1. Construct an example of one-way quarrel in a weighted voting game such that the hostile
player helps (possibly against her wish) the opponent player being hated by her.

2. The two power indexes $\phi$ and $\beta$ need not agree about the effect of quarreling. Illustrate
this phenomenon by considering a BC quarrel in $[5; 3, 2, 1, 1]$.

3. In $[7; 4, 3, 2, 1]$, consider the effect of one-way quarrels involving $A$ and $C$, or $A$
and $B$. For each case you consider, note carefully who is helped and who is hurt.

4. Consider the following weighted voting game:

$[10; 7, 3, 2, 1, 1, 1, 1, 1, 1]$. Show that there exists a bandwagon effect for any one of the uncommitted voters (each
with “1” vote) to join the major voter with “7” votes using the Shapeley-Shubik index.

5. Analyze the possibility of a bandwagon effect in $[5; 3, 2, 1, 1, 1]$ using the Banzhaf index,
where the first two players are not allowed to appear together in a winning coalition.

6. Consider a voting system consisting of 3 big states and 6 small states, passage of a bill
requires “yes” vote from all the big states and at least 2 “yes” votes from the 6 small
states.

   (a) Find the weighted voting vector of the above game, specifying the quota and the
number of votes held by each of the big states and small states.

   (b) Consider one of the big states, assuming the homogeneity assumption on voting
probabilities among all 9 states, find the probability $\pi_b(p)$ that the vote of this
particular big state makes a difference between approval or rejection of a bill. Here,
$p$ denotes the common homogeneous voting probability.

   (c) Using the above $\pi_b(p)$, or otherwise, compute the Shapley-Shubik index and Banzhaf
index for any one of the big states. Then deduce the values for the above two power
indexes for any one of the small states.

   (d) Suppose the 3 big states vote independently while the set of 6 smaller states vote as
a homogeneous group. Based on the Shapley-Shubik index calculations, determine
how the power is shared among the big states and small states?

7. Consider $[5; 3, 2, 1, 1]$.

$A B C D$

Show that

$$\pi_A(p) = p + (1 - p)p^2 = p + p^2 - p^3$$

$$\pi_B(p) = p(1 - p^2) = p - p^3$$

$$\pi_C(p) = p(1 - p)p = p^2 - p^3.$$
Hence, check that

\[
\beta = \left( \frac{5}{8}, \frac{3}{8}, \frac{1}{8}, \frac{1}{8} \right)
\]

\[
\phi = \left( \frac{7}{12}, \frac{3}{12}, \frac{1}{12}, \frac{1}{12} \right).
\]

8. The United States federal system has 537 voters in the system: 435 House of Representatives, 100 Senate members, the Vice President and the President. The Vice President plays the role of tie breaker in the Senate. The President has veto power that can be overridden by two-thirds vote of both the House and the Senate.

(a) Which of the following pairs are equally desirable (equivalent)? Give the full justification of your answers.
   
   Vice President and a Senator
   President and a House Representative

(b) Which of the following pairs are incomparable? Give the full justification of your answers.
   
   President and a Senator
   Vice President and a House Representative

9. Consider the yes-no voting system in which there are six voters: \( a, b, c, d, e, f \). Suppose the winning coalitions are precisely the ones containing at least two of \( a, b \) and \( c \) and at least two of \( d, e \) and \( f \).

(a) Show that \( a \) and \( b \) are equally desirable.

(b) Show that the desirability of \( a \) and \( d \) is incomparable.

10. Suppose that \( x \) and \( y \) are voters in a yes-no voting system and that \( x \approx y \). Suppose that \( Z' \) is a winning coalition to which both \( x \) and \( y \) belong. Assume that \( x \)'s defection from \( Z' \) is critical. Prove that \( y \)'s defection from \( Z' \) is also critical.

   \textit{Hint:} Assume, for contradiction, that \( y \)'s defection from \( Z' \) is not critical. Consider the coalition \( Z \) arrived at by deleting \( x \) and \( y \) from \( Z' \).

11. Suppose that \( x \) and \( y \) are voters in a yes-no voting system and that \( x \approx y \). Suppose that \( Z' \) is a coalition that contains \( x \) but not \( y \). Let \( Z'' \) be the coalition resulting from replacing \( x \) by \( y \) in \( Z' \).

(a) Prove that if \( Z' \) is winning, then \( Z'' \) is also winning.

(b) Prove that if \( Z' \) is losing, then \( Z'' \) is also losing.

   \textit{Hint:} Let \( Z \) be the result of deleting \( x \) from \( Z' \) and argue by contradiction.