MATH4823A – Mathematics and Social Choice Theory

Course outline – Fall 2012

1. **Instructor**
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2. **Teaching assistant**
   Name: Jingjing Wang
   Contact details: Office Room 3474, Tel: 2358-7453; E-mail: maariel

3. **Meeting time and Venue**
   Date / time: Tuesday and Thursday (9:00am – 10:20am)
   Venue: Room 1504

4. Course description
   Credit points: 3
   Pre-requisite: MATH144 or MATH241
   Exclusion: Nil

   In this course, we would like to illustrate the use of mathematical techniques to analyze voting systems and apportionment problems in social choice theory. The topics include: measurement of political power in voting systems, power indexes and analysis of electoral systems, apportionment of legislature seats according to populations or votes, and proportional representation,. The various social choice procedures of dealing with aggregation of individual preferences to a social choice preference will be demonstrated. The highlight is the proof of the renowned Arrow Impossibility Theorem: no voting system can convert the ranked preferences of individuals into a community-wide ranking while also meeting a certain set of reasonable criteria.

5. **Intended learning outcomes**

   Upon the completion of the course, students should be able to:

- Appreciate how to use quantitative tools to analyze issues related to achieving fair representation in voting systems.
- Recognize the importance of applying rigorous and numerate approach to analyze and solve problem in social choice theory.
- Apply mathematical modeling and analytic proofs, as well as statistical analyses, to describe and explain phenomena in voting theory.
- Communicate the solutions of mathematical models of voting theory and apportionment using mathematical terminology and English language.

6. **Assessment scheme**

One 80-minute test	40%
120-minute final examination	60%
4 sets of homework	0%

Proper submission of all homework sets may help improve a minor grade on marginal cases.

Date of mid-term test: 25 October (Thursday) during the lecture hour

7. Student Learning Resources

Textbook: "*Mathematics and Politics*," A.D. Taylor and A.M. Pacelli, second edition (2008) Springer. The text can be downloaded from the HKUST Library http://www.springerlink.com/content/p15061/

8. Teaching and Learning Activities

Scheduled activities: 3 hours of lecture per week

9. Course Schedule

Week 1 – Week 3

1. Voting systems and power indexes
   - 1.1 Weighted voting systems and yes-no systems
   - 1.2 Power indexes: Shapley-Shubik index and Banzhaf index
   - 1.3 Case studies of power indexes calculations
   - 1.4 Probabilistic characterization of power indexes

Week 4 – Week 6

2. Analysis of powers in voting systems
   - 2.1 Potential blocs, quarreling paradoxes and bandwagon effects
   - 2.2 Incomparability and desirability
   - 2.3 Power distribution in weighted voting systems

Week 7 – Week 10

3. Proportional representation and apportionment
   - 3.1 Quota method of the Greatest Remainder (Hamilton's method) and paradoxes
   - 3.2 Geometric characterization and apportionment simplex
   - 3.3 Divisor methods
   - 3.4 Huntington's family: pairwise comparison of inequity
   - 3.5 Analysis of bias
   - 3.6 Notion of marginal inequity measure

Week 11 – Week 13

4. Voting methods with more than two alternatives
   - 4.1 Social welfare procedures
   - 4.2 Desirable properties of voting systems
   - 4.3 Arrow Impossibility Theorem
   - 4.4 Cumulative voting and proportional representation