1. In a Nim game that starts with 4 pennies, each player may take 1 or 2 pennies from the pile. Suppose player I moves first. The game ends when there are no pennies left and the player who took the last penny pays 1 to the other player.

   (a) Draw the game as we did in 2 × 2 Nim.
   (b) Write down all the strategies for each player and then the game matrix.
   (c) Find the upper and lower values of the game, $v^+$, $v^-$. Would you rather be player I or player II?

2. Each of two players must choose a number between 1 and 5. If a player’s choice equals opposing player’s choice + 1, she loses $2; if a player’s choice $\geq$ opposing player’s choice + 2, she wins $1. If both players choose the same number, the game is a draw.

   (a) What is the game matrix?
   (b) Find $v^+$ and $v^-$ and determine whether a saddle point exists in pure strategies. If so, find it.

3. In the Russian roulette, suppose that if player I spins and survives and player II decides to pass, then I gets all of the additional money that II had to put into the pot in order to pass (instead of splitting the pot). Draw the game tree and find the game matrix. What are the upper and lower values of the game? Find the saddle point in pure strategies.

4. In a football game, the offense has two strategies: run or pass. The defense also has two strategies: defend against the run, or defend against the pass. A possible game matrix is

   $$ A = \begin{pmatrix} 3 & 6 \\ x & 0 \end{pmatrix}. $$

   This is the game matrix with the offense as the row player I. The numbers represent the number of yards gained on each play. The first row is run, the second is pass. The first column is defend the run and the second column is defend the pass. Assuming that $x > 0$, find the value of $x$ so that this game has a saddle point in pure strategies.

5. Let $f(x, y) = (x - y)^2$, the two intervals for $x$ and $y$ are $C = D = [-1, 1]$, respectively. Find $v^+ = \min_{y \in D} \max_{x \in C} f(x, y)$ and $v^- = \max_{x \in C} \min_{y \in D} f(x, y)$.

6. Nash and iterated dominance
   (a) Show that every iterated dominance equilibrium $s^*$ is a Nash equilibrium.
   (b) Show by counterexample that not every Nash equilibrium can be generated by iterated dominance.
(c) Is every iterated dominance equilibrium made up of strategies that are not weakly dominated?

7. Pareto dominance

(a) If a strategy profile $s^*$ is a dominant-strategy equilibrium, does that mean it weakly Pareto-dominates all other strategy profiles?

(b) If a strategy profile $s$ strongly Pareto-dominates all other strategy profiles, does that mean it is a dominant-strategy equilibrium?

(c) If a strategy profile $s$ weakly Pareto-dominates all other strategy profiles, then must it be a Nash equilibrium?

8. Discoordination

Suppose that a man and a woman each chooses whether to go to a prize fight or a ballet. The man would rather go to the prize fight, and the woman to the ballet. What is more important to them, however, is that the man wants to show up to the same event as the woman, but the woman wants to avoid him.

(a) Construct a game matrix to illustrate this game, choosing numbers to fit the preferences described verbally.

(b) If the woman moves first, what will happen?

(c) Does the game have a first-mover advantage?

(d) Show that there is no Nash equilibrium if the players move simultaneously.

9. Finding Nash equilibriums

Find the Nash equilibriums of the two games illustrated in the following tables. Can any of them be reached by iterated dominance?

(a) 

<table>
<thead>
<tr>
<th></th>
<th>Column</th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Left</td>
<td>Middle</td>
<td>Right</td>
</tr>
<tr>
<td>Up</td>
<td>10,10</td>
<td>0,0</td>
<td>−1,15</td>
</tr>
<tr>
<td>Row</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sideways</td>
<td>−12,1</td>
<td>8,8</td>
<td>−1,−1</td>
</tr>
<tr>
<td>Down</td>
<td>15,1</td>
<td>8,−1</td>
<td>0,0</td>
</tr>
</tbody>
</table>

Payoffs to: (Row, Column).

(b) 

<table>
<thead>
<tr>
<th></th>
<th>Brydox</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Flavor</td>
<td>Texture</td>
<td></td>
</tr>
<tr>
<td>Flavor</td>
<td>−2,0</td>
<td>0,1</td>
<td></td>
</tr>
<tr>
<td>Apex</td>
<td>−1,−1</td>
<td>0,−2</td>
<td></td>
</tr>
</tbody>
</table>

Payoffs to: (Apex, Brydox).
10. **Three-by-Three equilibriums**

   Identify any dominated strategies and any Nash equilibriums in pure strategies in the following game.

<table>
<thead>
<tr>
<th></th>
<th>Left</th>
<th>Middle</th>
<th>Right</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Up</strong></td>
<td>1, 4</td>
<td>5, −1</td>
<td>−0, 1</td>
</tr>
<tr>
<td><strong>Row</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Sideways</strong></td>
<td>−1, 0</td>
<td>−2, −2</td>
<td>−3, 4</td>
</tr>
<tr>
<td><strong>Down</strong></td>
<td>0, 3</td>
<td>9, −1</td>
<td>5, 0</td>
</tr>
</tbody>
</table>

   Payoffs to: (Row, Column).

11. **Selfish and altruistic social behavior**

   Two people enter a bus. Two adjacent cramped seats are free. Each person must decide whether to sit or stand. Sitting alone is more comfortable than sitting next to the other person, which is more comfortable than standing.

   (a) Suppose that each person cares only about her own comfort. Model the situation as a strategic game. Is this game the Prisoner’s Dilemma? Find its Nash equilibrium (equilibriums?).

   (b) Suppose that each person is altruistic, ranking the outcomes according to the other person’s comfort. However, out of politeness, one person prefers to stand than to sit if the other person stands. Model the situation as a strategic game. Is this game the Prisoner’s Dilemma? Find its Nash equilibrium (equilibriums?).

   (c) Compare the people’s comfort in the equilibriums of the two games.

12. **Voter participation**

   Two candidates, A and B, compete in an election. Of the n citizens, k support candidate A and m (= n − k) support candidate B. Each citizen decides whether to vote, at a cost, for the candidate she supports, or to abstain. A citizen who abstains receives the payoff of 2 if the candidate she supports wins, 1 if this candidate ties for first place, and 0 if this candidate loses. A citizen who votes receives the payoffs 2 − c, 1 − c and −c in these three cases, where the cost c satisfies 0 < c < 1.

   (a) For k = m = 1, name the type of the game.

   (b) For k = m, find the set of Nash equilibria. Is the action profile in which everyone votes a Nash equilibrium? Is there any Nash equilibrium in which candidates tie and not everyone votes? Is there any Nash equilibrium in which one of the candidates wins by one vote? Is there any Nash equilibrium in which one of the candidates wins by two or more votes?

   (c) What is the set of Nash equilibriums for k < m?

13. **Hawk - Dove**

   Two animals are fighting over some prey. Each can be passive or aggressive. Each prefers to be aggressive if its opponent is passive, and passive if its opponent is aggressive; given its own stance, it prefers the outcome in which its opponent is passive to that in which its opponent is aggressive. Formulate this situation as a strategic game and find its Nash equilibriums.
14. **Extension of the Stag Hunt model**

Extend the $n$-hunter *Stag Hunt* by giving each hunter $K$ (a positive integer) units of effort, which she can allocate between pursuing the stag and catching hares. Denote the effort hunter $i$ devotes to pursuing the stag by $e_i$, a nonnegative integer that is less than or equal to $K$. The chance that the stag is caught depends on the lowest expended effort among all the hunters, denoted by $\min_j e_j$. (“A chain is as strong as its weakest link.”) Hunter $i$’s payoff to the action profile $(e_1, \ldots, e_n)$ is $2 \min_j e_j - e_i$. The positive term corresponds to some multiple (taken to be 2 here) of the chance of success and she is worse off the more effort she devotes to pursuing the stag (which means she catches fewer hares).

(a) Is the action profile $(e, \ldots, e)$, in which every hunter devotes the same effort to pursuing the stag, a Nash equilibrium for any value of $e$? What is a player’s payoff to this profile? What is her payoff when she deviates to a lower or higher effort level?

(b) Is any action profile in which not all the players’ effort levels are the same a Nash equilibrium? Consider a player whose effort exceeds the minimum effort level of all players. What happens to her payoff if she reduces her effort level to the minimum?

15. Perform an analysis of the theory of moves version of Chicken that is analogous to what was done for Prisoner’s Dilemma.

16. Show that the theory of moves version of the following game is not finite. Assume that $(2, 3)$ is the initial position and Row goes first.

<table>
<thead>
<tr>
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<tbody>
<tr>
<td></td>
<td>C</td>
</tr>
<tr>
<td>Row</td>
<td>C</td>
</tr>
<tr>
<td></td>
<td>N</td>
</tr>
</tbody>
</table>

17. In the second-price sealed-bid auction, if two or more bidders are tied for high bid, a random device decides who will receive the object and the bidder receiving the object pays this tying bid. Show that the truth telling bidding strategy remains to be weakly dominating all other strategies.