MATH6380D - Advanced Topics in Derivative Pricing Models (Spring 2013)

Course objective
This course is directed to those students who would like to pursue more in-depth studies on derivative pricing theory beyond the first course. The mathematical model formulation of various structured products will be presented, like accumulators, dynamic fund protection, turbo warrants, guaranteed minimum withdrawal benefit, variance swaps, timer options and VIX derivatives. Mathematical solution to these pricing models will be presented. Pricing of derivatives under jump-diffusion models and stochastic volatility models will be discussed.

Prerequisite
Students should have taken at least one full course in option pricing theory, like MAFS5030 or MAFS5250. High level of sophistication in mathematics is required.

Instructor
Prof. Kwok Yue-Kuen
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Time and venue
Monday and Wednesday, 11:00am – 12:20pm; Room 2463

Reference text
Some parts of the lecture materials are based on "Mathematical models of financial derivatives," second edition, Chapters 4 and 5, Y.K. Kwok, Springer, 2008. The remaining parts are based on research papers on the relevant topics.

Course topics
1. Financial derivatives with embedded barrier features
   1.1 Product nature of barrier options
      - Accumulators
   1.2 Partial differential equation approach and method of images
      - Single-asset models
      - Extension to multistate models
   1.3 Probabilistic approach
      - Density functions of restricted Brownian motions
      - First passage time density functions
      - Two-sided barriers
   1.4 Approximation of probabilities of hitting time dependent barriers
      - Brownian bridge technique
   1.5 Barrier derivatives under stochastic interest rates
      - Fortet method
   1.6 Occupation time derivatives
      - Proportional step options
      - Transition density functions with killing rate

- Delayed barrier options and simple step options

1.7 Analytic pricing under double exponential jump diffusion models
- Jump diffusion processes
- Joint density functions of first passage time and jump
- Pricing formulas of barrier options

1.8 Discretely monitored barrier options
- Continuity correction formulas
- Fast Gauss transform algorithm
- Merton's jump diffusion model

2. Lookback style derivatives

2.1 Model formulation of lookback options
- Rollover strategy and strike bonus premium

2.2 Pricing formulas of European lookback options
- Floating strike lookback options
- Fixed strike lookback options

2.3 Partial differential equation approach
- Similarity transformation and Neumann boundary condition
- Multistate lookback options

2.4 Discretely monitored barrier options
- Continuity correction formulas
- Fast Gauss transform algorithm

2.5 Dynamic fund protection
- Finite number of resets

2.6 Turbo warrants
- Double exponential models

3. Derivatives with averaging style payoffs

3.1 Pricing models of Asian options
- Partial differential approach for continuous models
- Put-call parity relations and fixed-floating symmetry relations
- Closed form pricing formulas for discretely monitored models

3.2 Analytic approximation methods
- Lower bound: method of conditioning
- Upper bound: method of shifted lognormal approximation

3.3 Guaranteed minimum withdrawal benefit
- Static withdrawal policies
- Stochastic interest rates

4. Variance products and volatility derivatives

4.1 Volatility trading
- Pure exposure to volatility

4.2 Replication of variance swaps
- Log contract
- Portfolio of options

4.3 Discrete variance swaps
- Pricing under stochastic volatility models
- Third generation products

4.4 Volatility derivatives
- Convexity approximation formulas for options on volatility

4.5 Derivatives on VIX
- Tale of two indexes
- Forwards and options on VIX

4.6 Timer options
- Random termination date linked to realized variance

5. American options and optimal stopping models

5.1 Review of pricing properties of American options
- Asymptotic limits: near maturity and far from maturity

5.2 Pricing models of American options
- Variational inequalities and linear complementarity formulations
- Early exercise premium

5.3 Options with voluntary reset options
- Shout floors
- Reset-strike put options
- Multireset options

Assessment scheme

80-minute mid-test (April 8, Monday during lecture hour)	40%
120-minute final examination	60%

Both test and final examination allow open note.