**MATH4994 – Capstone Projects in Mathematics and Economics**

**Course outline – Fall 2019**

## **Instructor**

## Name: Professor Yue Kuen KWOK

## Contact details: Office Room 3445, Tel: 2358-7418; E-mail: maykwok

*Office hours: 3:00pm – 5:00pm on Wednesday, or by appointment*

1. **Teaching assistant**

*Name: Dr Chi Man LEUNG (E-mail: chimanleung@ust.hk)*

1. **Course description**

*Credit points: 3*

This is a lecture-presentation course that provides students an opportunity to integrate and apply mathematical tools to analyze problems in economics and social science. The topics include (i) fair allocations, (ii) matching schemes, (iii) voting methods and social choice theory, (iv) proportional representation and apportionment schemes.The model formulation and mathematical methods used for analysis of these problems are first presented in the lectures. Students are then required to do oral presentation of selected technical papers on these topics.

1. **Intended learning outcomes**

Upon completion of the course, students should be able to:

* Appreciate how to use quantitative tools to analyze issues related to various mathematical models in economics and social science, including fair allocation, voting methods, proportional representation and traffic flows.
* Recognize the importance of applying rigorous and numerate approach to analyze and solve problem in economics and social science.
* Apply mathematical modeling and analytic proofs to describe and explain phenomena in economics and social science.
* Communicate the solutions of mathematical models of economics and social science using mathematical terminology through oral presentation and written reports.

1. **Assessment scheme**

* 80-minute mid-term test 30%
* 120-minute final examination 45%
* Oral presentation of project 25%

(bonus for harder papers)

1. **Student Learning Resources**

*Textbook: “Mathematics and Politics*,” A.D. Taylor and A.M. Pacelli, second edition (2008) Springer. The text can be downloaded from the HKUST Library

http://www.springerlink.com/content/p15061/

1. **Teaching and Learning Activities**

Scheduled activities: 3 hours of lecture / students’ oral presentation and 1 hour of tutorial per week

1. **Course Content**
2. Fair allocations and matching schemes
   1. Criterion for fair divisions

* Proportionality, envy-freeness, equitability and efficiency

1.2 Procedures for two-player and multi-player cake-cutting

* Discrete cut-and-choose procedures
* Continuous moving-knife procedures

1.3 Adjusted winner for two-party allocation of discrete goods

* Point allocation procedures
* Pareto efficiency

1. Matching schemes
   1. Marriage problems

* Deferred acceptance algorithm
* Stable solution
  1. College admission and school choice problems

- Gale-Shapley student optimal stable mechanism

- Top trading cycles mechanism

- Boston school choice mechanism

2.3 Roommates problems

- Irving algorithm

1. Voting systems
   1. Social choice procedures

* Plurality voting
* Borda count
* Elimination procedure
* Sequential pairwise voting
  1. Paradoxes
* Condorcet paradox
* Chair paradox
  1. Desirable properties of voting methods
* Pareto condition
* Condorcet condition
* Monotonicity criterion
* Independence of irrelevant alternatives
  1. Condorcet voting methods
* Black method
* Nanson method
* Copeland method
  1. Social welfare functions
* May Theorem and quota system
* Weakly reasonable social welfare functions
  1. Arrow’s Impossibility Theorem
* Dictating set
  1. Single-peaked preferences
* Median Voter Theorem
  1. Cumulative voting
* Assuring a certain presentation
  1. Approval voting
* Positive aspects
* Characterization of election outcomes

1. Proportional representation and apportionment
   1. General issues of apportionment of legislature seats

* Gerrymandering
  1. Quota method of the Greatest Remainder (Hamilton’s method) and paradoxes
* Alabama paradox
* New State paradox
* Population monotonicity
  1. Divisor methods

- Choices of divisors

- Huntington approach: Pairwise comparison of inequity

- Rank index

- US history of apportionment

4.4 Analysis of bias

- Probabilistic approach

- Majorization ordering

- Population monotonicity

4.5 Proportionality in matrix apportionment

- Proportionality in districts and parties

- Greatest remainder biproportionality rounding method