1. Two simple games are equivalent (or isomorphic) if the players can be labeled in such a way that the winning coalitions are the same in both games. Show that the following three-person games are all equivalent.

   (i) \( N = \{A, B, C\} \). Approval is by majority vote, but \( A \) has a veto.

   (ii) \([3; 2, 1, 1]\).

   (iii) \([5; 3, 2, 2]\).

   (iv) \([17; 16, 1, 1]\).

2. Suppose the U.N. Security Council has eight nonpermanent members and three permanent members with passage of a bill requiring a total of seven votes subject to the veto power of each of the permanent members. Prove that this is a weighted voting system. (Include a discussion showing how you found the appropriate weights and quota.)

3. Which of the following properties about “winning coalition” are considered to be reasonable? Give your justification.

   (a) If \( X \) is a winning coalition and every voter in \( X \) is also in \( Y \), then \( Y \) is also a winning coalition.

   (b) If \( X \) and \( Y \) are winning coalitions, then so is the coalition consisting of voters in both \( X \) and in \( Y \).

   (c) If \( X \) is a winning coalition every voter in \( Y \) is also in \( X \), then \( Y \) is also a winning coalition.

   (d) If \( X \) and \( Y \) are disjoint (that is, have no voters in common), then at least one fails to be a winning coalition.

   (e) If \( X \) and \( Y \) are winning coalitions, then so is the coalition consisting of voters in either \( X \) or in \( Y \).

   (f) If \( X \) is the set of all players, then \( X \) is a winning coalition.

   (g) If \( X \) is the empty set, then \( X \) is a winning coalition.

   (h) If \( X \) is a winning coalition and \( X \) is split into two sets \( Y \) and \( Z \) so that every voter in \( X \) is in exactly one of \( Y \) and \( Z \), then either \( Y \) is a winning coalition or \( Z \) is a winning coalition.
4. Consider the “yes-no” voting system with minority veto where the 7 voters are classified into the majority group of 5 voters and the minority group of 2 voters. A bill is passed by requiring at least 4 votes from all voters and at least 1 vote from the minority voters.

(a) Show that the above “yes-no” system is swap robust but not trade robust.
(b) Show that this “yes-no” system is the intersection of two weighted voting systems, that is, a coalition is winning in the “yes-no” system if and only if it is winning in both weighted voting systems.
(c) Find the corresponding weights assigned to the voters and the quotas in the two weighted voting systems.
(d) Find the Shapley-Shubik index and the Banzhaf index for the voters in the majority group and the minority group.

5. Calculate the Shapley-Shubik and Banzhaf indices for the following weighted voting games:

(a) [4; 3, 1, 1, 1].
(b) [7; 4, 3, 2, 1].
(c) [5; 4, 2, 1, 1, 1].
(d) [9; 5, 4, 3, 2, 1].

6. A seven-person legislature has a three-person committee. Approval must be achieved by a majority of both the committee and the entire legislature. Denote the members by AAAAbbbbieb. Compute power indices. What is the ratio of power between a committee member and a noncommittee member?

7. Calculate the Shapley-Shubik and Banzhaf indices for the large stockholder with 40% of the shares if the remaining shares are split evenly among

(a) five other stockholders;
(b) seven other stockholders.

Why is the large stockholder less powerful in (b) than he is in (a)?

8. Show that in an oceanic majority game where there is just one major player $X$ who holds a fraction $x$ of the total vote, we have

$$\phi_X = \begin{cases} \frac{x}{1-x}, & \text{if } x \leq \frac{1}{2}, \\ 1, & \text{if } x \geq \frac{1}{2}. \end{cases}$$

9. For the general oceanic game with two major players, in which $X$ holds a fraction $x$ of the vote and $Y$ holds a fraction $y$ (assume $x < 1/2, y < 1/2$), calculate $\phi_X(x, y)$ and $\phi_Y(x, y)$. Note that different expressions will be obtained depending on whether $x + y \geq 1/2$ or $x + y \leq 1/2$.

10. Referring to calculation of the power of the major stockholders, calculate the Shapley-Shubik indices for the two oceanic games if 2/3’s vote is necessary to win.