at \( \text{LIBOR} + s^{TRS} \).

2. The TRS is marked to market at regular intervals, similar to a futures contract on the risky bond. The reference asset should be liquidly traded to ensure objective market prices for marking to market (determined using a dealer poll mechanism).

3. The TRS allows the receiver to leverage his position much higher than he would otherwise be able to (may require collateral). The TRS spread should not only be driven by the default risk of the underlying asset but also by the credit quality of the receiver.
Used as a financing tool

- The receiver wants financing to invest $100 million in the reference bond. It approaches the payer (a financial institution) and agrees to the swap.

- The payer invests $100 million in the bond. The payer retains ownership of the bond for the life of the swap and has much less exposure to the risk of the receiver defaulting (as compared to the actual loan of $100 million).

- The receiver is in the same position as it would have been if it had borrowed money at \( \text{LIBOR} + s^{TRS} \) to buy the bond. He bears the market risk and default risk of the underlying bond.
Motivation of the receiver

1. Investors can create new assets with a specific maturity not currently available in the market.

2. Investors gain efficient off-balance sheet exposure to a desired asset class to which they otherwise would not have access.

3. Investors may achieve a higher leverage on capital – ideal for hedge funds. Otherwise, direct asset ownership is on on-balance sheet funded investment.

4. Investors can reduce administrative costs via an off-balance sheet purchase.

5. Investors can access entire asset classes by receiving the total return on an index.
Motivation of the payer

The payer creates a hedge for both the price risk and default risk of the reference asset.

- A long-term investor, who feels that a reference asset in the portfolio may widen in spread in the short term but will recover later, may enter into a total return swap that is shorter than the maturity of the asset. She can gain from the price depreciation. This structure is flexible and does not require a sale of the asset (thus accommodates a temporary short-term negative view on an asset).
Differences between entering a total return swap and an outright purchase

(a) An outright purchase of the C-bond at $t = 0$ with a sale at $t = T_N$. $B$ finances this position with debt that is rolled over at Libor, maturing at $T_N$.

(b) A total return receiver in a TRS with the asset holder $A$.

1. $B$ receives the coupon payments of the underlying security at the same time in both positions.

2. The debt service payments in strategy (a) and the LIBOR part of the funding payment in the TRS (strategy (b)) coincide, too.
Payoff streams of a total return swap to the total return receiver $B$ (the payoffs to the total return payer $A$ are the converse of these).

<table>
<thead>
<tr>
<th>Time</th>
<th>Defaultable bond</th>
<th>TRS payments</th>
<th>Funding</th>
<th>Returns</th>
<th>Marking to market</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t = 0$</td>
<td>$-\overline{C}(0)$</td>
<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>$t = T_i$</td>
<td>$\overline{c}$</td>
<td>$-\overline{C}(0)(L_{i-1} + s^{TRS})$</td>
<td>$+\overline{c}$</td>
<td>$+\overline{C}(T_i) - \overline{C}(T_{i-1})$</td>
<td></td>
</tr>
<tr>
<td>$t = T_N$</td>
<td>$\overline{C}(T_N) + \overline{c}$</td>
<td>$-\overline{C}(0)(L_{N-1} + s^{TRS})$</td>
<td>$+\overline{C}(T_N) - \overline{C}(T_{N-1})$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Default</td>
<td>Recovery</td>
<td>$-\overline{C}(0)(L_{i-1} + s^{TRS})$</td>
<td>0</td>
<td>$-(\overline{C}(T_{i-1}) - \text{Recovery})$</td>
<td></td>
</tr>
</tbody>
</table>

The TRS is unwound upon default of the underlying bond. Day count fractions are set to one, $\delta_i = 1$
The source of value difference lies in the marking-to-market of the TRS at the intermediate intervals.

*Final payoff of strategy*

$B$ sells the bond in the market for $\overline{C}(T_N)$, and has to pay back his debt which costs him $\overline{C}(0)$. (The LIBOR coupon payment is already cancelled with the TRS.) This yields:

$$\overline{C}(T_N) - \overline{C}(0),$$

which is the amount that $B$ receives at time $T_N$ from following strategy (a), net of intermediate interest and coupon payments.
We decompose this total price difference between $t = 0$ and $t = T_N$ into the small, incremental differences that occur between the individual times $T_i$:

$$
\overline{C}(T_N) - \overline{C}(0) = \sum_{i=1}^{N} \overline{C}(T_i) - \overline{C}(T_{i-1}).
$$

This representation allows us to distribute the final payoff of the strategy over the intermediate time intervals and to compare them to the payout of the TRS position (b).
• Each time interval \([T_{i-1}, T_i]\) contributes an amount of

\[
\overline{C}(T_i) - \overline{C}(T_{i-1})
\]

to the final payoff, and this amount is directly observable at time \(T_i\).

• This payoff contribution can be converted into a payoff that occurs at time \(T_i\) by discounting it back from \(T_N\) to \(T_i\), reaching

\[
(\overline{C}(T_i) - \overline{C}(T_{i-1}))B(T_i, T_N).
\]
Conversely, if we paid $B$ the amount given in above equation at each $T_i$, and if $B$ reinvested this money at the default-free interest rate until $T_N$, then $B$ would have exactly the same final payoff as in strategy (a).

From the TRS position in strategy (b), $B$ has a slightly different payoff:

$$\overline{C}(T_i) - \overline{C}(T_{i-1})$$

at all times $T_i > T_0$ net of his funding expenses.
• The difference \((b) - (a)\) is:

\[
\left(\bar{C}(T_i) - \bar{C}(T_{i-1})\right)[1 - B(T_i, T_N)] = \Delta \bar{C}(T_i)[1 - B(T_i, T_N)].
\]

The above gives the excess payoff at time \(T_i\) of the TRS position over the outright purchase of the bond.

• This term will be positive if the change in value of the underlying bond \(\Delta \bar{C}(T_i)\) is positive. It will be negative if the change in value of the underlying bond is negative, and zero if \(\Delta \bar{C}(T_i)\) is zero.

• If the underlying asset is a bond, the likely sign of its change in value \(\Delta \bar{C}(T_i)\) can be inferred from the deviation of its initial value \(\bar{C}(0)\) from par. For example, if \(\bar{C}(0)\) is above par, the price changes will have to be negative on average.
• The most extreme example of this kind would be a TRS on a default-free zero-coupon bond with maturity $T_N$.

• If we assume constant interest rates of $R$, this bond will always increase in value because it was issued at such a deep discount.

• A direct investor in the bond will only realise this increase in value at maturity of the bond, while the TRS receiver effectively receives prepayments. He can reinvest these prepayments and earn an additional return.

Bonds that initially trade at a discount to par should command a positive TRS spread $s^{TRS}$, while bonds that trade above par should have a negative TRS spread $s^{TRS}$.
• TRS spreads that are observed in the market do not always follow this rule, because investors in TRS are frequently motivated by other concerns that were ignored here (leverage, having an off-balance-sheet exposure, counterparty risk, alternative refinancing possibilities) and are willing to adjust the prices accordingly.

• The TRS rate $s^{TRS}$ does not reflect the default risk of the underlying bond. If the underlying bond is issued at par and the coupon is chosen such that its price before default is always at par (assuming constant interest rates and spreads), then the TRS rate should be zero, irrespective of the default risk that the bond carries.